Tidal effects on gravitational waveforms in massless scalar-tensor theories of gravity

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Gravitational Universe: Challenges and Opportunities

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l'Observatoire de Paris — LUTH

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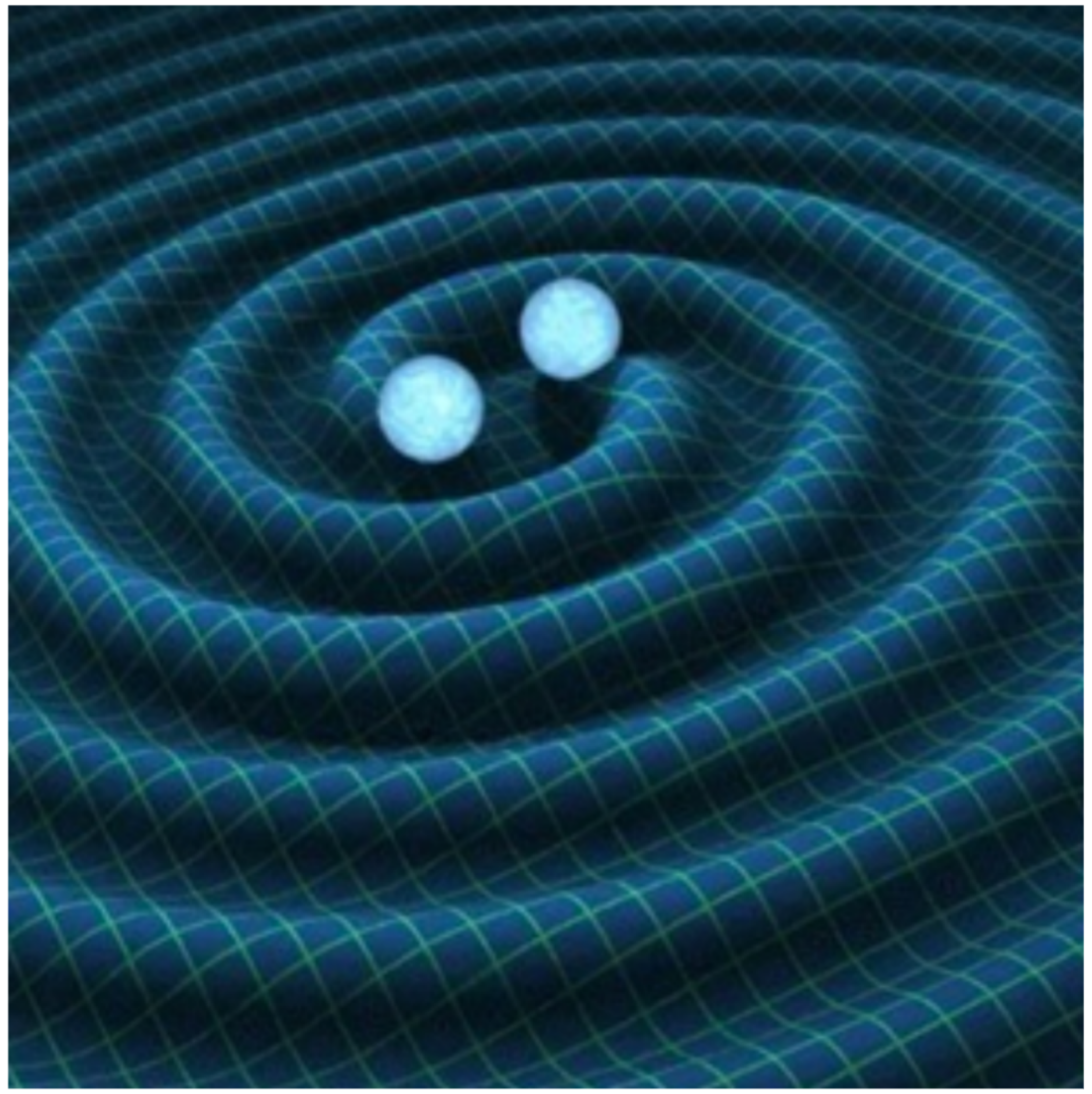
Overview



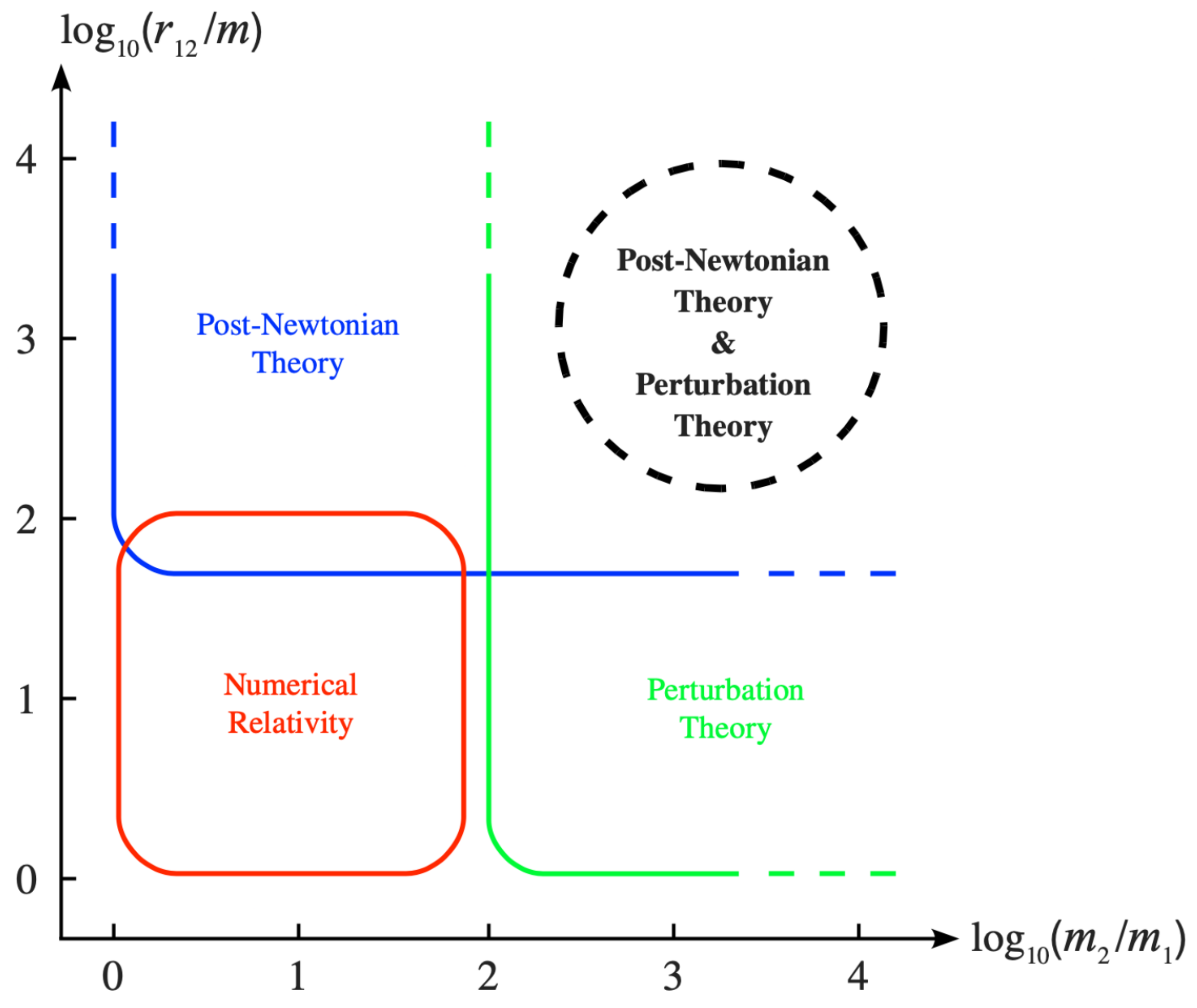
Analytical waveform modeling for inspiralling compact binary systems

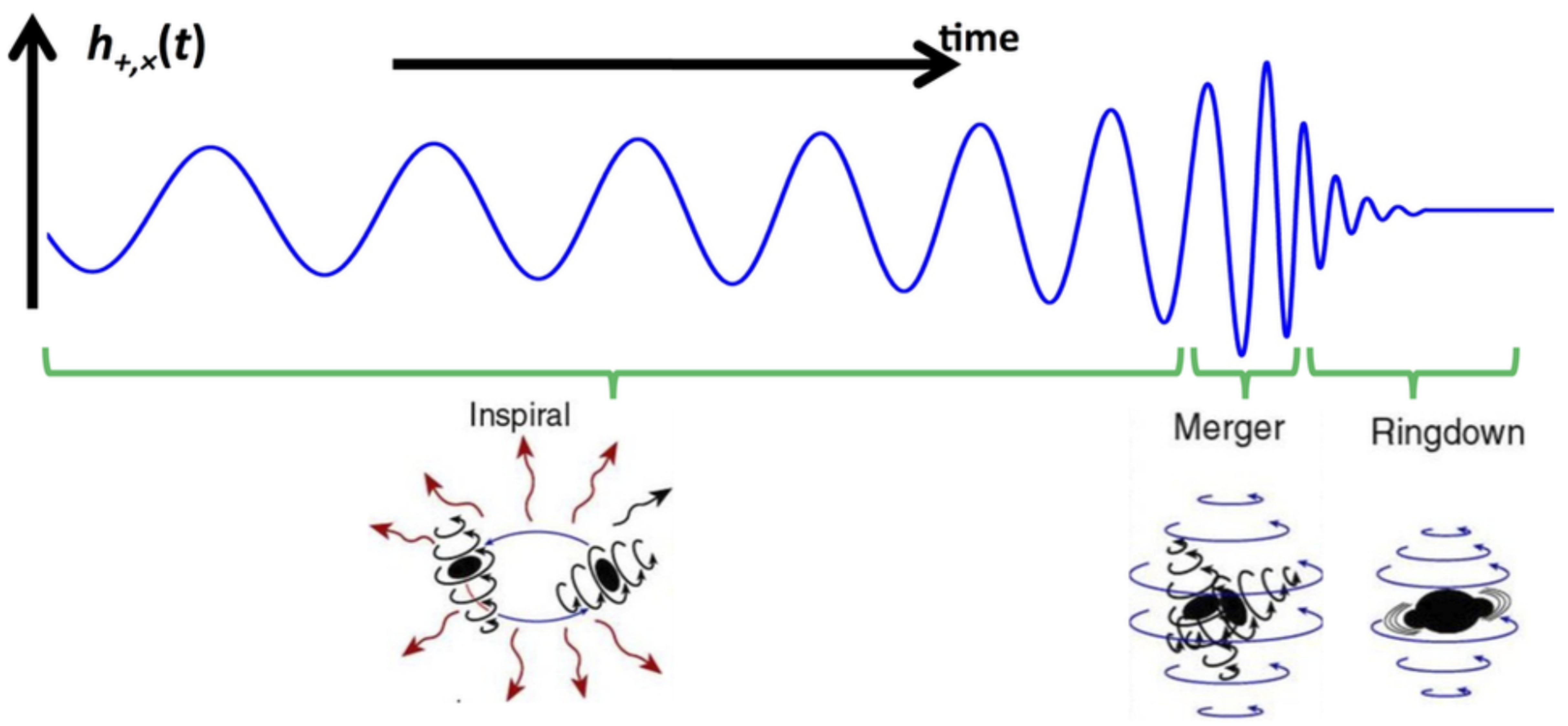
Two-body problem in massless scalar-tensor theories of gravity

Impact of modeling tidal effects

Detectability of tidal effects

Approaches to computing the waveform



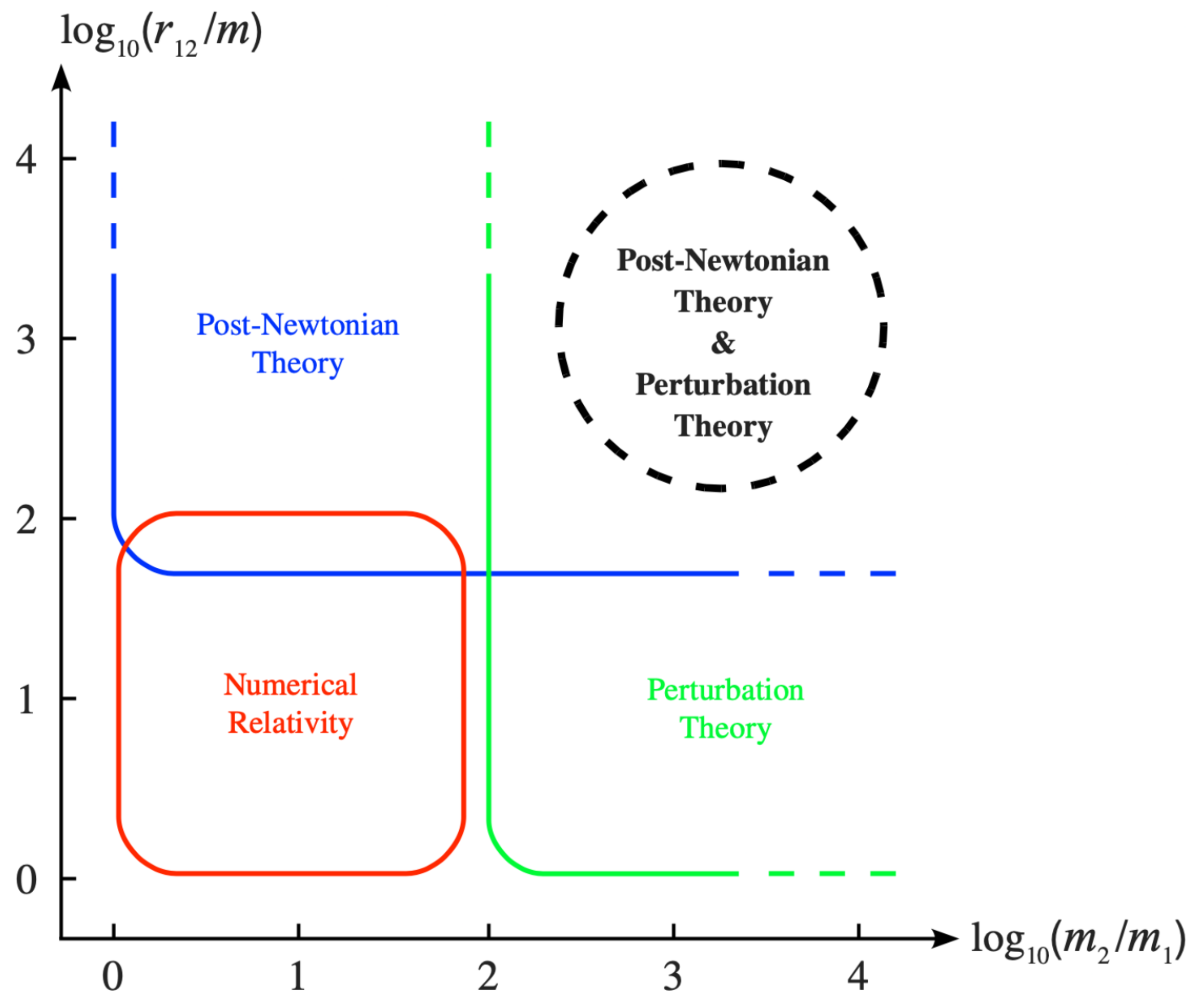


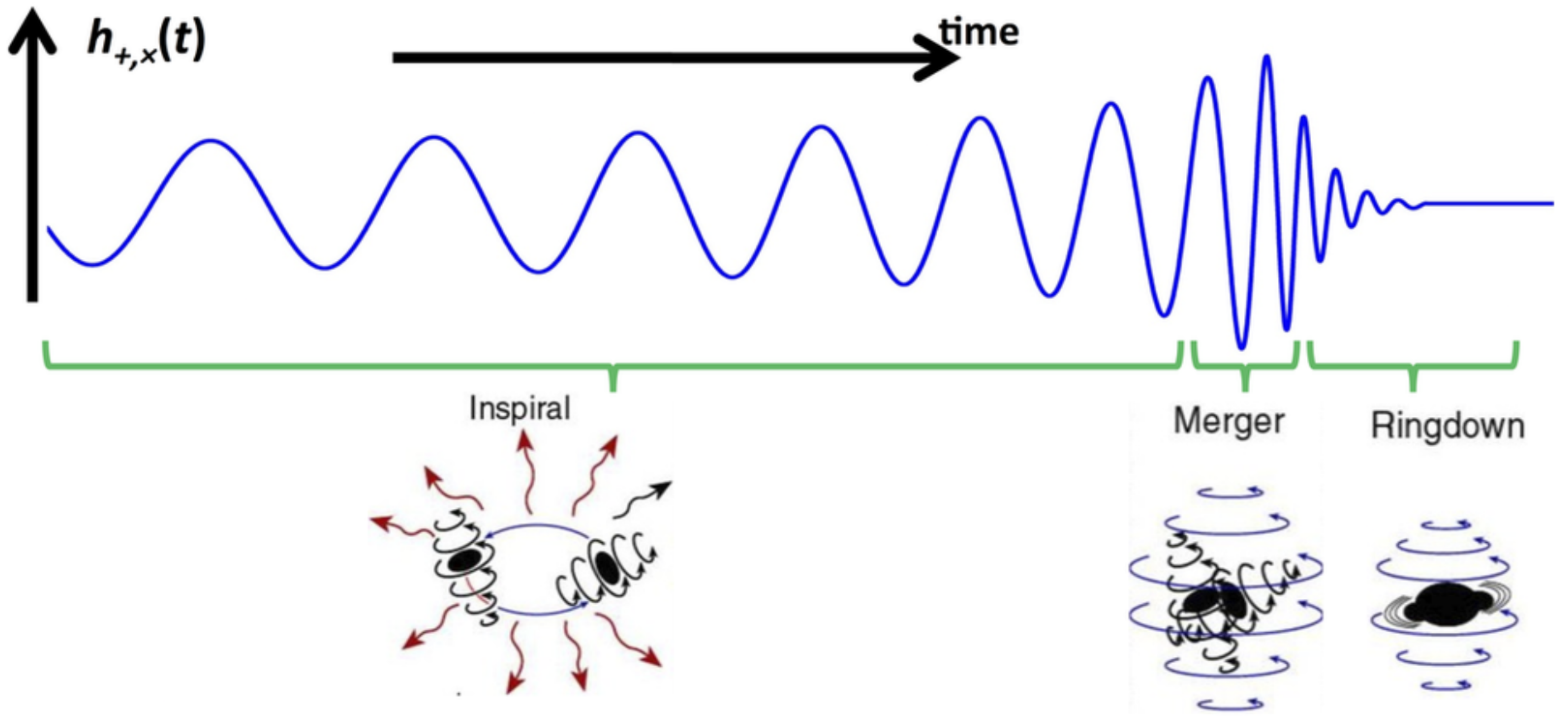
Post-Newtonian theory

Numerical relativity

Perturbation theory

Approaches to computing the waveform





Post-Newtonian theory

Numerical relativity

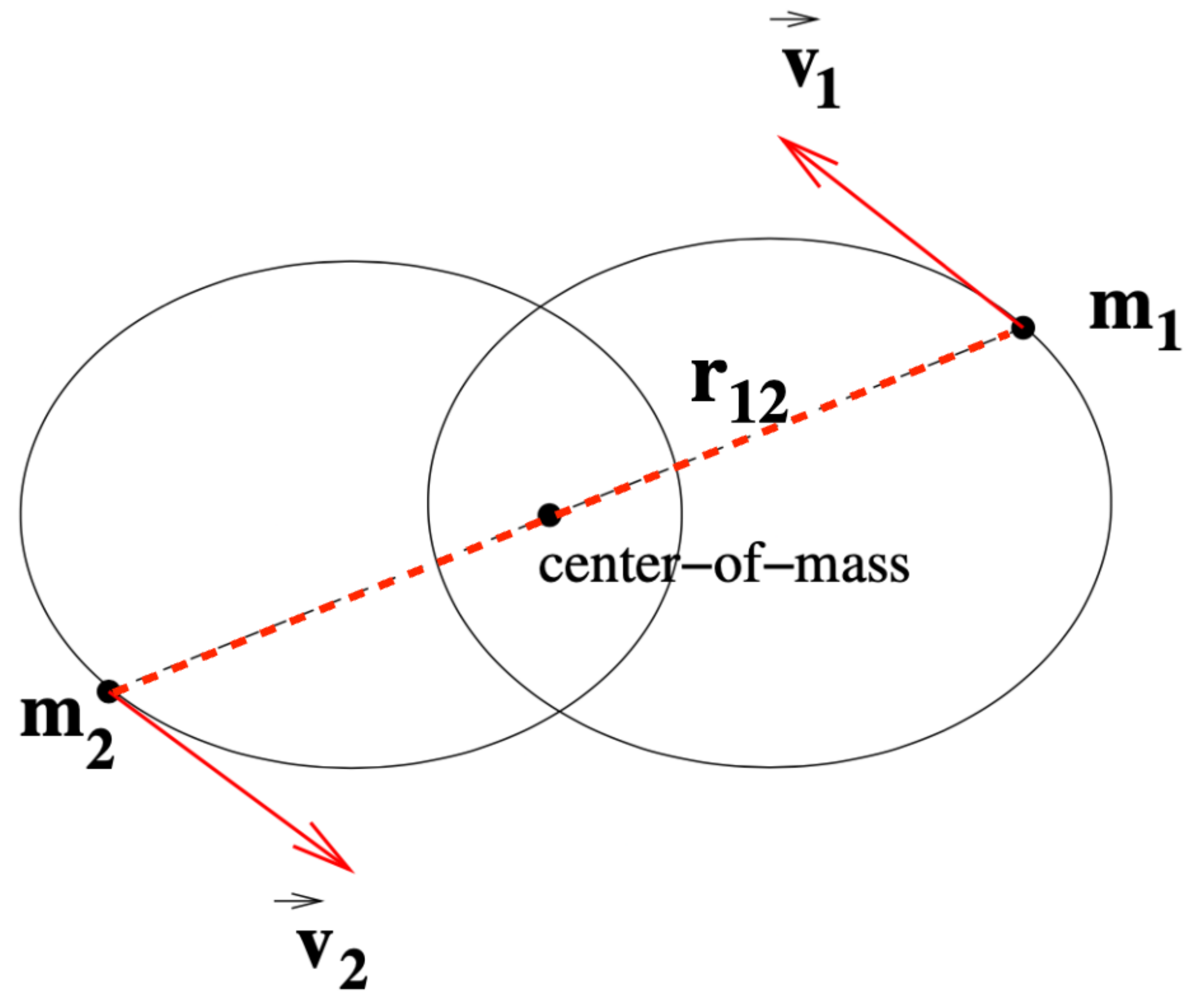
Perturbation theory

Post-Newtonian (PN) formalism

- **Slowly moving** compact objects forming a **weakly self-gravitating** system

- PN expansions in powers of a small parameter:

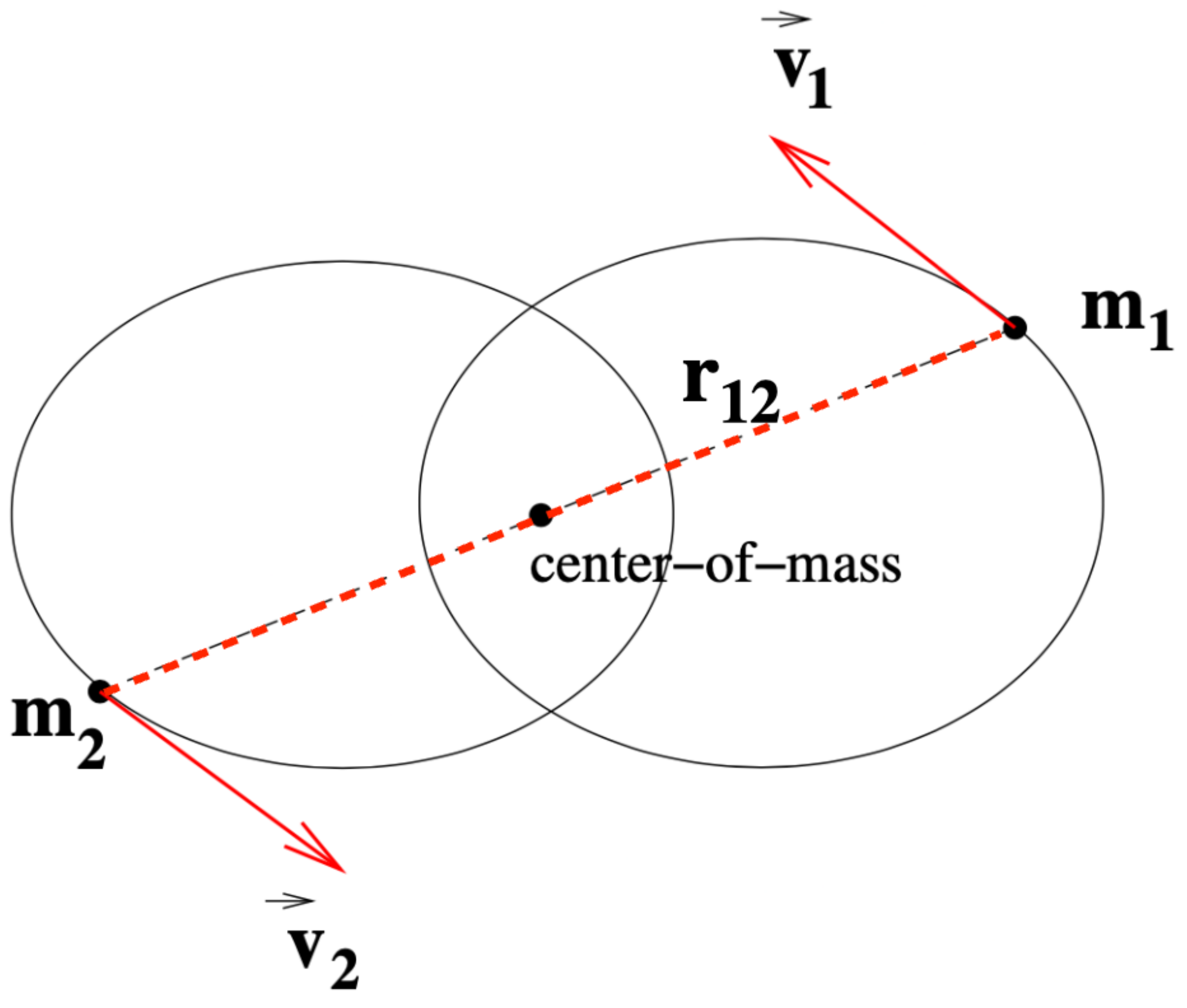
$$\varepsilon = \frac{v_{12}{}^2}{c^2} \sim \frac{Gm}{r_{12}\, c^2} \ll 1$$ (Virial theorem)



- nPN $= \mathcal{O}\left(\left(\frac{v_{12}}{c}\right)^{2n}\right)$

How to compute the waveform

Dynamical sector



- Any radiative effect is first discarded

- One can compute the equations of motion (EOM) of the binary system up to some PN order beyond the Newtonian acceleration

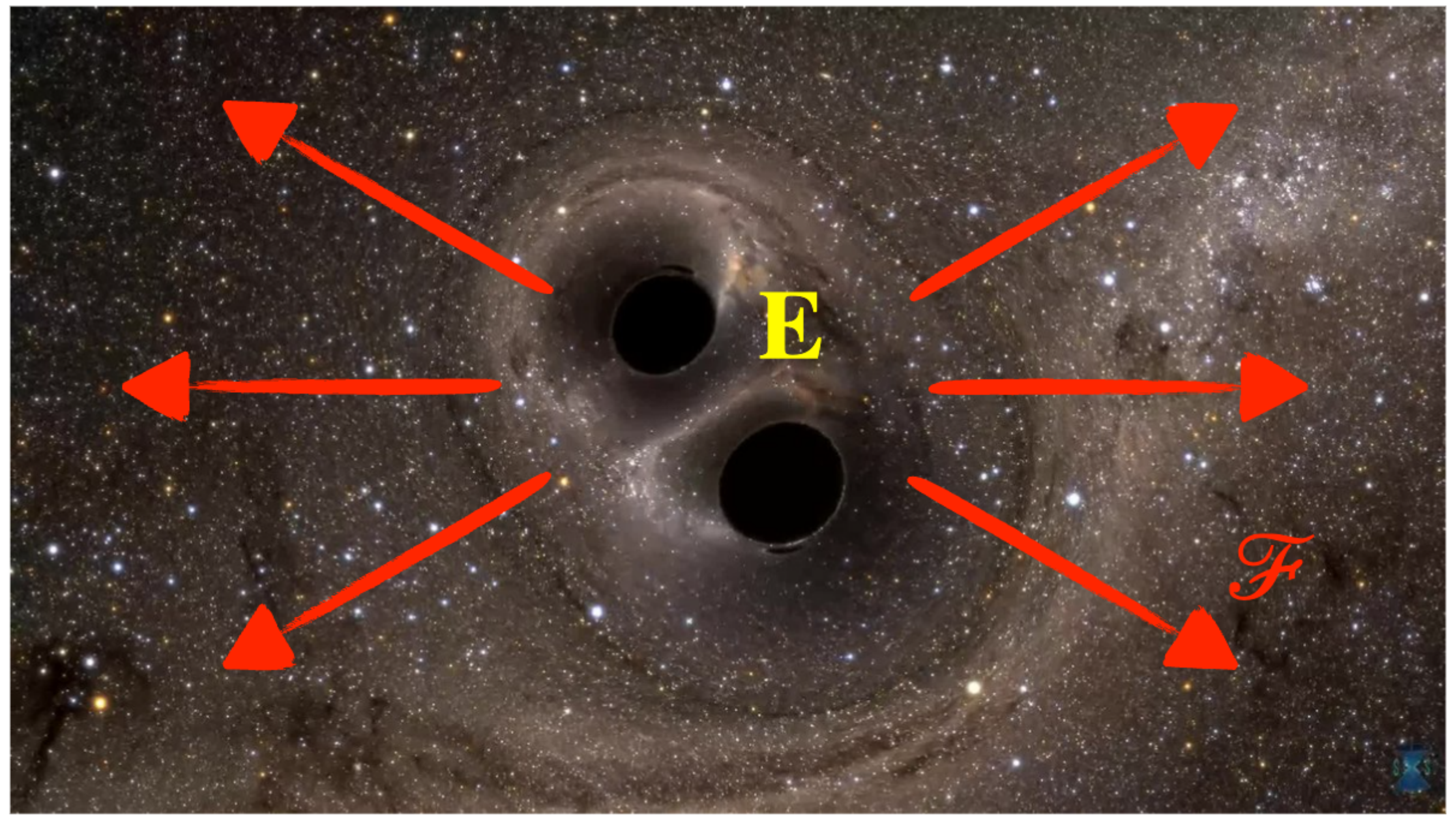
$$\frac{dv_1}{dt} = \frac{-G_{eff}\, m_2}{r_{12}{}^2} n_{12} + \frac{1}{c^2} A_1^{1PN} + \frac{1}{c^4} A_1^{2PN} + \frac{1}{c^6} A_1^{3PN} + \ldots$$

- The conserved energy E can be deduced from the Lagrangian

- The EOM are expected to be invariant under a global Lorentz-Poincaré transformation

How to compute the waveform

Radiative sector



- Use a gravitational wave generation formalism
  - **Multipolar Post-Minkowskian (MPM)** expansion of the external field to the isolated system
  - **Post-Newtonian (PN)** expansion of the internal field to the isolated system
  - Expansions connected in an **overlapping region** → yields to the energy flux $\mathscr{F}$

- Compute the dissipative part of the EOM appearing as odd powers in v/c

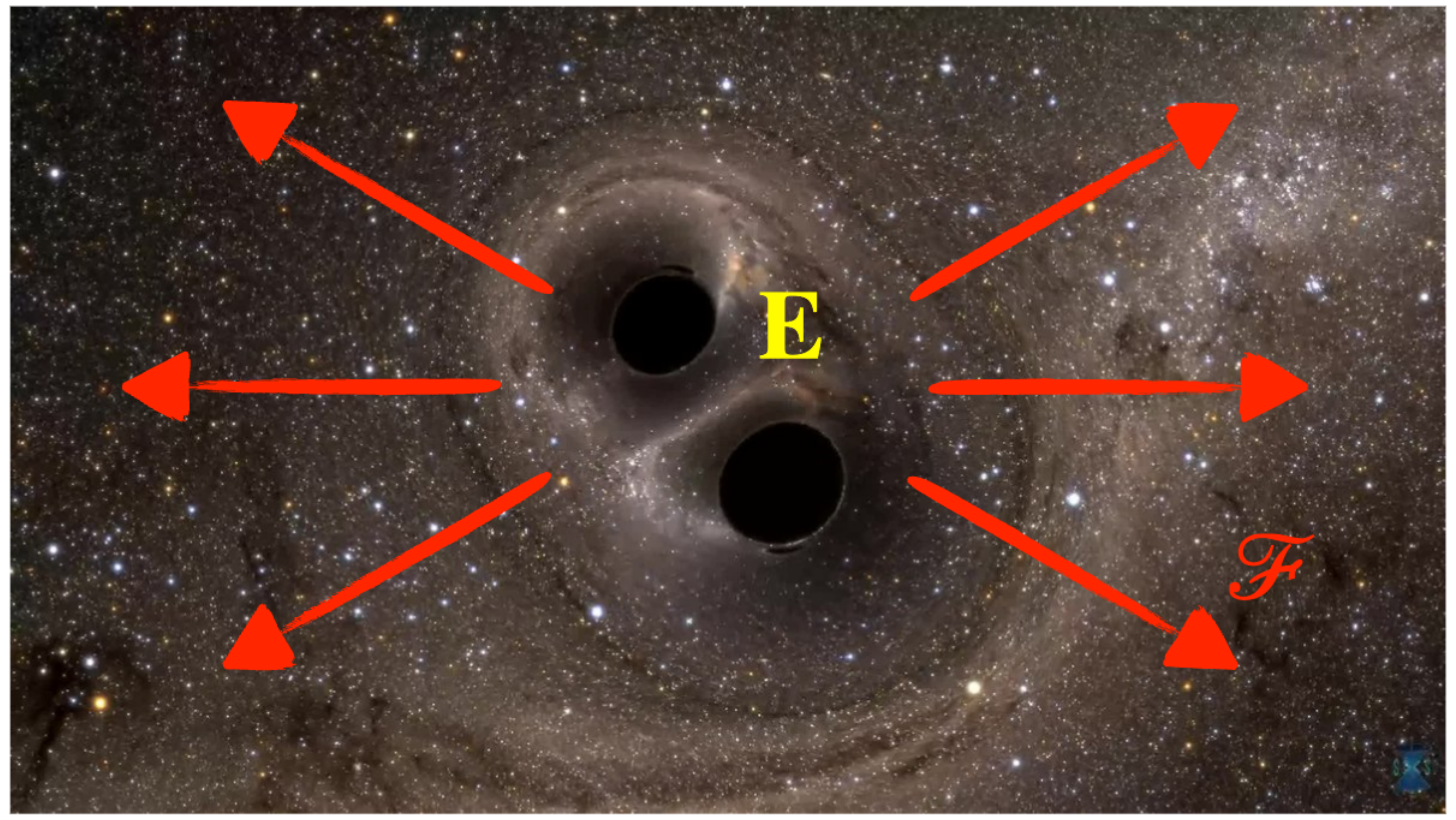
$$\frac{dv_1}{dt} = \frac{-G_{eff}\, m_2}{{r_{12}}^2} n_{12} + \frac{1}{c^2}A_1^{1PN} + \frac{1}{c^3}A_1^{1.5PN} + \frac{1}{c^4}A_1^{2PN} + \frac{1}{c^5}A_1^{2.5PN} + \frac{1}{c^6}A_1^{3PN} + \dots$$

How to compute the waveform

Flux-balance equation

$$\frac{dE(\omega)}{dt} = -\mathcal{F}(\omega)$$

- In (quasi-) circular orbits, E and $\mathcal{F}$ both depend on the orbital frequency ω



Orbital phase

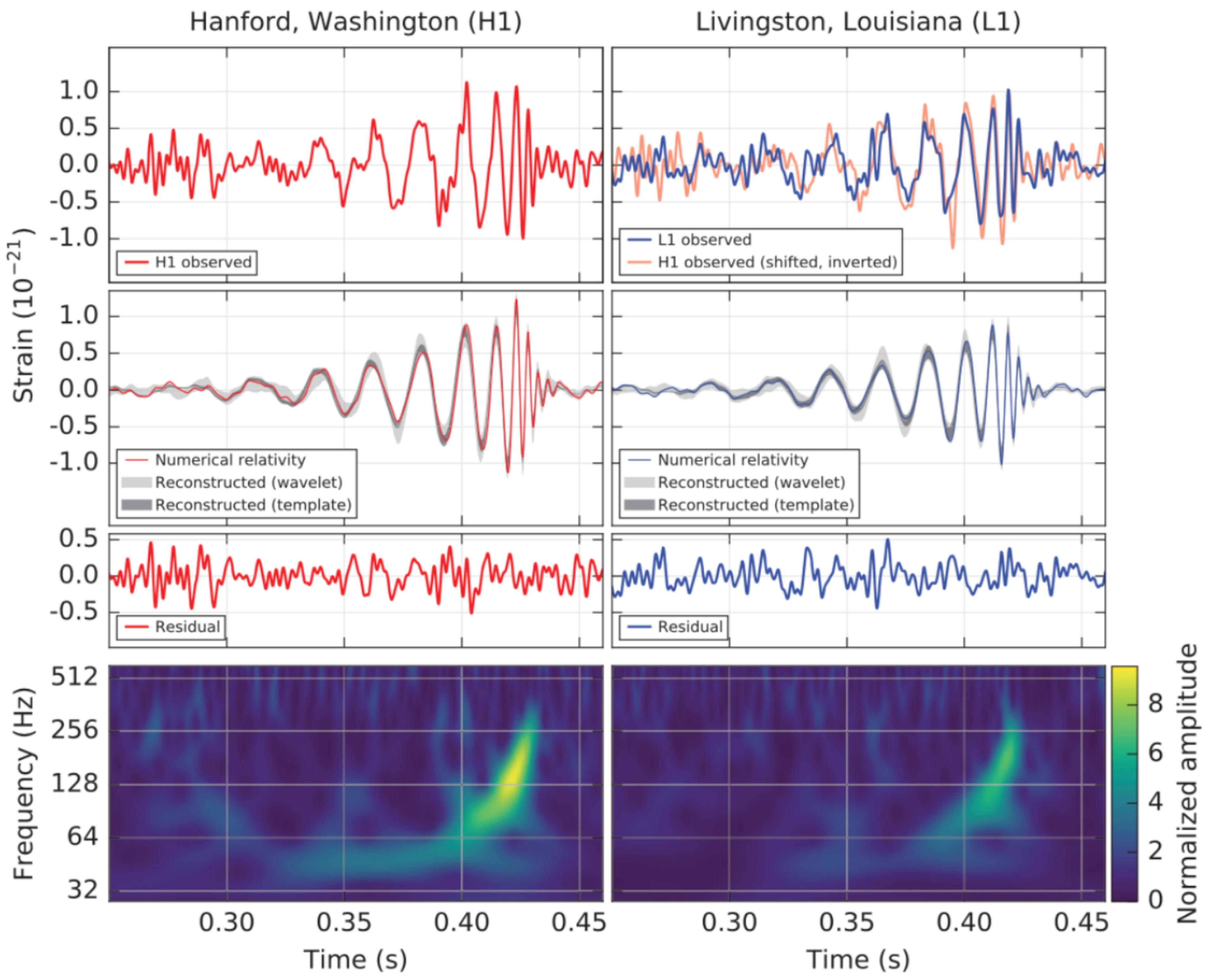
$$\phi = \int \omega dt = \int -\omega \frac{dE}{\mathcal{F}(\omega)}$$

- The orbital phase ϕ is simply obtain by an integration
- ϕ is an important observable for the gravitational signal analysis

Alternative theories of gravity

GR works already very well

GW150914



LIGO-Virgo Collaboration 2016

But ….

- How to explain that accelerated expansion of the Universe ?
- Is it due to the Dark Energy?
- Should we be looking for some possible modifications of GR ?

Alternative theories of gravity

GR	Alternative theories
One field quantity : the metric tensor	Several fields
Graviton is massless	Graviton is massive
Curvature appears linearly in S	Higher curvature theories
n = 4 dimensions	n > 4 dimensions
…	…

Alternative theories of gravity

GR	Alternative theories
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n = 4 dimensions	n > 4 dimensions
…	…

Massless scalar-tensor (ST) theories

Why scalar-tensor theories of gravity ?

- One of the most popular and well-posed class of theories
    - First introduced by Jordan (1955) and Brans-Dicke (1961)
    - Postulated as a low-energy limit of string theory
    - F(R) class of theories, constructed to explain the accelerated expansion of the Universe
- It passes weak field tests (in the solar system and in binary pulsar systems)
- Deviations from GR are expected for neutron stars
- It is simple enough to compute GW

Massless scalar-tensor theories

The action

$$S\left(g_{\alpha\beta},\phi\right)=\frac{c^3}{16\pi G}\int d^4x\sqrt{-g}\left[\phi R-\frac{\omega(\phi)}{\phi}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\right]+S_m\left(\mathfrak{m},g_{\alpha\beta}\right)$$

- Scalar field ϕ and scalar function $\omega(\phi)$
- Matter field $\mathfrak{m}$
- The scalar field coupled to the physical metric $g_{\alpha\beta}$ but not directly to the matter

The matter action

$$S_m\left(\mathfrak{m},g_{\alpha\beta}\right)=-\sum_A\int dt\,m_A(\phi)\,c^2\sqrt{-(g_{\alpha\beta})_A\frac{v_A^\alpha v_A^\beta}{c^2}}\qquad (\mathrm{A}=1,2)$$

- **Eardley** (1975) : for self-gravitating bodies, the masses $m_A(\phi)$ depend on the scalar field
- $(g_{\alpha\beta})_A$ indicates that the metric is evaluated at the position of the body A

Massless scalar-tensor theories

Conformal transformation

$$\tilde{g}_{\alpha\beta} = \frac{\phi}{\phi_0} g_{\alpha\beta} = \varphi g_{\alpha\beta}$$

- Far from the system, $\phi = \phi_0$ (constant)
- Decouple the scalar and tensor degrees of freedom

PN formalism in scalar-tensor theories

- Perturbed metric

$$h^{\alpha\beta} \equiv \sqrt{-\tilde{g}}\,\tilde{g}^{\alpha\beta} - \eta^{\alpha\beta} \equiv \tilde{\mathfrak{g}}^{\alpha\beta} - \eta^{\alpha\beta}$$

- Scalar perturbation

$$\psi \equiv \varphi - 1$$

- Metric decomposition

$$h \equiv \left(\bar{h}^{00ii} = \bar{h}^{00} + \bar{h}^{ii}\,,\ \bar{h}^{0i}\,,\ \bar{h}^{ij}\ ;\ \psi\right)$$

$$\begin{cases} \bar{h}^{00ii} &= -\frac{4}{c^2} V - \frac{8}{c^4} V^2 + \mathcal{O}(\frac{1}{c^6}) \\ \bar{h}^{0i} &= -\frac{4}{c^3} V_i + \mathcal{O}(\frac{1}{c^5}) \\ \bar{h}^{ij} &= -\frac{4}{c^4}\left(W_{ij} - \frac{1}{2}\delta_{ij} W^k_{-k}\right) + \mathcal{O}(\frac{1}{c^6}) \\ \bar{\psi} &= -\frac{2}{c^2}\psi_{(0)} + \frac{2}{c^4}\left(1 - \frac{\phi_0\omega_0'}{3+2\omega_0}\right)\psi_{(0)}{}^2 + \frac{1}{c^6}\left[\frac{-4}{3}\left(1 - \frac{4\phi_0\omega_0'}{3+2\omega_0} + \frac{\phi_0^2\left(4\,\omega_1^2 - (3+2\omega_0)\,\omega_2\right)}{\left(3+2\omega_0\right)^2}\right)\psi_{(0)}{}^3 + \psi_{(1)}\right] + \mathcal{O}(\frac{1}{c^8}) \end{cases}$$

$$\begin{cases} \Box V &= -4\pi G\sigma \\ \Box V_i &= -4\pi G\sigma_i \\ \Box W_{ij} &= -4\pi G\left(\sigma_{ij} - \delta_{ij}\sigma_{kk}\right) - \partial_i V \partial_j V - (3+2\omega_0)\,\partial_i\psi_{(0)}\,\partial_j\psi_{(0)} \\ \Box \psi_{(0)} &= 4\pi G\sigma_s \end{cases}$$

Tidal effects

Why adding these physical effects in our models ?

$$\nabla^{\perp}_{\mu} = \perp^{\nu}_{\mu} \nabla_{\nu} = (\delta^{\nu}_{\mu} + u_{\mu}u^{\nu}) \nabla_{\nu}$$

- Go beyond the point particle approximation

$$S_m = -\sum_A \int d\tau_A \left\{ m_A(\phi)\, c^2 + \sum_{l=2}^{\infty} \frac{1}{2l!} \left[c_A^{(l)}(\phi)\, G^A_{\mu_1 \ldots \mu_l} G_A^{\mu_1 \ldots \mu_l} + \frac{l}{(l+1)c^2} d_A^{(l)}(\phi)\, H^A_{\mu_1 \ldots \mu_l} H_A^{\mu_1 \ldots \mu_l} \right] \right. \quad \text{Gravitational}$$

$$\left. + \sum_{l=1}^{\infty} \frac{1}{2} \lambda_A^{(s),(l)}(\phi) \left(\nabla^{\perp}_{\mu_1 \ldots \mu_l} \varphi \right)_A \left(\nabla_{\perp}^{\mu_1 \ldots \mu_l} \varphi \right)_A + \ldots \right\} \quad \text{Scalar}$$

- $c_A^{(l)}$ and $d_A^{(l)}$: **mass-type and current-type Tidal Love Numbers** → characterize the deformability and polarizability of the body A due to the presence of its companion

$$c_A^{(l)} \propto \frac{\tilde{c}_A^{(l)} R_A^{2l+1}}{G} \qquad d_A^{(l)} \propto \frac{\tilde{d}_A^{(l)} R_A^{2l+1}}{G}$$

- $\lambda_A^{(s),(l)}$: **scalar Tidal Love Number** (sTLN) → characterize the deformability of the body A due to the scalar field ϕ

$$\frac{\lambda_A^{(s),(l)}}{c^2} \propto \frac{\tilde{\lambda}_A^{(s),(l)} R_A^{2l+1}}{G}$$

Tidal effects

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Gravitational

$$\left. + \sum_{l=1}^{\infty} \frac{1}{2} \lambda_A^{(s),(l)}(\phi) \left(\nabla^{\perp}_{\mu_1\ldots\mu_l}\varphi \right)_A \left(\nabla_{\perp}^{\mu_1\ldots\mu_l}\varphi \right)_A + \ldots \right\}$$

Scalar

- Challenges:
  - Learn how **Neutron Star (NS) nuclear physics** influence the gravitational signal
  - **Test deviations from GR:** we look for GW signatures that may arise in ST theories to lift degeneracies when observing deviations from GR in the signals

Tidal effects

State-of-the-art

- Go beyond the point particle approximation

$$S_m = -\sum_A \int d\tau_A \left\{ m_A(\phi)\, c^2 + \mathcal{O}\left(\frac{1}{c^{10}}\right) \right.$$

$$\left. + \frac{1}{2}\, \lambda_A^{(s),(1)}(\phi)\ \left[(g^{\mu\nu})_A + u_A^\mu u_A^\nu\right](\nabla_\mu \varphi)_A (\nabla_\nu \varphi)_A \right\}$$

Gravitational

Scalar

- **What have been done :** Leading-order (LO) tidal correction [Bernard, 2019]

$$\frac{d\vec{v_1}}{dt} = \underbrace{-\frac{G_{eff} m_2}{{r_{12}}^2}\vec{n_{12}}}_{\text{Conservative term}} + \underbrace{\frac{1}{c^6}\vec{{A_1}^{LO}}}_{\text{Scalar tidal effects at the LO (3PN)}}$$

Tidal effects

State-of-the-art

- Go beyond the point particle approximation

$$S_m = -\sum_A \int d\tau_A \left\{ m_A(\phi)\, c^2 + \frac{1}{4} c_A^{(2)}(\phi)\, G^A_{\mu\nu}\, G^{\mu\nu}_A \right.$$ Gravitational

$$\left. + \frac{1}{2}\, \lambda_A^{(s),(1)}(\phi) \left[(g^{\mu\nu})_A + u^\mu_A u^\nu_A \right] (\nabla_\mu \varphi)_A (\nabla_\nu \varphi)_A + \frac{1}{2} \lambda_A^{(s),(2)}\, (\phi) (\nabla^\perp_{\mu\nu} \varphi)_A\, (\nabla^{\mu\nu}_\perp \varphi)_A \right\}$$ Scalar

- **Work in progress :** Next-to-next-to-leading order (NNLO) tidal correction

$$\frac{d\vec{v_1}}{dt} = \underbrace{-\frac{G_{eff} m_2}{{r_{12}}^2} \overrightarrow{n_{12}} + \frac{1}{c^2} \overrightarrow{{A_1}^{1PN}} + \frac{1}{c^4} \overrightarrow{{A_1}^{2PN}}}_{\text{Conservative terms}} + \underbrace{\frac{1}{c^6} \overrightarrow{{A_1}^{LO}} + \frac{1}{c^8} \overrightarrow{{A_1}^{NLO}} + \frac{1}{c^{10}} \overrightarrow{{A_1}^{NNLO}}}_{\text{Scalar and gravitational tidal effects up to the NNLO (5PN)}}$$

Fokker action for a tidal perturbation

- The action and solutions to the field equations are developed as :

$$S\left[\mathbf{y}_A, \mathbf{v}_A, g, \phi\right] = S^{(0)}\left[\mathbf{y}_A, \mathbf{v}_A, g, \phi\right] + \varepsilon\, \overbrace{S^{(1)}\left[\mathbf{y}_A, \mathbf{v}_A, g, \phi\right]}^{\Delta S_{(fs)}}$$

$$g_{sol}\left[\mathbf{y}_A, \mathbf{v}_A, \ldots\right] = {g_{sol}}^{(0)} + \varepsilon\, {g_{sol}}^{(1)} \qquad (\varepsilon \ll 1)$$

$$\phi_{sol}\left[\mathbf{y}_A, \mathbf{v}_A, \ldots\right] = {\phi_{sol}}^{(0)} + \varepsilon\, {\phi_{sol}}^{(1)}$$

- It is sufficient to replace in the Fokker action the **non-perturbed solutions** ${g_{sol}}^{(0)}$ and ${\phi_{sol}}^{(0)}$

$$S_{Fokker} = S\left[\mathbf{y}_A, \mathbf{v}_A, \left({g_{sol}}^{(0)} + \varepsilon\, {g_{sol}}^{(1)}\right), \left({\phi_{sol}}^{(0)} + \varepsilon\, {\phi_{sol}}^{(1)}\right)\right]$$
$$= S\left[\mathbf{y}_A, \mathbf{v}_A, {g_{sol}}^{(0)}, {\phi_{sol}}^{(0)}\right] + \mathcal{O}(\varepsilon^2)$$

- It gives the same dynamics as the original action

Conservative sector

- Get the NNLO tidal correction to:
  - To the generalized Lagrangian $L = L\left[\vec{y}_A(t), \vec{v}_A(t), \vec{a}_A(t), \vec{b}_A(t), \vec{c}_A(t), \ldots\right]$
  - To the conservative EOM using the generalized Euler-Lagrange equations:

$$\frac{\delta L}{\delta y_A^i} = \frac{\partial L}{\partial y_1^i} - \frac{d}{dt}\frac{\partial L}{\partial v_1^i} + \frac{d^2}{dt^2}\frac{\partial L}{\partial a_1^i} - \ldots = 0$$

  - To the 10 Noetherian conserved quantities : $E, P^i, J^i, G^i - P^i t$

- Rewrite these quantities in the center-of-mass (COM) frame by solving $G^i = 0$
- Reduce to the case of circular orbits using the PN variable $x = \left(\frac{\alpha \tilde{G} m \omega}{c^3}\right)^{2/3}$

Radiative sector

Other piece needed : the energy flux

Source moments

- From the PN - MPM formalism: the mass-type and current-type multipole moments

$$I_L(u) = \underset{B=0}{\mathrm{FP}} \int \mathrm{d}^3\mathbf{x}\, \tilde{r}^B \int_{-1}^{1} \mathrm{d}z \left[\delta_\ell(z)\hat{x}_L\Sigma - \frac{4(2\ell+1)}{c^2(\ell+1)(2\ell+3)}\delta_{\ell+1}(z)\hat{x}_{iL}\Sigma_i^{(1)} + \frac{2(2\ell+1)}{c^4(\ell+1)(\ell+2)(2\ell+5)}\delta_{\ell+2}(z)\hat{x}_{ijL}\Sigma_{ij}^{(2)} \right](\mathbf{x}, u+zr/c)\,,$$

$$J_L(u) = \underset{B=0}{\mathrm{FP}} \int \mathrm{d}^3\mathbf{x}\, \tilde{r}^B \int_{-1}^{1} \mathrm{d}z\, \varepsilon_{ab\langle i_\ell} \left[\delta_\ell(z)\hat{x}_{L-1\rangle a}\Sigma_b - \frac{2\ell+1}{c^2(\ell+2)(2\ell+3)}\delta_{\ell+1}(z)\hat{x}_{L-1\rangle ac}\Sigma_{bc}^{(1)} \right](\mathbf{x}, u+zr/c)$$

- In ST theories : an additional scalar source moment

$$I_L^s(u) = \underset{B=0}{\mathrm{FP}} \int \mathrm{d}^3\mathbf{x}\, \tilde{r}^B \int_{-1}^{1} \mathrm{d}z\, \delta_\ell(z)\,\hat{x}_L \Sigma^s(\mathbf{x}, u+zr/c)\,.$$

$$\Sigma \equiv \frac{\bar{\tau}^{00}+\bar{\tau}^{ii}}{c^2}\,, \qquad \Sigma_i \equiv \frac{\bar{\tau}^{0i}}{c}\,, \qquad \Sigma_{ij} \equiv \bar{\tau}^{ij}$$

$$\Sigma^s \equiv -\frac{\bar{\tau}_s}{c^2}$$

Energy densities

Radiative sector

Other piece needed : the energy flux

Source moments

- From the PN - MPM formalism: the mass-type and current-type multipole moments

$$I_L(u) = \underset{B=0}{\mathrm{FP}} \int \mathrm{d}^3\mathbf{x}\, \tilde{r}^B \int_{-1}^{1} \mathrm{d}z \Big[\delta_\ell(z)\hat{x}_L \Sigma - \frac{4(2\ell+1)}{c^2(\ell+1)(2\ell+3)} \delta_{\ell+1}(z)\hat{x}_{iL}\Sigma_i^{(1)} + \frac{2(2\ell+1)}{c^4(\ell+1)(\ell+2)(2\ell+5)} \delta_{\ell+2}(z)\hat{x}_{ijL}\Sigma_{ij}^{(2)} \Big](\mathbf{x}, u + zr/c)\,,$$

$$J_L(u) = \underset{B=0}{\mathrm{FP}} \int \mathrm{d}^3\mathbf{x}\, \tilde{r}^B \int_{-1}^{1} \mathrm{d}z\, \varepsilon_{ab\langle i_\ell} \Big[\delta_\ell(z)\hat{x}_{L-1\rangle a}\Sigma_b - \frac{2\ell+1}{c^2(\ell+2)(2\ell+3)} \delta_{\ell+1}(z)\hat{x}_{L-1\rangle ac}\Sigma_{bc}^{(1)} \Big](\mathbf{x}, u + zr/c)$$

- In ST theories : an additional scalar source moment

$$I_L^s(u) = \underset{B=0}{\mathrm{FP}} \int \mathrm{d}^3\mathbf{x}\, \tilde{r}^B \int_{-1}^{1} \mathrm{d}z\, \delta_\ell(z)\, \hat{x}_L \Sigma^s(\mathbf{x}, u + zr/c)\,.$$

$$\int_{-1}^{1} \mathrm{d}z\ \delta_\ell(z)\, \Sigma(\mathbf{x}, t + zr/c) = \sum_{k=0}^{+\infty} \frac{(2\ell+1)!!}{(2k)!!(2\ell+2k+1)!!} \left(\frac{r}{c}\right)^{2k} \Sigma^{(2k)}(\mathbf{x}, t)$$

Infinite PN series

Radiative sector

Energy flux at 1PN with the NNLO tidal correction

$$\mathscr{F} = \frac{G\phi_0}{c^5}\left(\frac{1}{5}\overset{(1)}{\mathcal{U}}_{ij}\overset{(1)}{\mathcal{U}}_{ij} + \frac{1}{c^2}\left[\frac{1}{189}\overset{(1)}{\mathcal{U}}_{ijk}\overset{(1)}{\mathcal{U}}_{ijk} + \frac{16}{45}\overset{(1)}{\mathscr{V}}_{ij}\overset{(1)}{\mathscr{V}}_{ij}\right] + \mathcal{O}\left(\frac{\lambda_A}{c^6}\right)\right)$$

$$\mathscr{F}^s = \frac{G\phi_0(3+2\omega_0)}{c}\left(\overset{(1)}{\mathcal{U}^s}\overset{(1)}{\mathcal{U}^s} + \frac{1}{3\,c^2}\overset{(1)}{\mathcal{U}^s_i}\overset{(1)}{\mathcal{U}^s_i} + \frac{1}{30\,c^4}\overset{(1)}{\mathcal{U}^s_{ij}}\overset{(1)}{\mathcal{U}^s_{ij}} + \frac{1}{630\,c^6}\overset{(1)}{\mathcal{U}^s_{ijk}}\overset{(1)}{\mathcal{U}^s_{ijk}} + \mathcal{O}\left(\frac{\lambda_A}{c^{10}}\right)\right)$$

- The scalar energy flux **starts at -1PN** wrt to GR
- The scalar tides **start at 0PN** wrt to GR
- The radiative moment required at the highest PN order are the **scalar monopole** and **dipole**

$$\mathcal{U}^s = I^s$$

$$\mathcal{U}^s_i = \overset{(1)}{I^s_i} + \underbrace{\frac{2GM}{\phi_0 c^3}\int_{-\infty}^{U} \mathrm{d}V\,\overset{(3)}{I^s_i}(V)\left[\ln\left(\frac{U-V}{2b}\right)+1\right]}_{\text{scalar tail term}}$$

scalar tail term
(enters at 0.5PN in the flux wrt to GR)

→ The scalar monopole and dipole I^s and I^s_i needed at the **NNLO**

Radiative sector

Energy flux at 1PN with the NNLO tidal correction

$$\mathscr{F} = \frac{G\phi_0}{c^5}\left(\frac{1}{5}\overset{(1)}{\mathscr{U}}_{ij}\overset{(1)}{\mathscr{U}}_{ij} + \frac{1}{c^2}\left[\frac{1}{189}\overset{(1)}{\mathscr{U}}_{ijk}\overset{(1)}{\mathscr{U}}_{ijk} + \frac{16}{45}\overset{(1)}{\mathscr{V}}_{ij}\overset{(1)}{\mathscr{V}}_{ij}\right] + \mathscr{O}\left(\frac{\lambda_A}{c^6}\right)\right)$$

$$\mathscr{F}^s = \frac{G\phi_0(3+2\omega_0)}{c}\left(\overset{(1)}{\mathscr{U}^s}\overset{(1)}{\mathscr{U}^s} + \frac{1}{3\,c^2}\overset{(1)}{\mathscr{U}^s_i}\overset{(1)}{\mathscr{U}^s_i} + \frac{1}{30\,c^4}\overset{(1)}{\mathscr{U}^s_{ij}}\overset{(1)}{\mathscr{U}^s_{ij}} + \frac{1}{630\,c^6}\overset{(1)}{\mathscr{U}^s_{ijk}}\overset{(1)}{\mathscr{U}^s_{ijk}} + \mathscr{O}\left(\frac{\lambda_A}{c^{10}}\right)\right)$$

- Higher order radiative moments need less correction terms

$$\mathcal{U}_{ij} = \overset{(2)}{I}_{ij}$$
$$\mathcal{U}^s_{ij} = \overset{(2)}{I^s_{ij}}$$

→ The mass-type and scalar quadrupole I_{ij} and I^s_i needed at the **NLO**

$$\mathscr{U}_{ijk} = \overset{(3)}{I}_{ijk}$$
$$\mathscr{U}^s_{ijk} = \overset{(3)}{I^s_{ijk}}$$
$$\mathscr{V}_{ij} = \overset{(2)}{J}_{ij}$$

→ The mass-type and scalar octupole I_{ijk} and I^s_{ijk} and the current-type quadrupole needed at the **LO**

- Times derivatives are performed by injecting the EOM previously obtained

Radiative sector

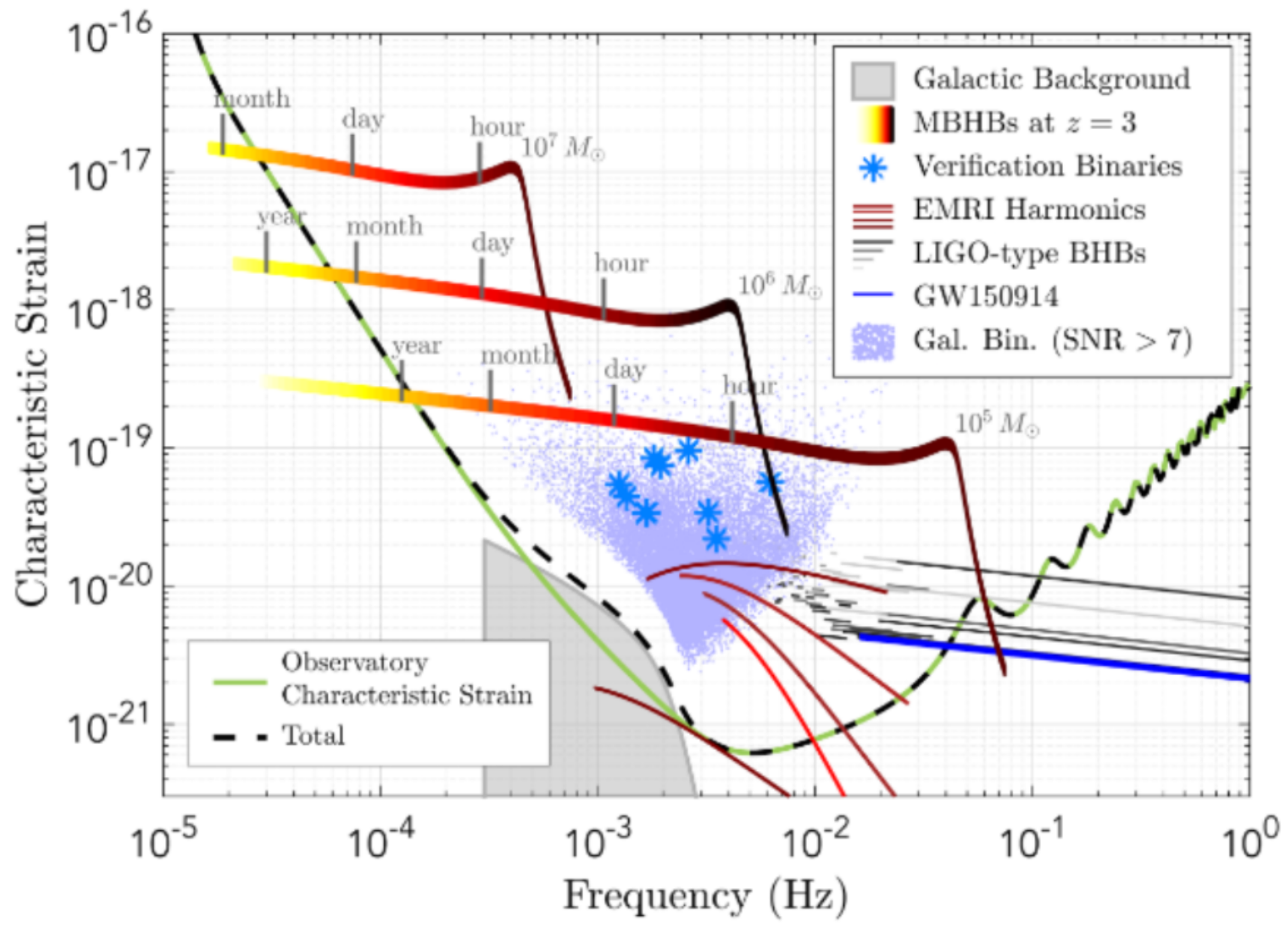
Orbital phase

- E and $\mathcal{F} + \mathcal{F}^s$ reduced to the circular orbits with $x = \left(\alpha\tilde{G}m\omega/c^3\right)^{2/3}$
- Orbital frequency $\omega = \dfrac{c^3}{\alpha\tilde{G}m}x^{3/2}$
- Using the flux balance equation $\dfrac{dE_{circ}}{dt} = -\left(\mathcal{F}_{circ} + \mathcal{F}^s_{circ}\right)$ we find the phase evolution:

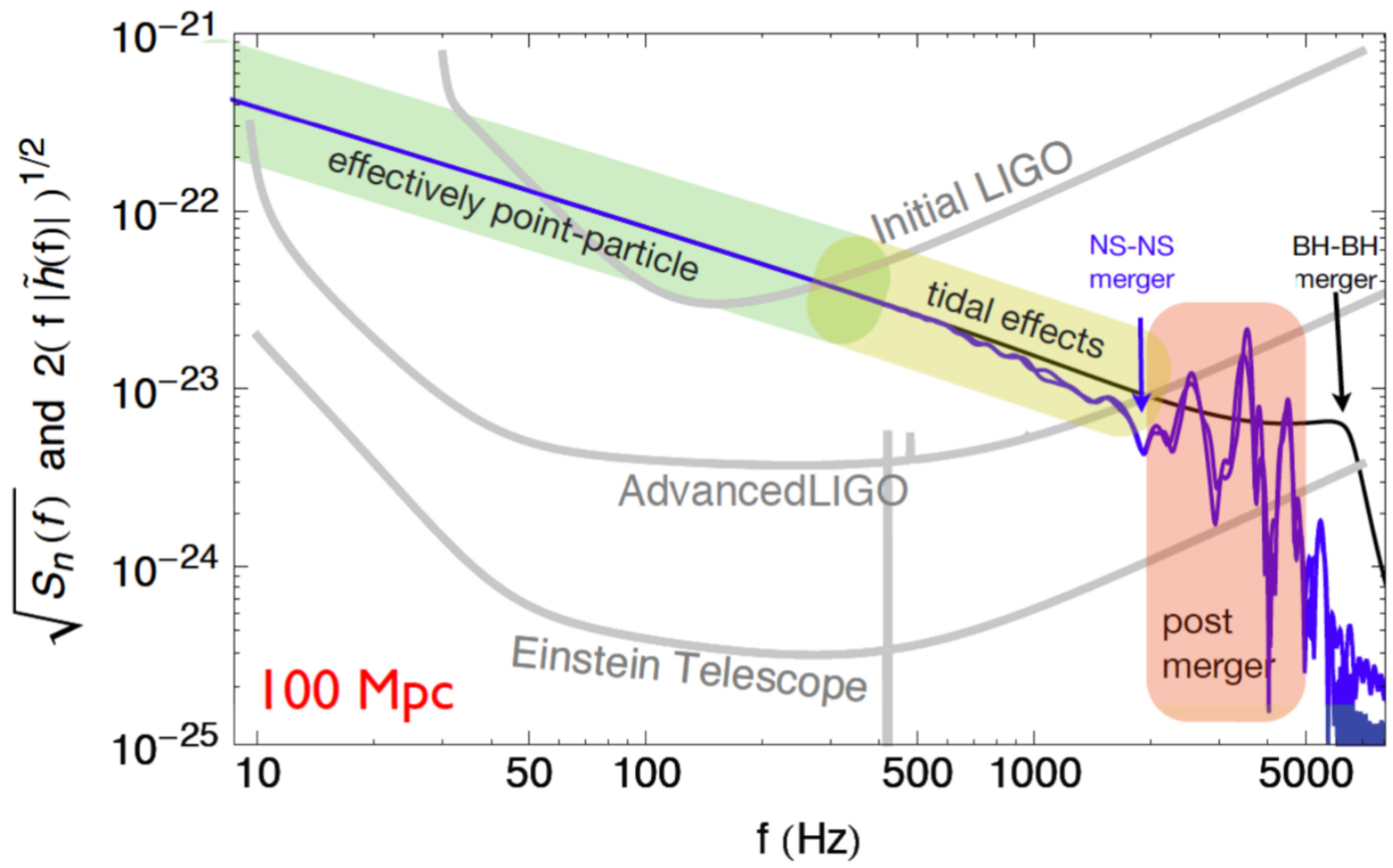
$$\frac{d\Psi}{dx} = \frac{dt}{dx}\omega = \frac{dE/dx}{dE/dt}\omega = -\frac{c^3x^{3/2}}{\alpha\tilde{G}m}\frac{dE/dx}{\left(\mathcal{F}_{circ} + \mathcal{F}^s_{circ}\right)(x)}$$

- To find an analytical result for the orbital phase :
  - re-expand the ratio $(dE/dx)/\left(\mathcal{F}_{circ} + \mathcal{F}^s_{circ}\right)$ in x
  - truncate at the NNLO in the tidal effects
  - integrate term by term

Detectability of tidal effects



LISA Astro2020 white paper



ET Science case 2019