The threshold of gravitational collapse in vacuum

David Hilditch

CENTRA, IST Lisbon

Trieste, June 2023

Gravity at the Edge

We wish to find and understand solutions to the (vacuum) field equations $R_{ab} = 0$ at their most extreme.

Highly dynamical, non-linear PDEs. Strategy?

- Mathematical relativity; the use of rigorous mathematics to understand the solution space, where possible demonstrating specific, sharp, estimates.
- Numerical relativity; use of numerical methods to obtain *in hand* approximate solutions that converge to the continuum solution as you use more computational resources.

The two compliment each other's strengths and can be combined to unearth the truth.

The cosmic censorship conjectures

The most important open conjectures in GR for asymptotically flat spacetimes are those of cosmic censorship:

- Weak: generically singularities lie inside black holes. (Global existence outside BHs).
- Strong: maximal Cauchy developments are inextendible. (Global uniqueness).

(Proper formulation needs care).
How can we attack this?

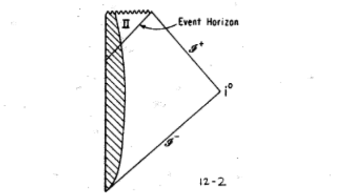


Fig. 12.2. Another representation of the closure, $\overline{M}$, of the physical spacetime depicted in Figure 12.1. As in Figure 12.1, the angular dimensions are suppressed so each point in this diagram (except those at $r = 0$ and the point i^0) represents a 2-sphere.

Wald 1984

Weak cosmic censorship: spherical symmetry

- Christodoulou 1987-1997.
- Choptuik, 1992, Collapse of spherical scalar field:
  - i). Small blackholes can be created, $M \propto (p - p_\star)^\gamma$.
  - ii). γ universal.
  - iii). $p = p_\star$ self-similar, unique.

Critical phenomena in gravitational collapse!

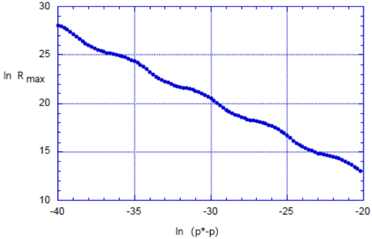


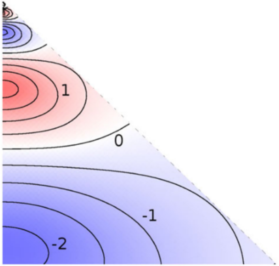
FIG. 1. ln R_{max} is plotted vs. $\ln(p* - p)$. The result is a line with slope -2γ and a periodic wiggle with period $\Delta/(2\gamma)$

Garfinkle & Duncan 1998: clever coordinates. No AMR.

Weak cosmic censorship: spherical symmetry

- Christodoulou 1987-1997.
- Choptuik, 1992, Collapse of spherical scalar field:
  - i). Small blackholes can be created, $M \propto (p - p_\star)^\gamma$.
  - ii). γ universal.
  - iii). $p = p_\star$ self-similar, unique.

  Critical phenomena in gravitational collapse!



Reiterer & Trubowitz 2012: Choptuik spacetime exists (and is real analytic)!

Weak cosmic censorship conjecture: beyond spherical

Vacuum. Consider initial data with no trapped surface.

- Small waves disperse. Christodoulou & Klainerman, 1993.
- Proof: trapped region can form. Christodoulou, 2008.
- Numerics: Abrahams & Evans, 1993.

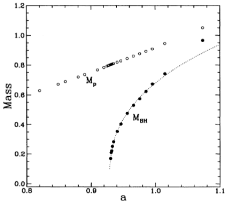


FIG. 2. Critical behavior of black-hole mass. Black-hole masses $M_{\rm BH}$ derived from a sequence of simulations are plotted (filled circles) as a function of initial wave packet amplitude. The quasilocal (Brill) masses M_p of the wave packets are shown (open circles) for comparison. The dotted curve represents the best-fit power law $M_{\rm BH} = C(a - a^\star)^\beta$ with values $a^\star = 0.928$, $C = 1.750$, and $\beta = 0.369$. Wave packet width $L = 2\pi$ is chosen to normalize the mass scale.

Despite multiple attempts, nobody has reproduced the latter.

Enter bamps: Pseudospectral NR code

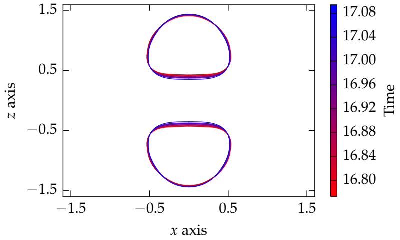
DH, A. Weyhausen & B.Brügmann [Phys.Rev.D93, (2016) 063006, Phys.Rev.D96, (2017) 104051].

Method *vastly* superior than earlier approaches. But problems persist.

Brill Waves. Key difficulties:

- No evidence of scaling in first peak. Hard to pass.
- Null-infinity?
- Resolving fine features.
- Coordinate singularities.

Off-center BHs.



Enter `bamps`: Pseudospectral NR code

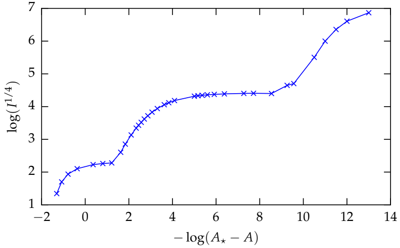
DH, A. Weyhausen & B.Brügmann [Phys.Rev.D93, (2016) 063006, Phys.Rev.D96, (2017) 104051].

Method *vastly* superior than earlier approaches. But problems persist.

Brill Waves. Key difficulties:

- No evidence of scaling in first peak. Hard to pass.
- Null-infinity?
- Resolving fine features.
- Coordinate singularities.

Scaling of Kretschmann.



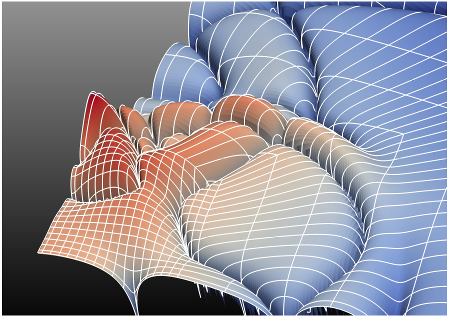
Enter `bamps`: Pseudospectral NR code

DH, A. Weyhausen & B.Brügmann [Phys.Rev.D93, (2016) 063006, Phys.Rev.D96, (2017) 104051].

Method *vastly* superior than earlier approaches. But problems persist.

Brill Waves. Key difficulties:

- No evidence of scaling in first peak. Hard to pass.
- Null-infinity?
- Resolving fine features.
- Coordinate singularities.

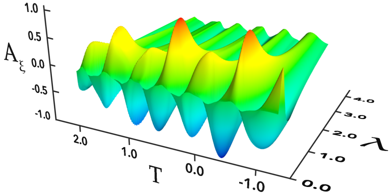


Aspherical toy models I: `SphGR` - Maxwell

Baumgarte, Gundlach & DH. [Phys.Rev.Lett.123, (2019) 171103].

To what extent are our difficulties caused by the lack of symmetry? In scalarfield collapse with large asphericity the Choptuik solution suffers from a second decay channel. What else can we do?

- Maxwell minimally coupled to GR: dynamical solutions aspherical.
- Curvature quantities still scale near the threshold.
- But the threshold solution seems no longer unique!



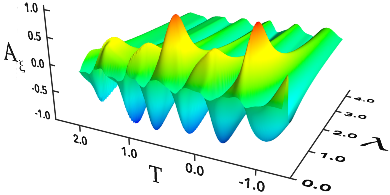
Spacetime plot of field variable at threshold.

Aspherical toy models I: SphGR - Maxwell

Baumgarte, Gundlach & DH. [Phys.Rev.Lett.123, (2019) 171103].

To what extent are our difficulties caused by the lack of symmetry? In scalarfield collapse with large asphericity the Choptuik solution suffers from a second decay channel. What else can we do?

- Maxwell minimally coupled to GR: dynamical solutions aspherical.
- Curvature quantities still scale near the threshold.
- But the threshold solution seems no longer unique!



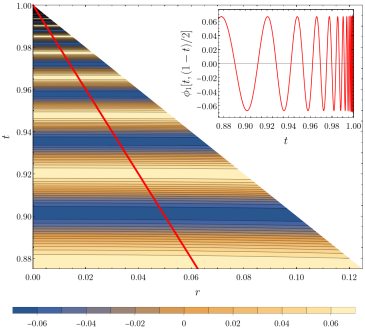
Spacetime plot of field variable at threshold.

Aspherical toy models II: Semi-linear wave equations

Suárez Fernández, Vicente & DH. [Phys.Rev.D103, (2021) 044016].

What is the most stupid model that we can concoct? Surprisingly just applying a *deformation* function $\phi = \mathcal{D}[\varphi]$ to solutions of the wave equation solves many riddles!

- Spherical: complete critical phenomenology recovered.
- General: power law scaling persists.
- General: uniqueness of threshold solutions lost!



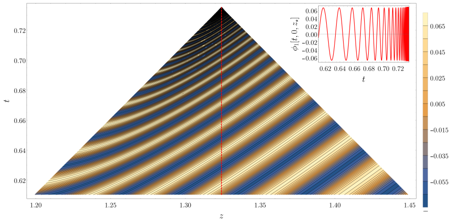
Spherical threshold solution.

Aspherical toy models II: Semi-linear wave equations

Suárez Fernández, Vicente & DH. [Phys.Rev.D103, (2021) 044016].

What is the most stupid model that we can concoct? Surprisingly just applying a *deformation* function $\phi = \mathcal{D}[\varphi]$ to solutions of the wave equation solves many riddles!

- Spherical: complete critical phenomenology recovered.
- General: power law scaling persists.
- General: uniqueness of threshold solutions lost!



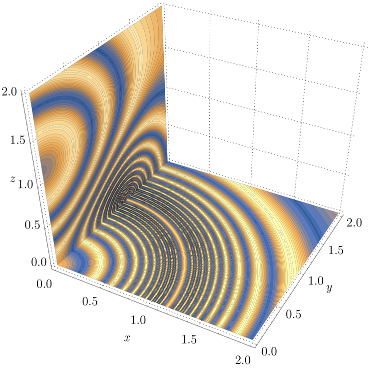
Axisymmetric threshold solution.

Aspherical toy models II: Semi-linear wave equations

Suárez Fernández, Vicente & DH. [Phys.Rev.D103, (2021) 044016].

What is the most stupid model that we can concoct? Surprisingly just applying a *deformation* function $\phi = \mathcal{D}[\varphi]$ to solutions of the wave equation solves many riddles!

- Spherical: complete critical phenomenology recovered.
- General: power law scaling persists.
- General: uniqueness of threshold solutions lost!



Another axisymmetric (near) threshold solution!

Prague - universal curvature features

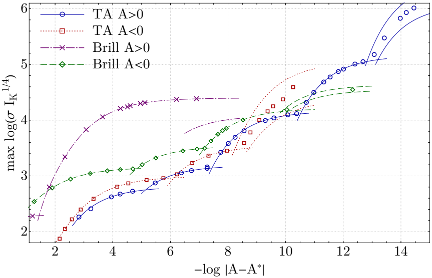
Khirnov, Ledvinka. [Class.Quant.Grav.35, (2018) 215003], Ledvinka, Khirnov, [Phys.Rev.Lett.127, (2021) 011104]

The Prague code:

- Cartoon, FD within ET.
- 'Quasimaximal' slicing.
- Optimized in assembly!

Tuning to threshold:

- Power-laws; not universal.
- Repeated curvature features.



Prague - universal curvature features

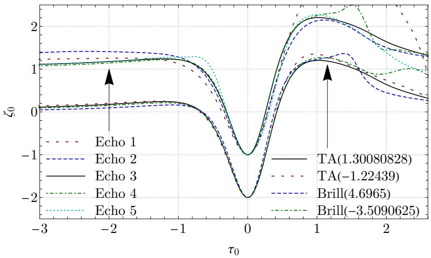
Khirnov, Ledvinka. [Class.Quant.Grav.35, (2018) 215003], Ledvinka, Khirnov, [Phys.Rev.Lett.127, (2021) 011104]

The Prague code:

- Cartoon, FD within ET.
- 'Quasimaximal' slicing.
- Optimized in assembly!

Tuning to threshold:

- Power-laws; not universal.
- Repeated curvature features.



bamps - off-center Brill waves

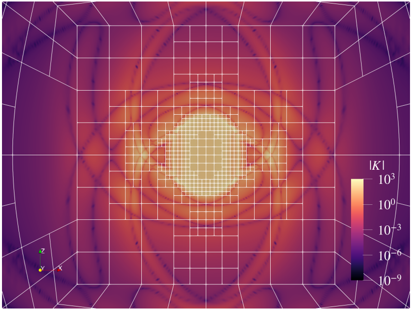
Suárez Fernández, Cors, Renkhoff, Brügmann, DH. [Phys.Rev.D106, (2022) 024036],

Renkhoff, Cors, Brügmann, DH. [Phys.Rev.D107, (2023) 104043].

hp-refinement bamps upgrade reduces cost by $O(10)$ factor. Complete rewrite of AH finder.

Tuning to threshold:

- Off-center AHs retained.
- Power-laws; not universal.
- Repeated curvature features.



bamps - off-center Brill waves

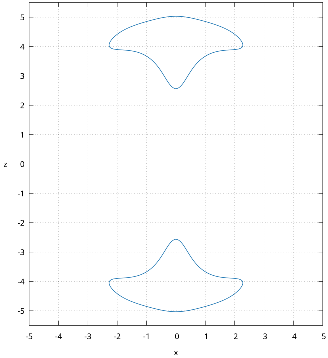
Suárez Fernández, Cors, Renkhoff, Brügmann, DH. [Phys.Rev.D106, (2022) 024036],

Renkhoff, Cors, Brügmann, DH. [Phys.Rev.D107, (2023) 104043].

hp-refinement bamps upgrade reduces cost by $O(10)$ factor. Complete rewrite of AH finder.

Tuning to threshold:

- Off-center AHs retained.
- Power-laws; not universal.
- Repeated curvature features.



bamps - off-center Brill waves

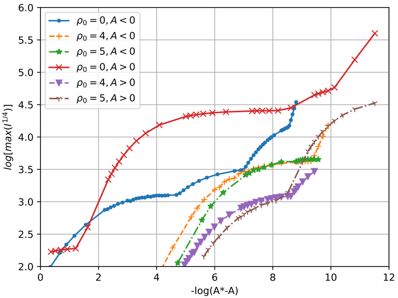
Suárez Fernández, Cors, Renkhoff, Brügmann, DH. [Phys.Rev.D106, (2022) 024036],

Renkhoff, Cors, Brügmann, DH. [Phys.Rev.D107, (2023) 104043].

hp-refinement bamps upgrade reduces cost by $O(10)$ factor. Complete rewrite of AH finder.

Tuning to threshold:

- Off-center AHs retained.
- Power-laws; not universal.
- Repeated curvature features.



bamps - off-center Brill waves

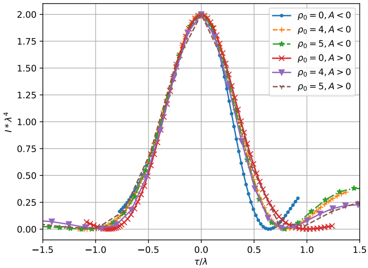
Suárez Fernández, Cors, Renkhoff, Brügmann, DH. [Phys.Rev.D106, (2022) 024036],

Renkhoff, Cors, Brügmann, DH. [Phys.Rev.D107, (2023) 104043].

hp-refinement bamps upgrade reduces cost by $O(10)$ factor. Complete rewrite of AH finder.

Tuning to threshold:

- Off-center AHs retained.
- Power-laws; not universal.
- Repeated curvature features.



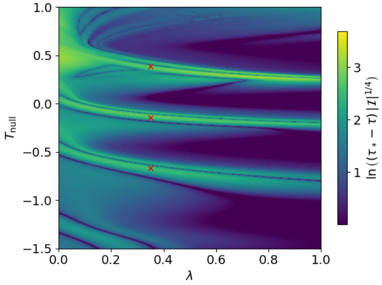
SphGR - quadrupolar & hexadecapolar waves

Baumgarte, DH. [Phys.Rev.D106, (2022) 044014], Baumgarte, Gundlach, DH [Phys.Rev.D107, (2023) 084012].

Shock-avoiding slicing condition allows better tuning with SphGR.

Families with accumulation at center?

- Yes, with *approximate* DSS.
- Observe power-law with wiggle.



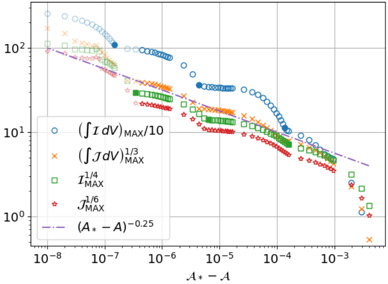
SphGR - quadrupolar & hexadecapolar waves

Baumgarte, DH. [Phys.Rev.D106, (2022) 044014], Baumgarte, Gundlach, DH [Phys.Rev.D107, (2023) 084012].

Shock-avoiding slicing condition allows better tuning with SphGR.

Families with accumulation at center?

- Yes, with *approximate* DSS.
- Observe power-law with wiggle.

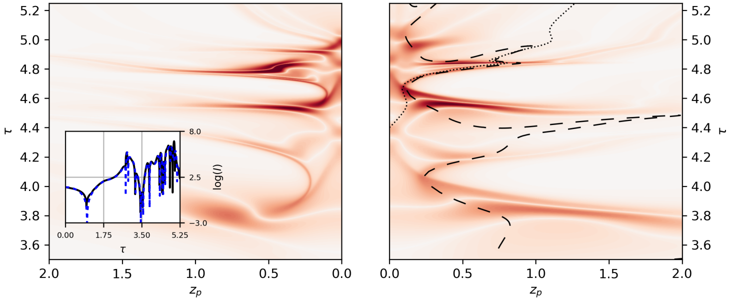


The end of universality

Baumgarte, Brügmann, Cors, Gundlach, DH, Renkhoff, Khirnov, Ledvinka, Suárez Fernández. [arXiv:2305.17171].

Code comparison $\{\texttt{bamps}, \texttt{prague}, \texttt{sphGR}\}$:

- Quantitative agreement in canonical coordinates.
- Uniqueness of threshold solutions lost.
- No evidence for exact DSS.

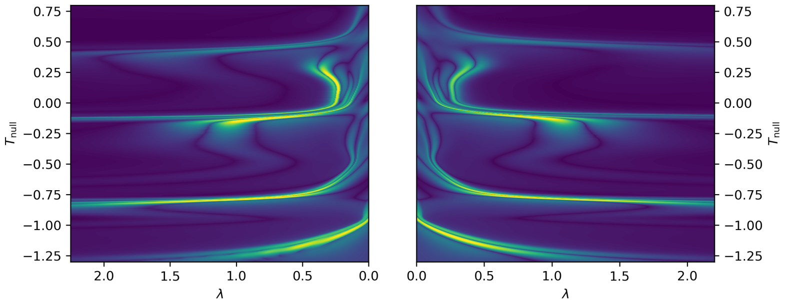


The end of universality

Baumgarte, Brügmann, Cors, Gundlach, DH, Renkhoff, Khirnov, Ledvinka, Suárez Fernández. [arXiv:2305.17171].

Code comparison $\{\texttt{bamps}, \texttt{prague}, \texttt{sphGR}\}$:

- Quantitative agreement in canonical coordinates.
- Uniqueness of threshold solutions lost.
- No evidence for exact DSS.

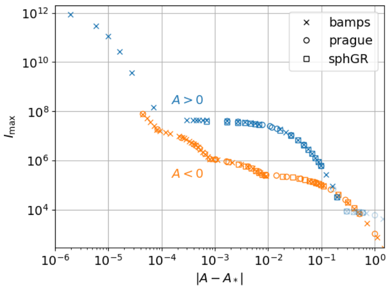


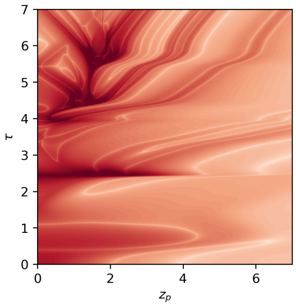
The end of universality

Baumgarte, Brügmann, Cors, Gundlach, DH, Renkhoff, Khirnov, Ledvinka, Suárez Fernández. [arXiv:2305.17171].

Code comparison $\{\texttt{bamps}, \texttt{prague}, \texttt{sphGR}\}$:

- Quantitative agreement in canonical coordinates.
- Uniqueness of threshold solutions lost.
- No evidence for exact DSS.





Conclusions

We seek a complete picture of the threshold of blow-up in vacuum!

- Mathematical and numerical relativity offer complimentary approaches to our understanding of GR. Extreme spacetimes require *ever more sophisticated* methods. We attack both.
- In spherical symmetry the threshold of gravitational collapse is well understood. In the more general setting *a new picture* is emerging. Limited aspects of the phenomenology survive.
- The deliverables of our research program have broad utility for extreme spacetimes, compact binaries and gravitational wave astronomy.