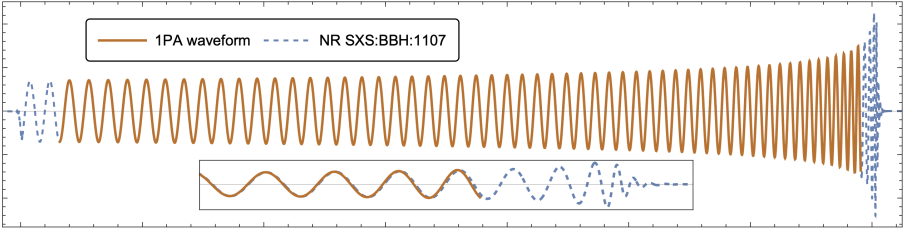
Status of post-adiabatic EMRI modelling



Wardell, Pound, Warburton, Durkan, Miller, Le Tiec [arXiv:2112.12265]

Leor Barack (Southampton)

Self-force and adiabatic expansion 3-layer structure

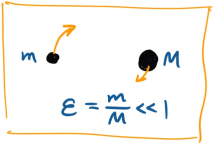
$$G_{\mu\nu}(g_{\alpha\beta})=0 \quad /. \quad g_{\alpha\beta}=g^{\mathrm{KERR}}_{\alpha\beta}+\varepsilon h^{(1)}_{\alpha\beta}+\varepsilon^2 h^{(2)}_{\alpha\beta}+\dots$$

SELF-FORCE THEORY

from PDEs to point-particle orbits

$$\ddot{x}^\alpha=\varepsilon F^\alpha_{(1)}+\varepsilon^2 F^\alpha_{(2)}+\dots$$

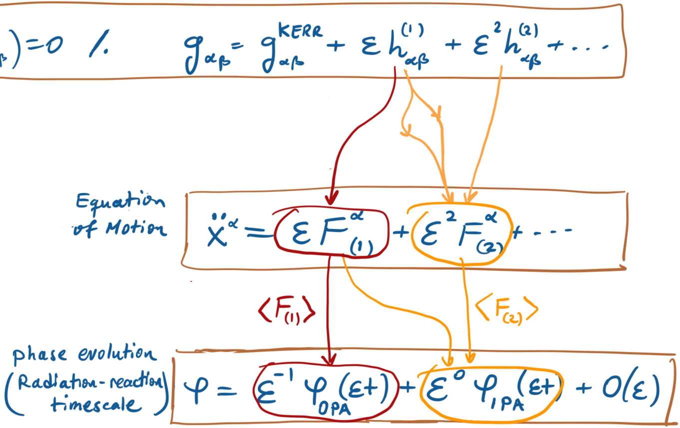
TWO-TIMESCALE/ADIABATIC EXPANSION



$$\varphi=\varepsilon^{-1}\varphi_{0PA}(\varepsilon t)+\varepsilon^{0}\varphi_{1PA}(\varepsilon t)+O(\varepsilon)$$

Should suffice for parameter extraction if ε sufficiently small

Self-force and adiabatic expansion



Rapid waveforms

- Treat $h^{(n)}_{\alpha\beta}$ as functions on extended manifold:

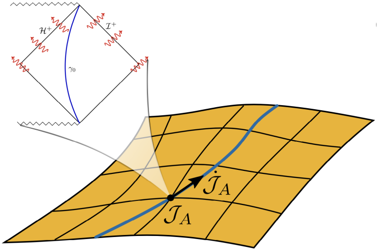
$$h_{\alpha\beta}(t,x^i) \to \sum_{n=1}^{\infty} \epsilon^n h^{(n)}_{\alpha\beta}(x^i;\mathcal{J}_A,\varphi_A)$$

where $\mathcal{J}_A = (J_A, M_{\rm BH}, J_{\rm BH}, \ldots)$.

- With a suitable choice of (J_A,φ_A):

$$h^{(n)}_{\alpha\beta} = \sum_{k^A} h^{(n)\Omega_k}_{\alpha\beta}(x^i;\mathcal{J}_A)e^{-ik^A\varphi_A}$$

with $\Omega_k := k^A\dot{\varphi}_A$.



Offline step: Solve field equations for amplitudes $h^{(n)\Omega_k}_{\alpha\beta}$ on grid of $\mathcal{J}_A$ values.

Online step (FEW): Rapidly evolve through parameter space

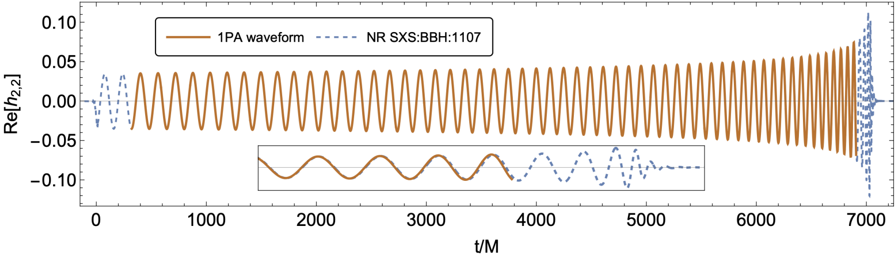
$$\begin{aligned}\dot{\varphi}_A &= \Omega_A(\mathcal{J}_B)\\ \dot{\mathcal{J}}_A &= \epsilon\tilde{F}^{(0)}(\mathcal{J}_B) + \epsilon^2\tilde{F}^{(1)}(\mathcal{J}_\mathcal{B}) + \cdots\end{aligned}$$

Status summery

- **OPA:**
  - Full-SF generic inspirals in Kerr are available (numerical $h^{(0)\Omega_k}_{\alpha\beta}$).
  - FEW implemented in Schwarzschild with full SF
  - FEW implemented in Kerr with 5.5PN fluxes and AAK waveform amplitudes
  - ("Soon") FEW for equatorial inspirals in Kerr with full SF
- **1PA:**
  - Quasicircular inspiral in Schwarzschild, generic/precessing CO spin
  - Quasicircular inspiral with slowly-spinning primary, aligned CO spin
- **Synergies**
  - With PN: High-order PA augmentation of 1PA SF model
  - With EOB: benchmarking & calibration for EOB
  - With NR: benchmarking for SF; NR+SF worldtube excision model
- **Misc.**
  - Advanced methods for solving field equations
  - Work on transition to plunge, merger & ringdown—relevant for IMRIs.

1PA waveforms

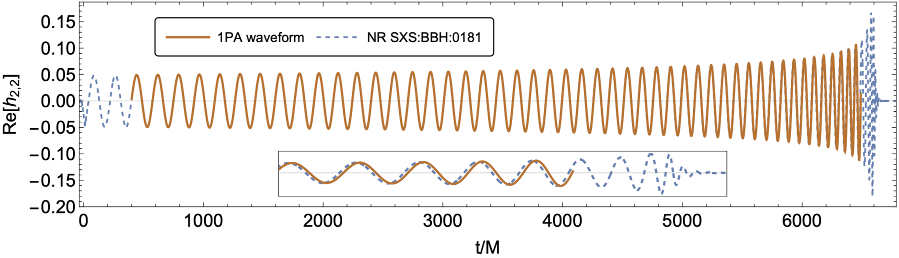
Mass ratio $\epsilon = 1/10$



(Wardell, Pound, Warburton, Durkan, Miller, Le Tiec)

1PA waveforms

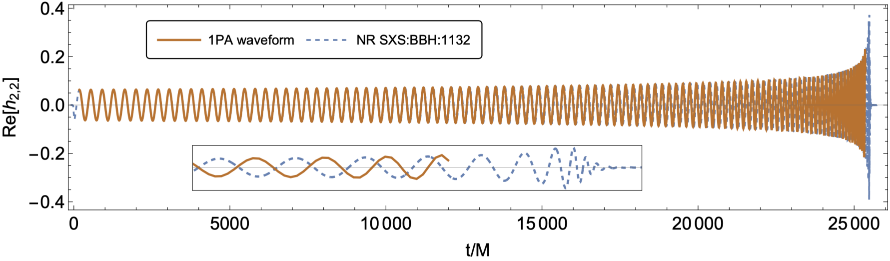
Mass ratio $\epsilon = 1/6$



(Wardell, Pound, Warburton, Durkan, Miller, Le Tiec)

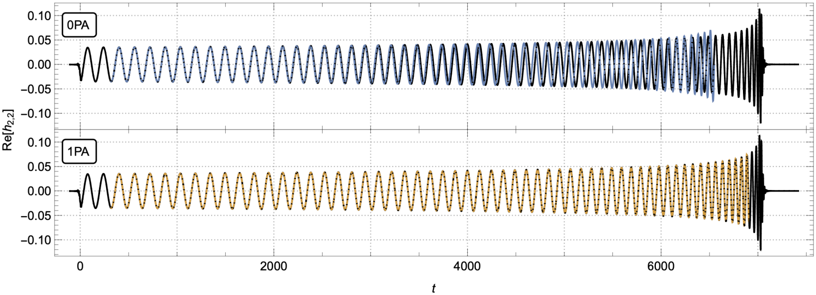
1PA waveforms

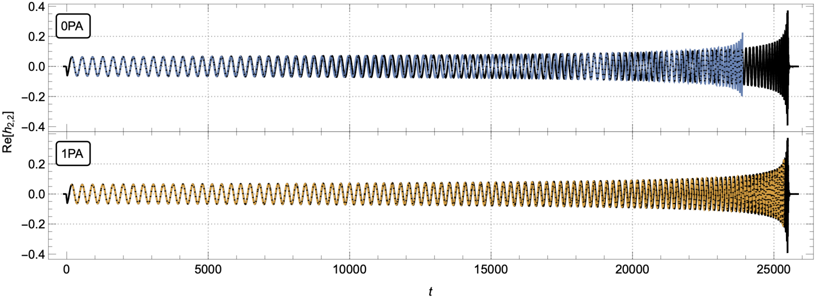
Mass ratio $\epsilon = 1$



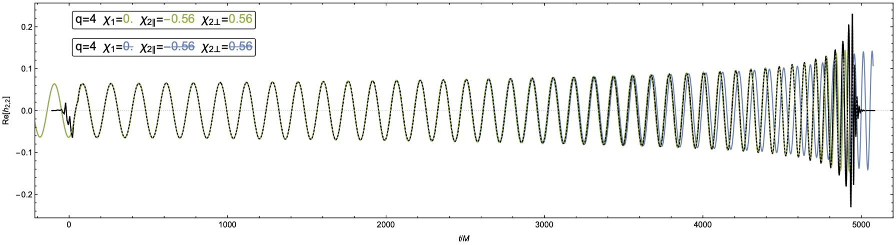
Error estimate: $\sim 7.5\epsilon$ rad from $R = 20M$ to ISCO

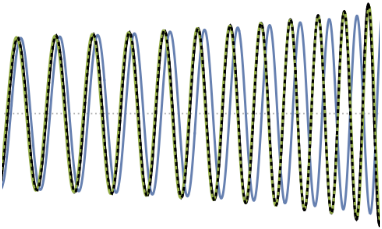
0PA vs. 1PA [$\epsilon = 1:10$ (top), $1:1$ (bottom)]

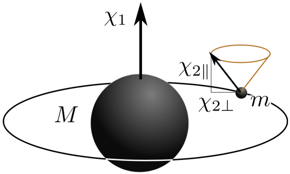




1PA inspiral with CO spin

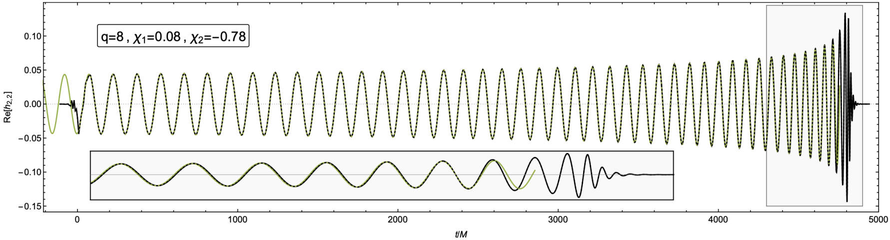


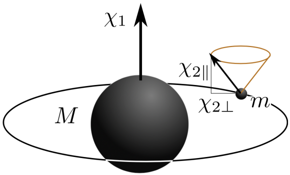




(Mathews, Pound, Wardell)

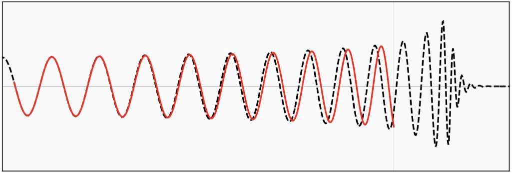
1PA inspiral with small MBH spin and (anti-)aligned CO spin



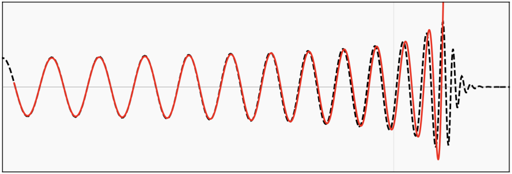


(Mathews, Pound, Wardell)

Transition to plunge



$$\frac{d\Omega}{dt} = F^{(0)}_{\Omega}$$
$$h_{lm} = \epsilon h^{(1)}_{lm}(\Omega) e^{-im\phi_p}$$

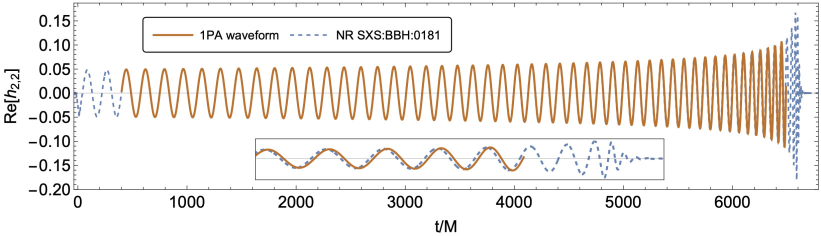


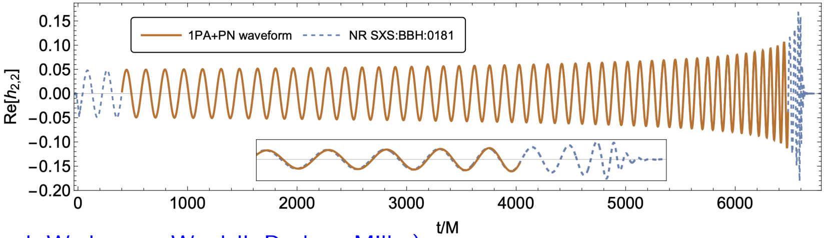
$$\frac{d\Delta\tilde{\Omega}}{dt} = \epsilon^{1/5}\left(F^{(0)}_{\Delta\tilde{\Omega}} + \epsilon^{2/5} F^{(2)}_{\Delta\tilde{\Omega}}\right)$$
$$h_{lm} = \epsilon\Big(j^{(0)}_{lm} + \epsilon^{2/5} j^{(2)}_{lm} + \epsilon^{3/5} j^{(3)}_{lm}\Big) e^{-im\phi_p}$$

(Compere, Durkan, Kuchler, Pound)

5PA PN augmentation of 1PA SF model

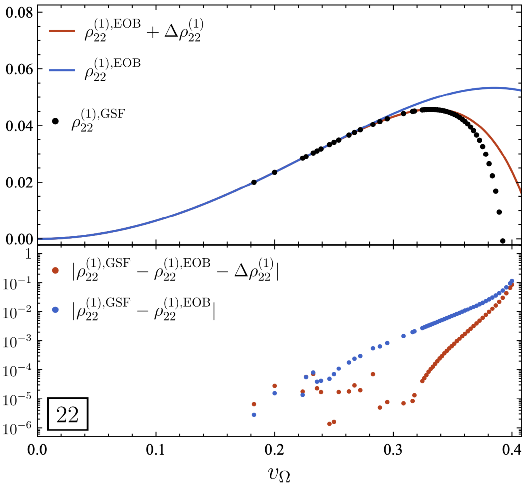
Mass ratio $\epsilon = 1/6$





(Pound, Warburton, Wardell, Durkan, Miller)

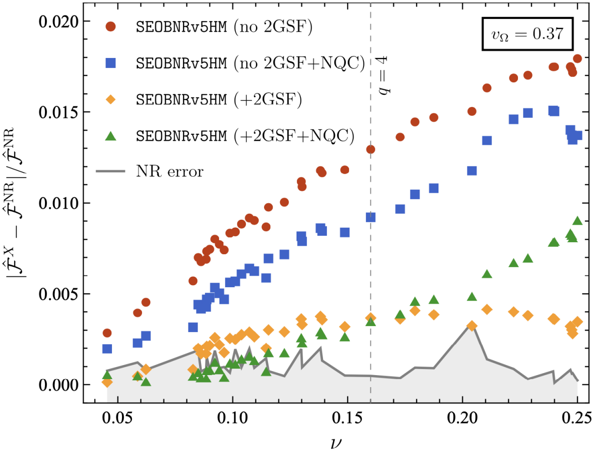
EOB Calibration modal flux, quasicircular inspiral



(Van de Meent et al 2303.18026)

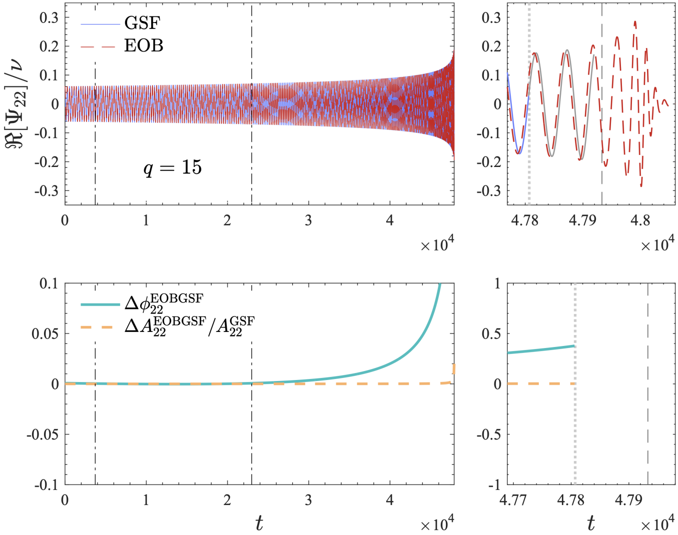
$$\Delta\rho_{22}^{(1)} = 21.2 v_\Omega^8 - 411 v_\Omega^{10}$$

EOB Calibration modal flux, quasicircular inspiral



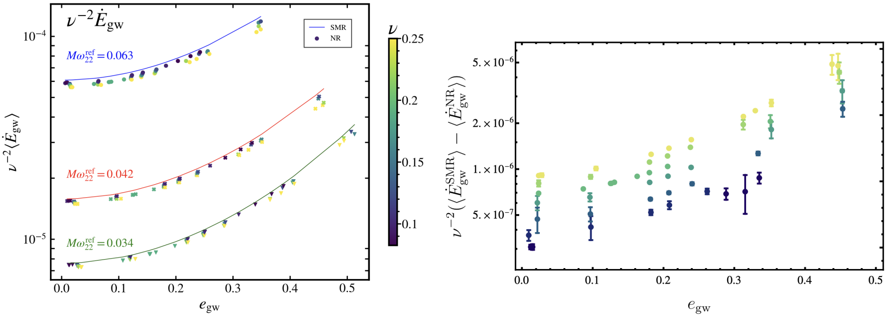
(Van de Meent et al 2303.18026)

TEOBResumS benchmarking



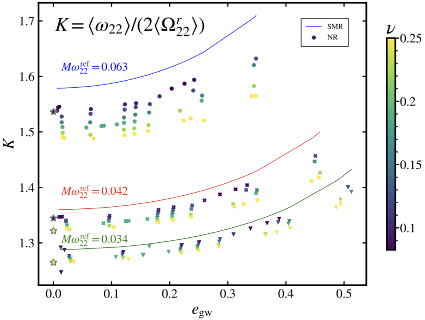
(Albertini et al 2208.01049, 2208.02055)

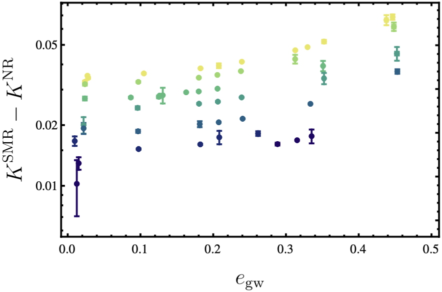
SF benchmarking using NR energy flux, eccentric inspiral



(A. Ramos-Buades et al. 2209.03390)

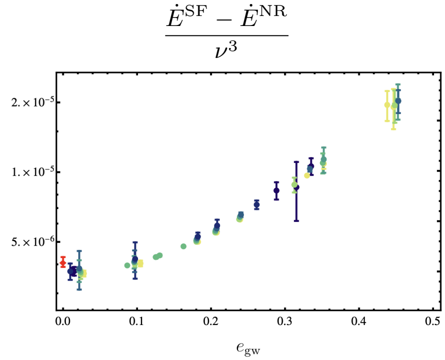
SF benchmarking using NR perisatron advance, eccentric inspiral

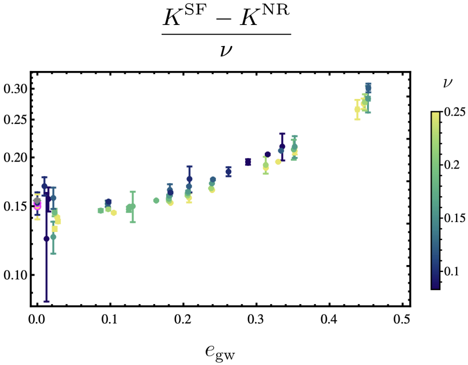




(A. Ramos-Buades et al. 2209.03390)

$\geq$ 2PA contributions are strangely small



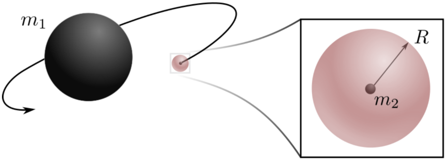


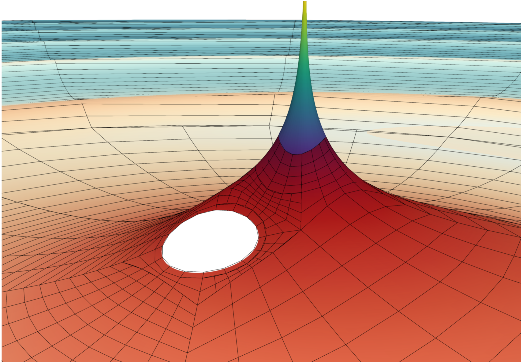
$$\frac{\dot{E}^{\rm SF} - \dot{E}^{\rm NR}}{\nu^3} = f_{\rm 1PA}(e) + f_{\rm 2PA}(e)\nu + f_{\rm 3PA}(e)\nu^2 + \cdots$$

Weak ν dependence $\Rightarrow$ $f_{\geq \rm 2PA}$ very small

Putting SF and NR together: worldtube excision for IMRIs

(Dhesi, Wittek, LB, Pfeiffer, Pound, Rüter)





PA modelling and Environmental Effects

- Variety of EEs potentially discernible at 1PA
- Some EEs can be incorporated as sourcing terms within existing PA scheme
- But EEs can also come with new time/lengthscales, requiring new types of multiple-scale expansions
- Degeneracy with/between EEs resolved with multiple EMRI observations
- Whether EEs are signal to be extracted or noise to be removed, 1PA-accuracy models of clean EMRIs will be crucial
- (Also good value for money, given ≥ 2PA terms appear to be negligible)