Dynamical friction in Gravitational Atoms

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1st Trieste meeting on the Physics of GWs

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Gravitational Atoms @ University of Amsterdam:

H.S. Chia, R. Porto, D. Baumann, G. Bertone, J. Stout, G.M.T., T. Spieksma

"Probing Ultralight Bosons with Binary Black Holes"
arXiv:1804.03208 PRD

"The Spectra of Gravitational Atoms"
arXiv:1908.10370, JCAP

"Gravitational Collider Physics"
arXiv:1912.04932, PRD

"Ionization of Gravitational Atoms"
arXiv:2112.14777, PRD

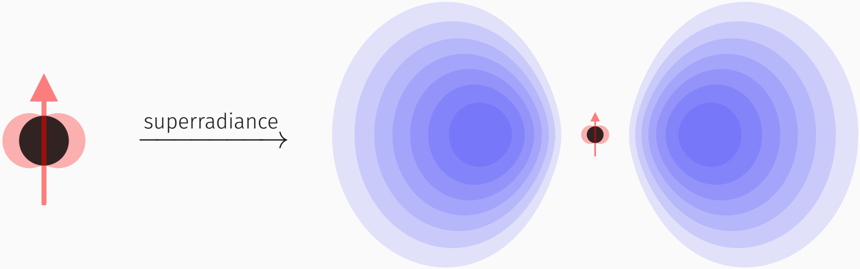
"Sharp Signals of Boson Clouds in Black Hole Binary Inspirals"
arXiv:2206.01212, PRL

"Dynamical Friction in Gravitational Atoms"
arXiv:2305.15460

Outline

- Review of ionization of gravitational atoms.
- Discussion on dynamical friction.
- New results on eccentric orbits.
- New results on inclined orbits.
- New results on dynamical capture cross section.

The Gravitational Atom

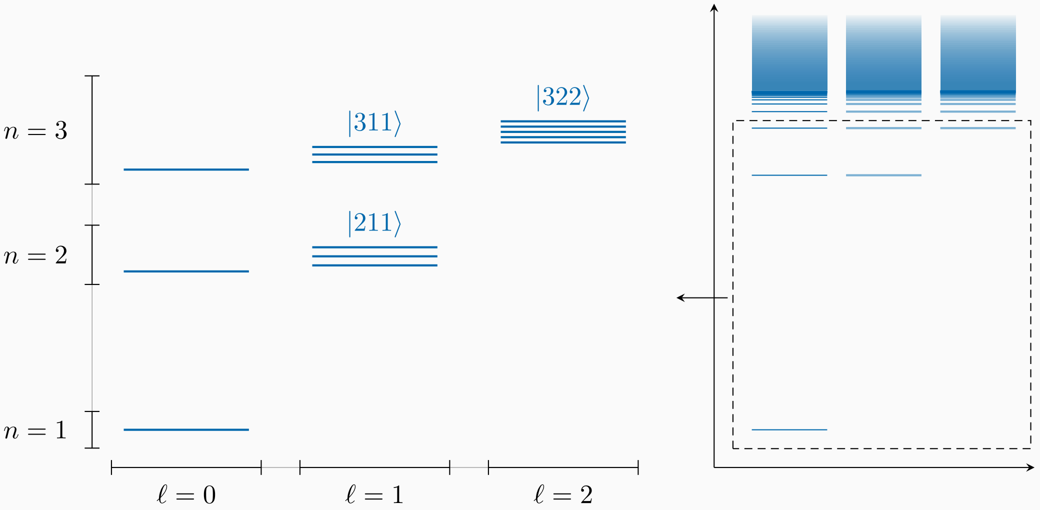


$$(\Box - \mu^2)\Phi = 0 \quad \longrightarrow \quad i\frac{\mathrm{d}\psi}{\mathrm{d}t} \approx \left(-\frac{1}{2\mu}\nabla^2 - \frac{\alpha}{r}\right)\psi$$

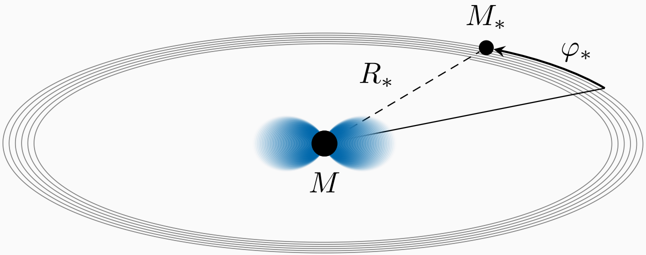
Gravitational fine structure constant: $\alpha = \mu M \sim \mathcal{O}(0.1)$.

[Zeldovich '72; Starobinsky '73; Dolan '07; Arvanitaki et al. '09]

The spectrum



Perturbation with slowly increasing frequency:

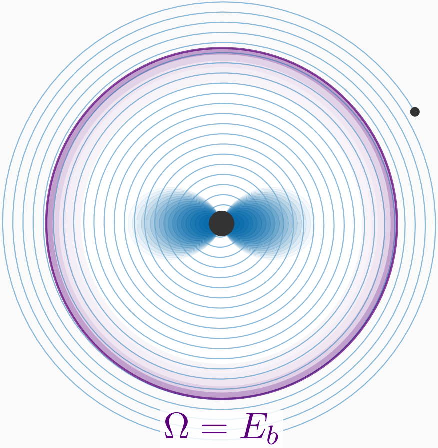


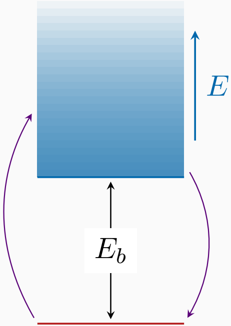
$$i\frac{\mathrm{d}\psi}{\mathrm{d}t} = \left(-\frac{1}{2\mu}\nabla^2 - \frac{\alpha}{r} + \underbrace{V_*(R_*, \varphi_*)}_{\text{perturbation}}\right)\psi$$

Level mixing:

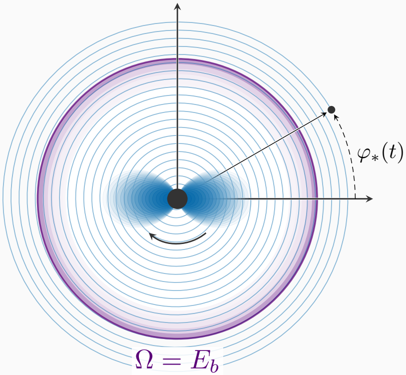
$$\langle a|\,V_*(t)|b\rangle = \sum_g \eta^{(g)} e^{-ig\Omega t}$$

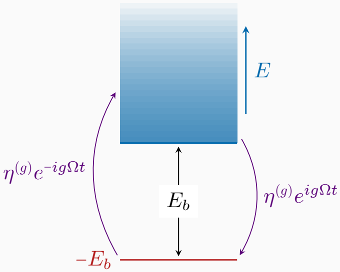
Orbital frequency above **threshold** to excite **transitions to unbound states**





Fermi's Golden Rule





The transition rate (per unit energy) is given by Fermi's Golden Rule:

$$\mathrm{d}\Gamma = \mathrm{d}E \underbrace{|\eta^{(g)}|^2}_{\text{Level mixing}} \delta(\underbrace{E - E_b - g\Omega}_{E - E_*^{(m)}})$$

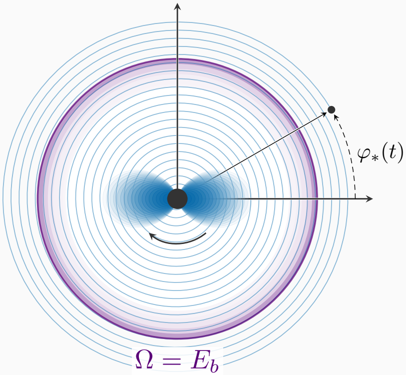
$$\mathrm{d}\Gamma_{\ell m} = \mathrm{d}E \underbrace{|\eta^{(g)}|^2}_{\text{Level mixing}} \delta(\underbrace{E - E_b - g\Omega(t)}_{E - E_*^{(m)}})$$

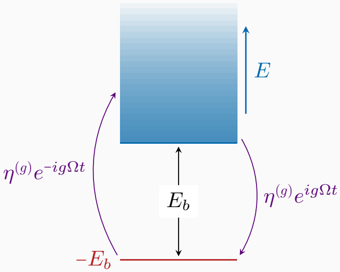
Multipole expansion:

$$\begin{aligned}\langle E; \ell m | V_*(t, \vec{r}) | n_b \ell_b m_b \rangle &= \sum_{\ell_*, m_*} \frac{4\pi\alpha q}{2\ell_* + 1} Y_{\ell_* m_*}(\theta_*, \varphi_*) \times I_r(R_*) \times I_\Omega \\ &= \sum_g \eta^{(g)} e^{-ig\Omega t}\end{aligned}$$

On **equatorial quasi-circular** orbits, $g = \pm(m - m_b)$.

Ionization Power

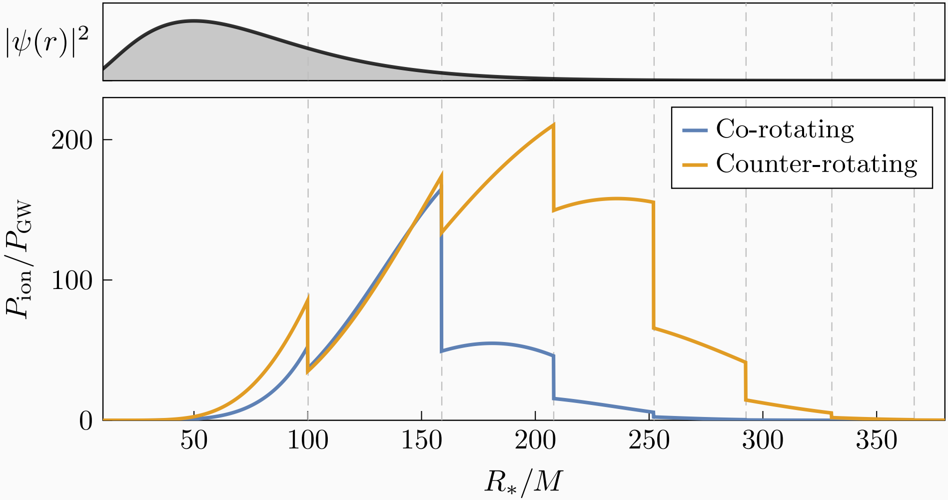




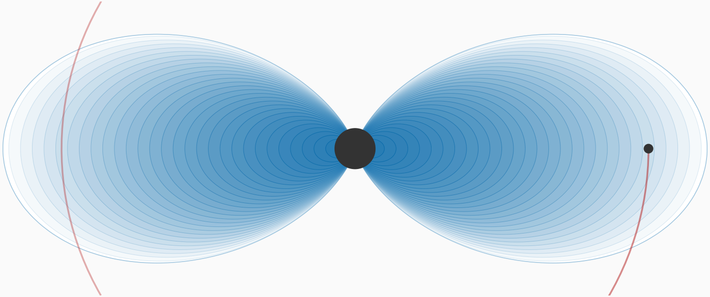
Summing over all bound states gives the total **ionization power**:

$$P_{\text{ion}} = \frac{M_c}{\mu} \sum_{\ell,m} g\Omega \, |\eta^{(g)}|^2 \, \Theta(E_*^{(m)})$$

Ionization plot

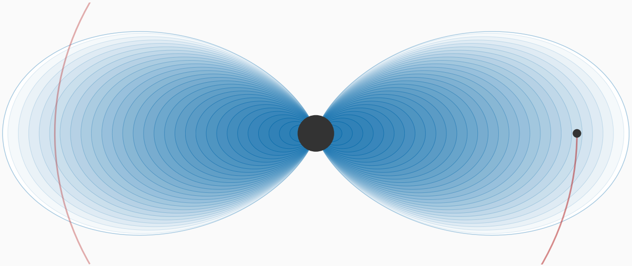


$[|211\rangle, \alpha = 0.2, M_c/M = 0.01, q = 10^{-3}]$



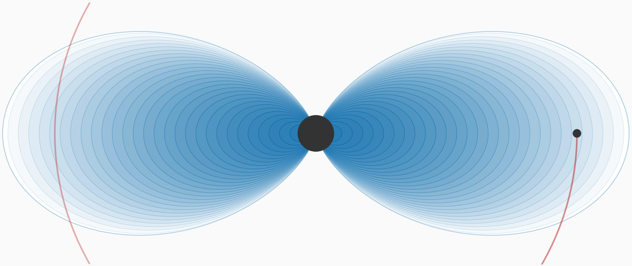
Ionization or **dynamical friction**?

Adapting P_{DF}



$$P_{\text{DF}} = \frac{4\pi M_*^2 \rho}{v} \log(v\mu b_{\text{max}})$$

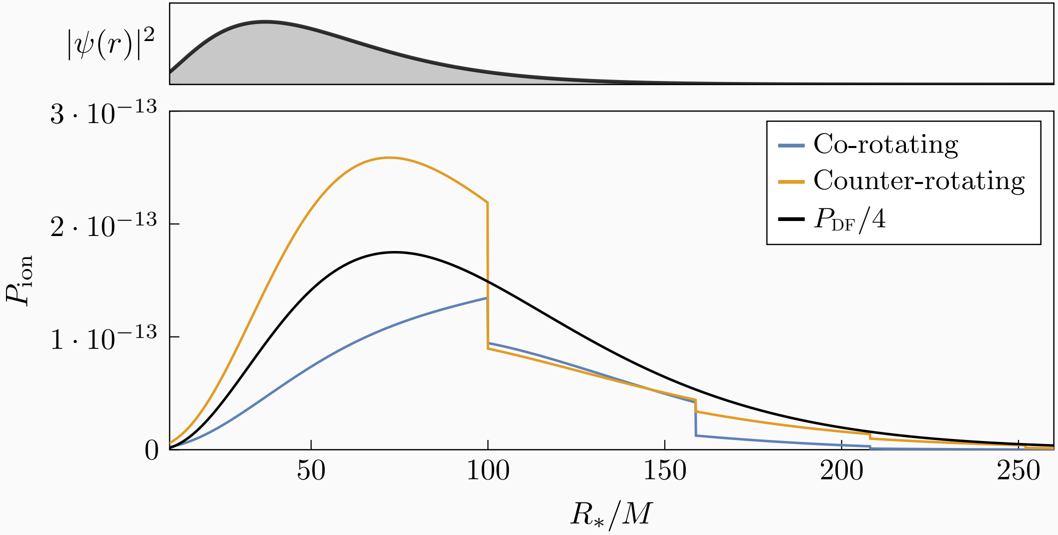
ADAPTING P_{DF}



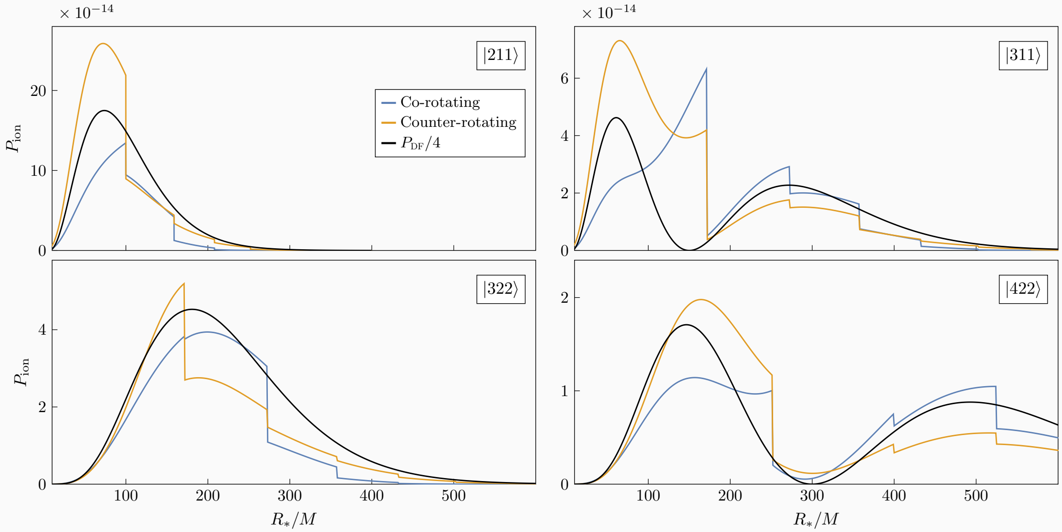
$$P_{\text{DF}} = \frac{4\pi M_*^2 \rho}{v} \log(v\mu b_{\max})$$

Need to fix: ρ, v, $b_{\max}$.

P_{ion} VS P_{DF}: NUMERICAL



P_{ion} VS P_{DF}: NUMERICAL



P_{ion} vs P_{DF}: physical arguments

- $P_{\mathrm{ion}}/P_{\mathsf{DF}}$ roughly independent of the state;
- $P_{\mathrm{ion}}/P_{\mathsf{DF}}$ independent of the parameters:

$$P_{\mathsf{DF}} \sim P_{\mathrm{ion}} = \alpha^5 q^2 \frac{M_{\mathrm{c}}}{M} \mathcal{P}(\alpha^2 R_*/M)$$

- Same physical interpretation:

$$P_{\mathsf{DF}} \sim P_{\mathrm{ion}} = \int_{\partial V} T^{0i} \, \mathrm{d}S$$

- What does P_{DF} fail to describe?

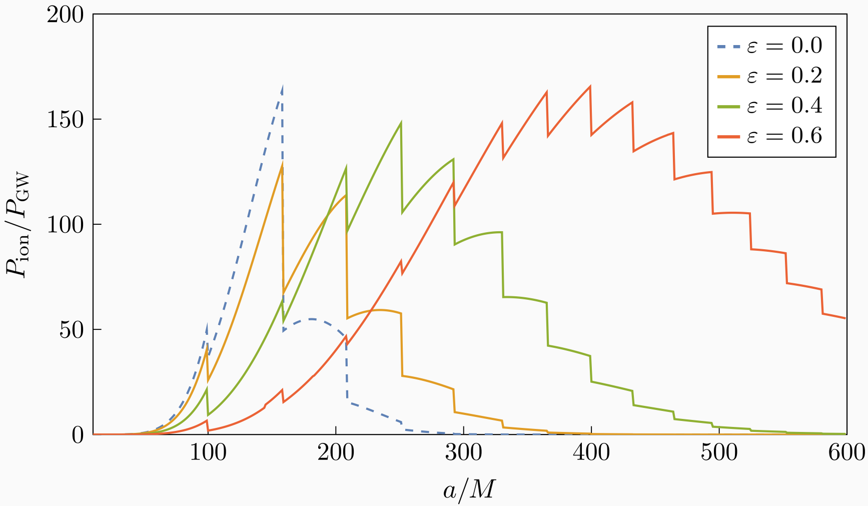
Ionization on eccentric orbits

$$\langle E;\ell m|V_*(t,\vec{r})|n_b\ell_b m_b\rangle = \sum_{\ell_*,m_*}\frac{4\pi\alpha q}{2\ell_*+1}\overbrace{Y_{\ell_* m_*}(\theta_*,\varphi_*)\times I_r(R_*)}^{\text{not monochromatic}}\times I_\Omega$$
$$= \sum_g \eta^{(g)}\, e^{-ig\varphi_*(t)}$$

- g is now independent of m;
- no (simple) formula for $\eta^{(g)}$.

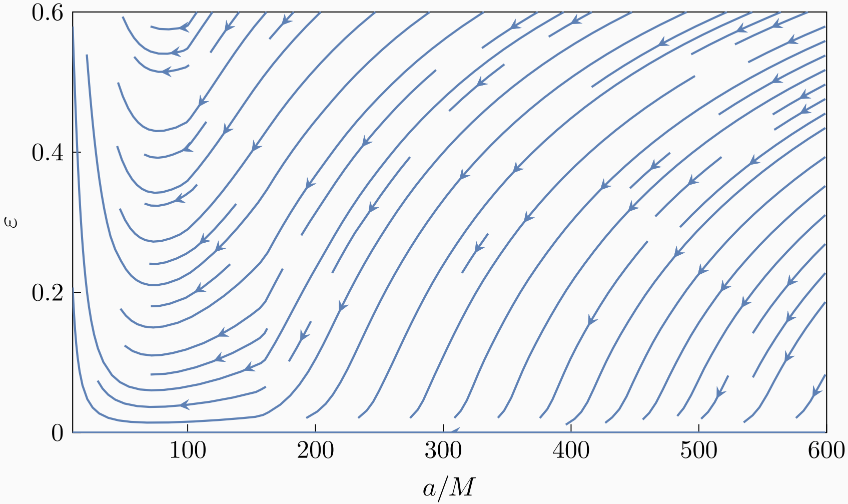
$$P_{\text{ion}} = \frac{M_{\text{c}}}{\mu}\sum_{\ell,m,g} g\Omega\ |\eta^{(g)}|^2\ \Theta(E_*^{(m)})$$

Ionization plot on eccentric orbits



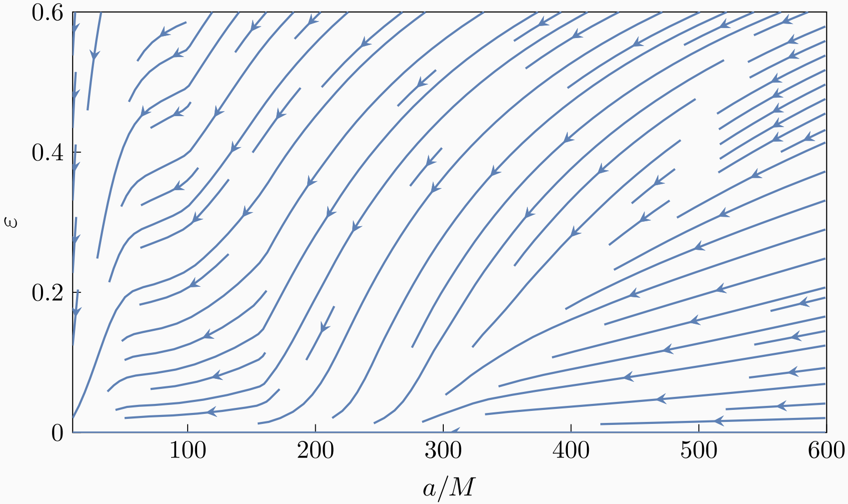
$[|211\rangle$, $\alpha = 0.2$, $M_c/M = 0.01$, $q = 10^{-3}$, equatorial co-rotating]

Evolution of eccentricity (no GWs)



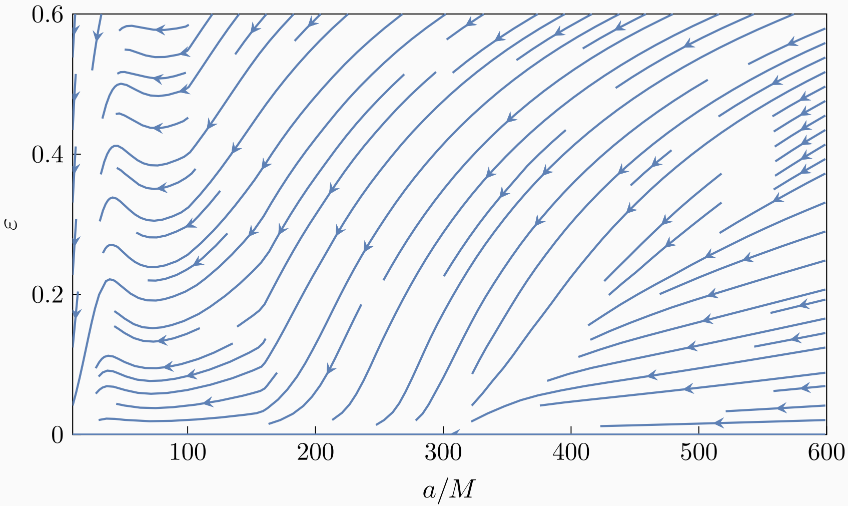
[$|211\rangle$, $\alpha = 0.2$, $q = 10^{-3}$, equatorial co-rotating]

Evolution of eccentricity



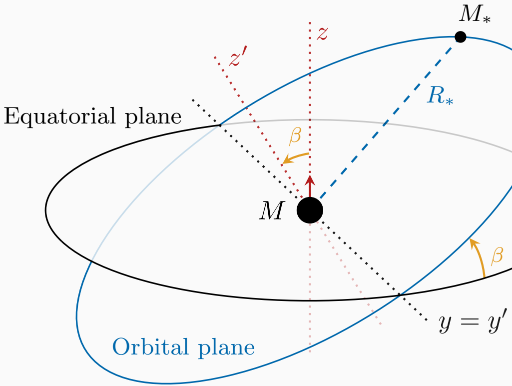
[$|211\rangle$, $\alpha = 0.2$, $M_c/M = 0.01$, $q = 10^{-3}$, equatorial co-rotating]

Evolution of eccentricity (higher mass)



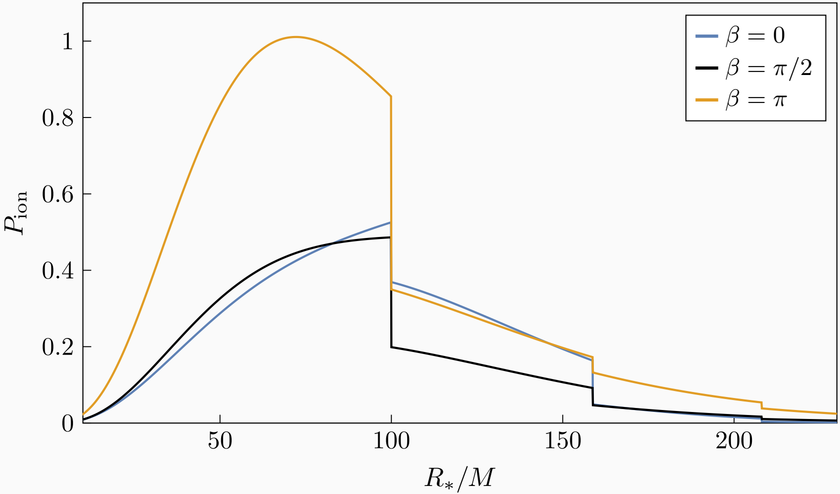
[$|211\rangle$, $\alpha = 0.2$, $M_c/M = 0.1$, $q = 10^{-3}$, equatorial co-rotating]

Ionization on inclined orbits

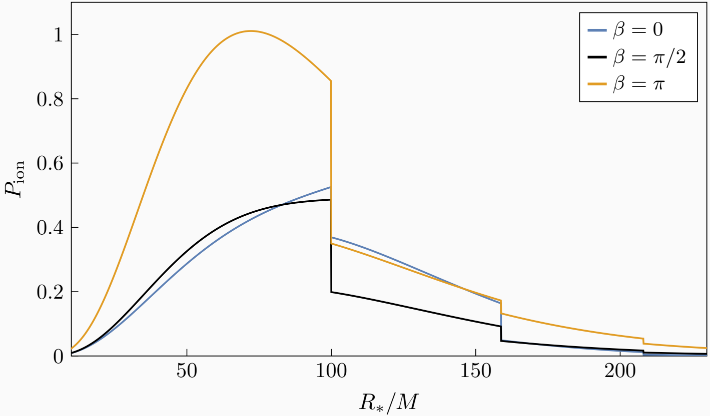


Precession? Evolution of β?

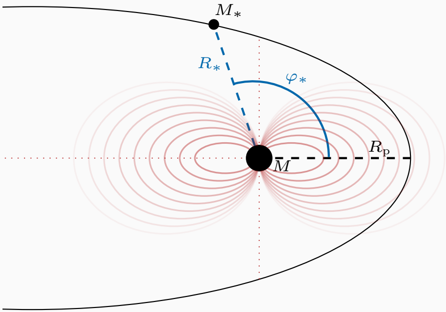
Ionization plot on inclined orbits



Ionization plot on inclined orbits



No precession! Negligible variation of β.

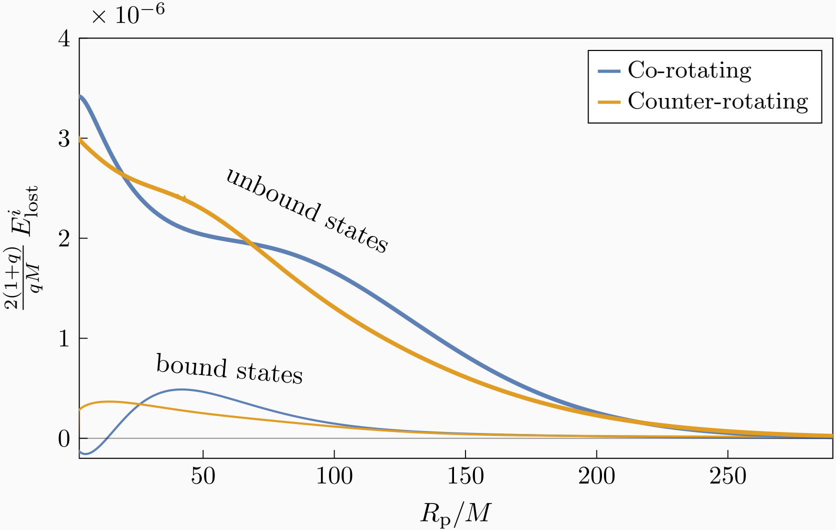


Eccentricity $\to 1$

$$\sigma_{\text{GW}} = 2\pi M^2 \left(\frac{85\pi}{6\sqrt{2}}\right)^{2/7} q^{2/7}(1+q)^{10/7} v^{-18/7}$$

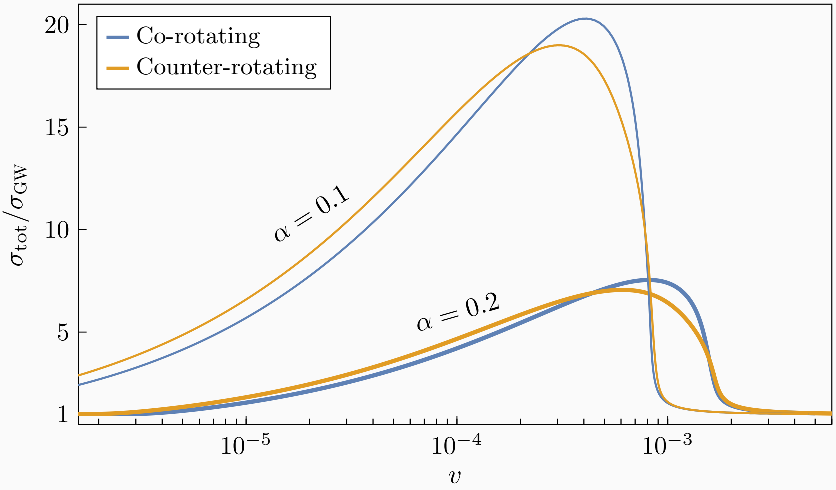
The cloud opens up a new channel for **energy loss**!

Energy lost



$\times 10^{-6}$
4
3
2
1
0
50
100
150
200
250
R_p/M
$\frac{2(1+q)}{qM} E^i_\mathrm{lost}$
Co-rotating
Counter-rotating
unbound states
bound states

Dynamical capture cross section



Summary

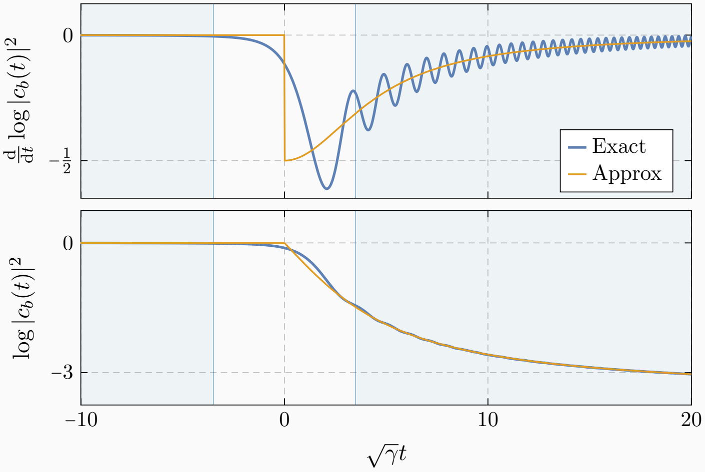
- (The backreaction of) **ionization** $=$ **dynamical friction**.
- Ionization generally circularizes orbits.
- Ionization generally doesn't affect the orbital plane.
- Dynamical capture cross section increases by $\gtrsim \mathcal{O}(10)$.

- This is a non-relativistic analysis.
- Resonances to be taken into account...

Backup

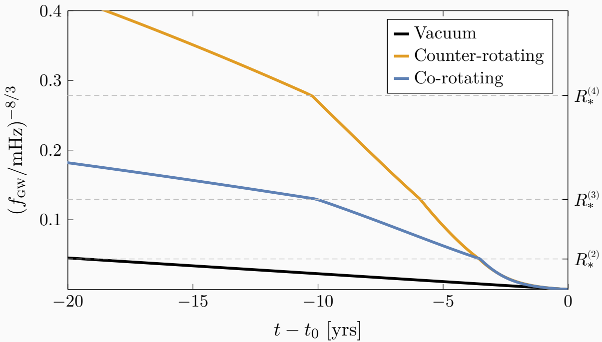
Discontinuities?

When $\Omega(t) \approx \Omega_0 + \gamma t$ "hits" the continuum, the deoccupation starts.



Kinks in the frequency

Kinks in the frequency evolution: **signature** of the cloud!



$$f_{\rm GW}^{(g)} = \frac{6.45\,\text{mHz}}{g}\left(\frac{10^4 M_\odot}{M}\right)\left(\frac{\alpha}{0.2}\right)^3\left(\frac{2}{n}\right)^2$$

$[M = 10^4 M_\odot$, $|211\rangle$, initial: $R_* = 400M$, $M_*/M = 10^{-3}$, $M_c/M = 10^{-2}]$

Ionization plot on inclined orbits