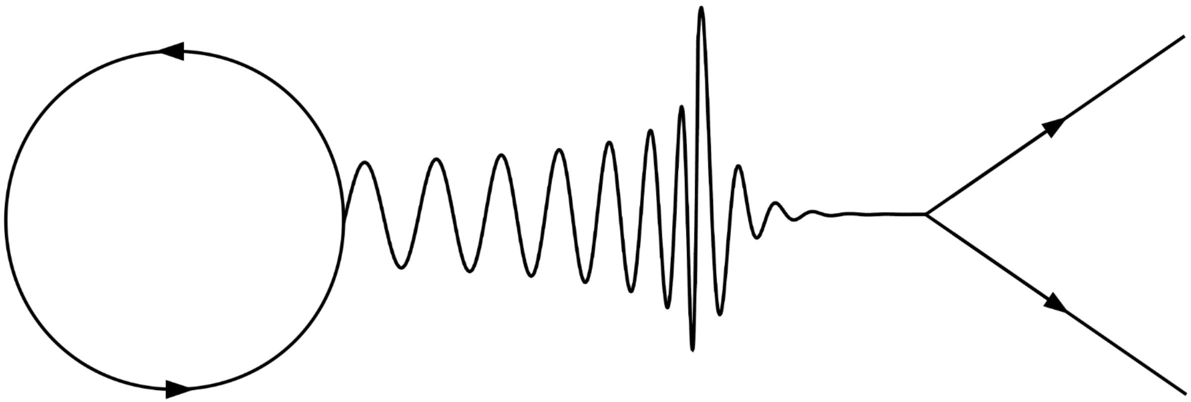
Black hole spectroscopy

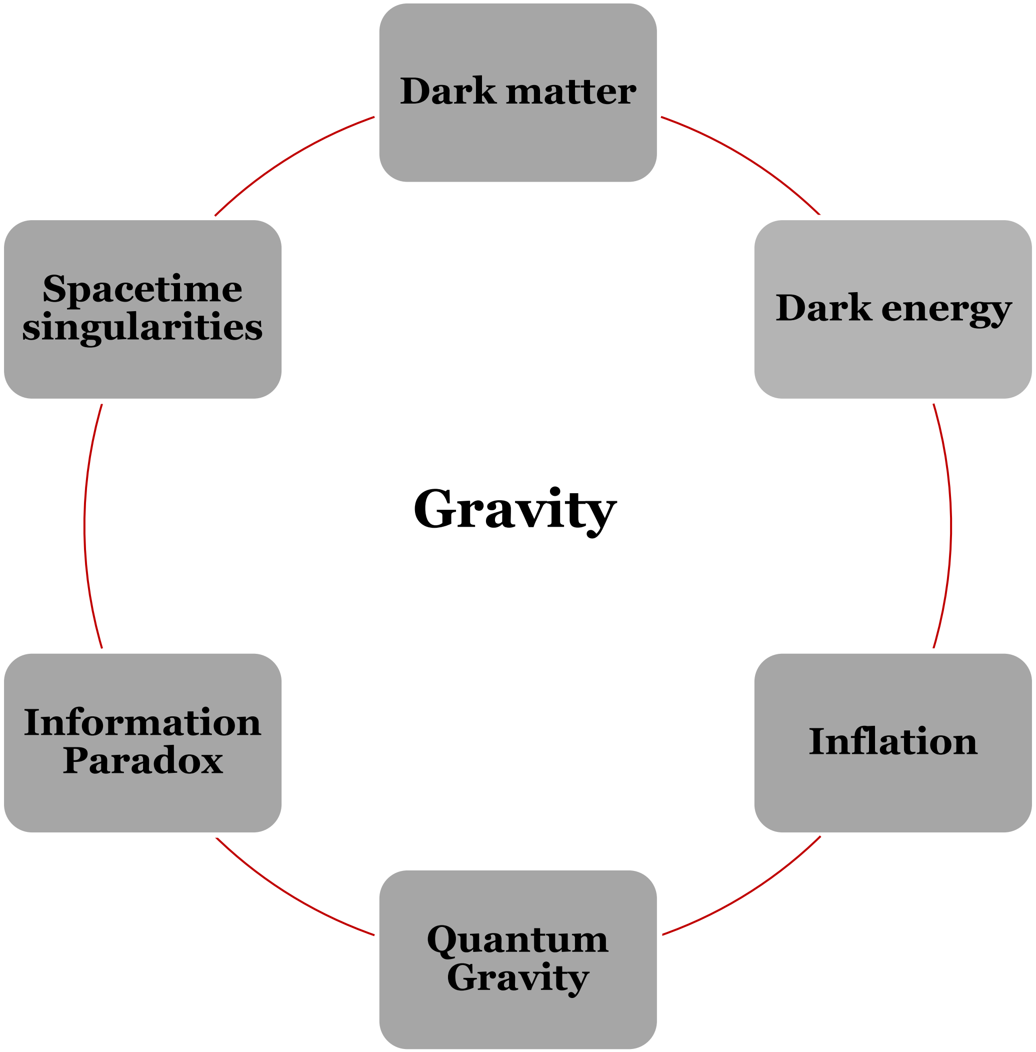


Vítor Cardoso

[Niels Bohr Institute & Técnico Lisbon]

Image: Cardoso & Pani
CERN Courier (2016)

strong. niels bohr institute
grit. gravitation in técnico
KØBENHAVNS UNIVERSITET
Danmarks Grundforskningsfond Danish National Research Foundation
VILLUM FONDEN
erc European Research Council


Dark matter
Dark energy
Inflation
Quantum Gravity
Information Paradox
Spacetime singularities
Gravity

Singularities & Cosmic Censorship

Theorem (Penrose 1965; 1969):

For "reasonable" matter, trapped surface formation results in "singularity," where at least one of the following holds:

a. Negative local energy occurs.
b. Einstein's equations are violated.
c. The space-time manifold is incomplete.
d. The concept of space-time loses its meaning at very high curvatures – possibly because of quantum phenomena.

Conjecture (Penrose 1969):

No singularity is visible from future null infinity (weak CCC)
General Relativity is deterministic (strong CCC)

Uniqueness: the Kerr solution

Theorem (Carter 1971; Robinson 1975; Chrusciel, Costa & Heusler 2012):
A stationary, asymptotically flat, *vacuum* BH solution must be Kerr

$$
\begin{aligned}
ds^2 \quad = \quad & \frac{\Delta - a^2 \sin^2\theta}{\Sigma} dt^2 + \frac{2a(r^2 + a^2 - \Delta)\sin^2\theta}{\Sigma} dt d\phi \\
& - \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2\theta}{\Sigma} \sin^2\theta d\phi^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2
\end{aligned}
$$

$$
\Sigma = r^2 + a^2 \cos^2\theta \,, \quad \Delta = r^2 + a^2 - 2Mr
$$

Describes a rotating BH with mass M and angular momentum J=aM, iff a<M

"In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein's equations of general relativity provides the *absolutely exact representation* of untold numbers of black holes that populate the universe."

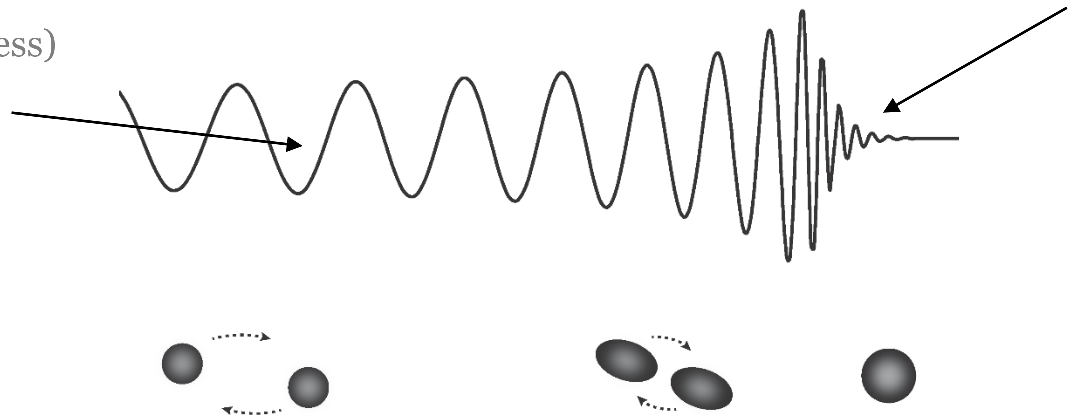
S. Chandrasekhar, The Nora and Edward Ryerson lecture, Chicago April 22 1975

Black holes as perfect laboratories

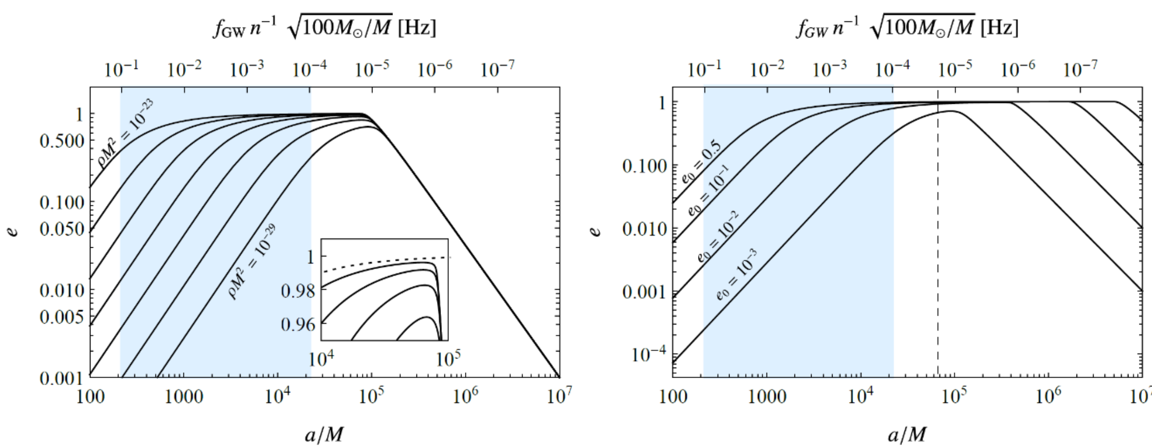
Simple spin-induced multipoles (uniqueness)

Tidal heating

Tidal Love number (BHs don't polarize)



Simple relaxation of final state (uniqueness)



Peters PR136: B1224 (1964);
Cardoso & Pani, Nature Astronomy 1: 586 (2017); Living Reviews in Relativity 22: 1 (2019)
Cardoso, Macedo & Vicente PRD103: 023015 (2021)

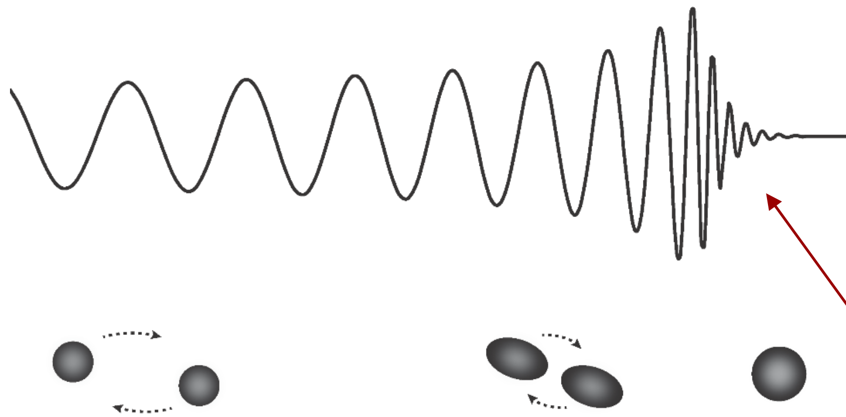
The next decimal place

“New realms will become visible by improving the accuracy of the numerical measurement of quantities with which one has long been familiar.“ [Maxwell, Scientific papers]

Test black hole & Kerr paradigm with precision & understand environments

Cardoso, Nature Reviews Physics (2019)
Cardoso and Pani, Living Reviews in Relativity 22: 1 (2019)

Black hole spectroscopy



Ringdown

$$\frac{\partial^2 \Psi}{\partial r_*^2} - \frac{\partial^2 \Psi}{\partial t^2} - V(r_*)\Psi = \mathrm{S} \qquad \sim e^{-i\omega t}$$

$$\left(\omega^2 - \mathcal{L}\right)\psi = s \rightarrow \psi = \left(\omega^2 - \mathcal{L}\right)^{-1} s$$

poles = QNMs

$$h = \sum_{n\ell m} A_{n\ell m} e^{-t/\tau_{n\ell m}} \sin(2\pi f_{n\ell m} t) Y_{\ell m}(\theta, \phi)$$

Leaver PRD34:384 (1986); Cardoso & Gualtieri CQG33: 174001 (2016); Baibhav + arXiv:2302.03050

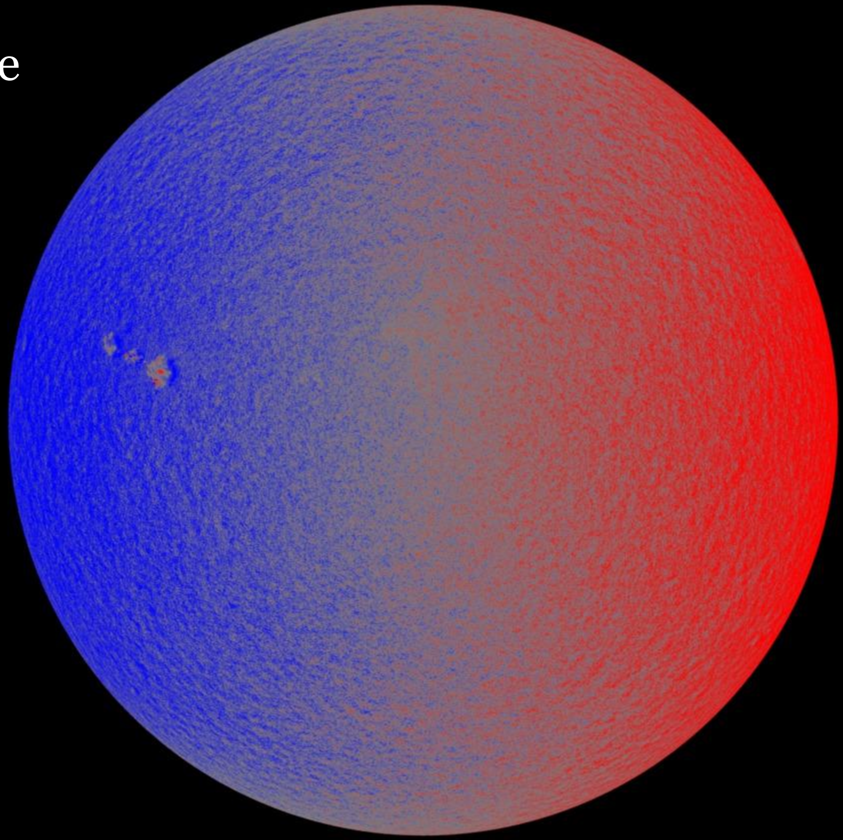
Helioseismology

Solar spectrum sensitive to radial structure, opacity, temperature…

More than 10^6 modes with periods between 4 and 6 minutes

Quality factors of order $10^6 - 10^9$

Driven by turbulence



Holeseismology...

The Astrophysical Journal, **239**:292–295, 1980 July 1
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BLACK HOLES AND GRAVITATIONAL WAVES. III. THE RESONANT FREQUENCIES OF ROTATING HOLES

Steven Detweiler
Physics Department, Yale University
Received 1979 October 15; accepted 1979 December 27

ABSTRACT

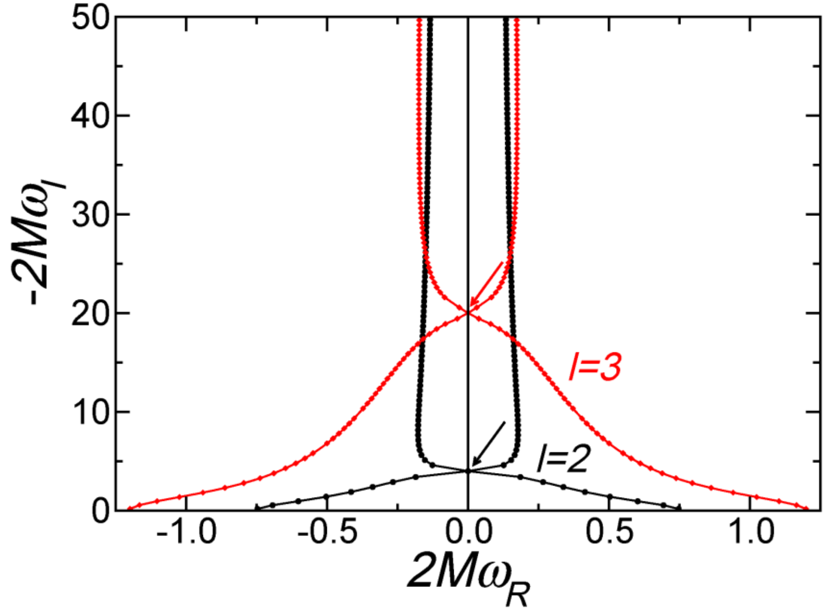
The free oscillations of rotating black holes are studied. Values of the complex resonant frequencies are given as functions of the Kerr parameter a for a variety of spherical harmonic indices l and m. The Appendix uses analytic methods to show that the maximally rotating Kerr solution is, in some sense, marginally unstable.

Subject headings: black holes — gravitation

IV. CONCLUSIONS

Lately black holes have been at the center of much scientific attention. But from the time of Schwarzschild's solution of Einstein's equations in 1915 and of Chandrasekhar's derivation of the maximum mass of white dwarfs in 1931 through the last decade of stimulating observational and theoretical research, all of the evidence for the existence of black holes has been indirect. There is evidence for some small but massive objects at the centers of some active galactic nuclei and others in binary systems associated with X-ray sources. But there is no direct observational evidence that these must be black holes.

In various theoretical studies, gravitational waves generated in the vicinity of black holes have shown a characteristic wave form of a damped sinusoid. In this paper we have given the complex frequencies of the free oscillations of rotating black holes for different spherical harmonic indices. After the advent of gravitational wave astronomy, the observation of these resonant frequencies might finally provide direct evidence of black holes with the same certainty as, say, the 21 cm line identifies interstellar hydrogen.



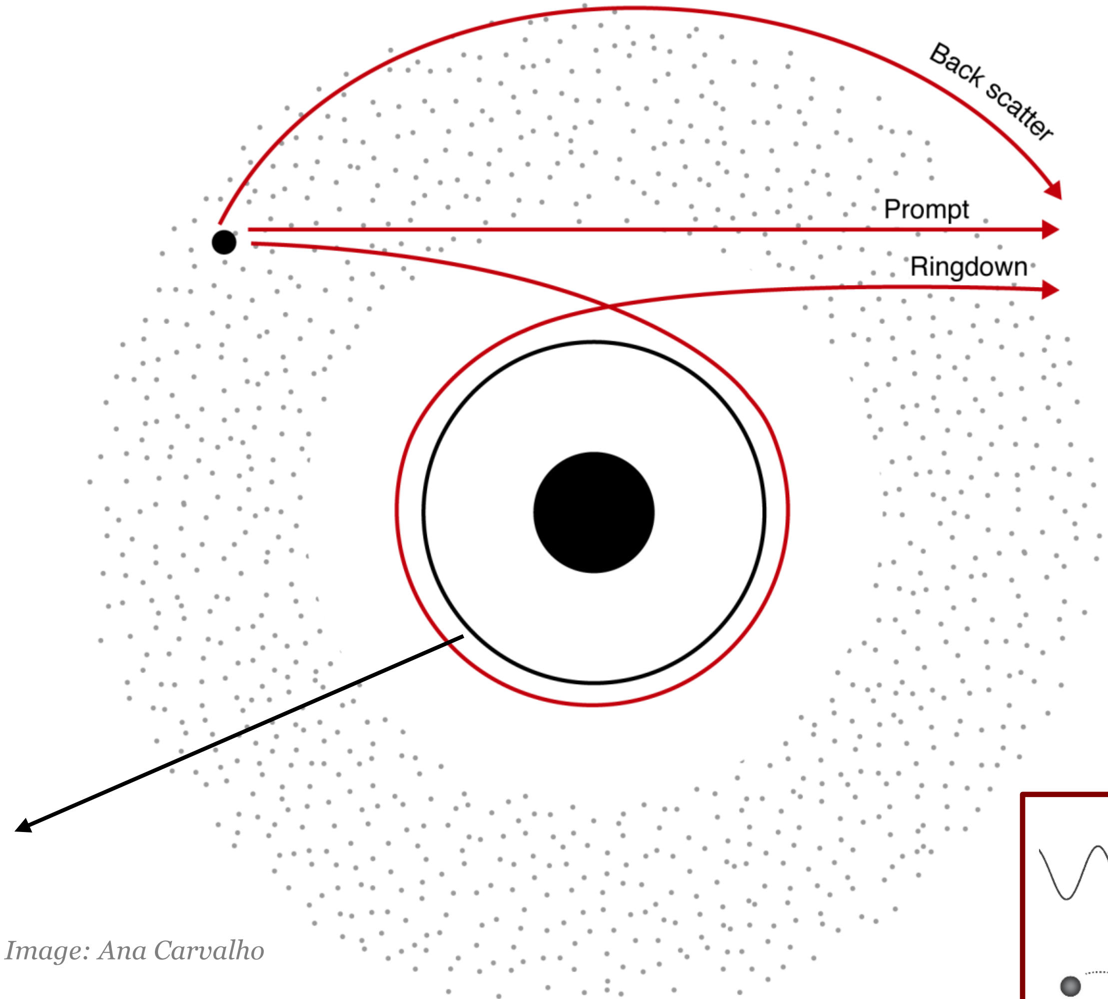
$$f = \omega_R/2\pi = 1.207 \left(\frac{10\ M_\odot}{M}\right) \text{kHz}$$

$$\tau = 1/|\omega_I| = 0.5537 \left(\frac{M}{10\ M_\odot}\right) \text{ms}$$

Quality factors of ~3. Black holes don't ring, they relax with a thud.

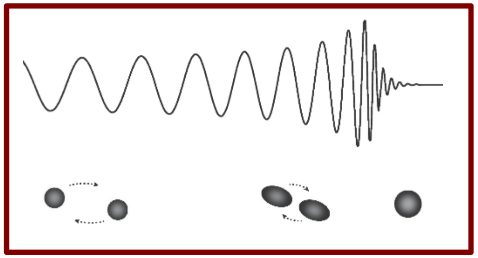
Berti, Cardoso and Starinets CQG 26: 163001 (2009)

When is a *linear* ringdown description valid?



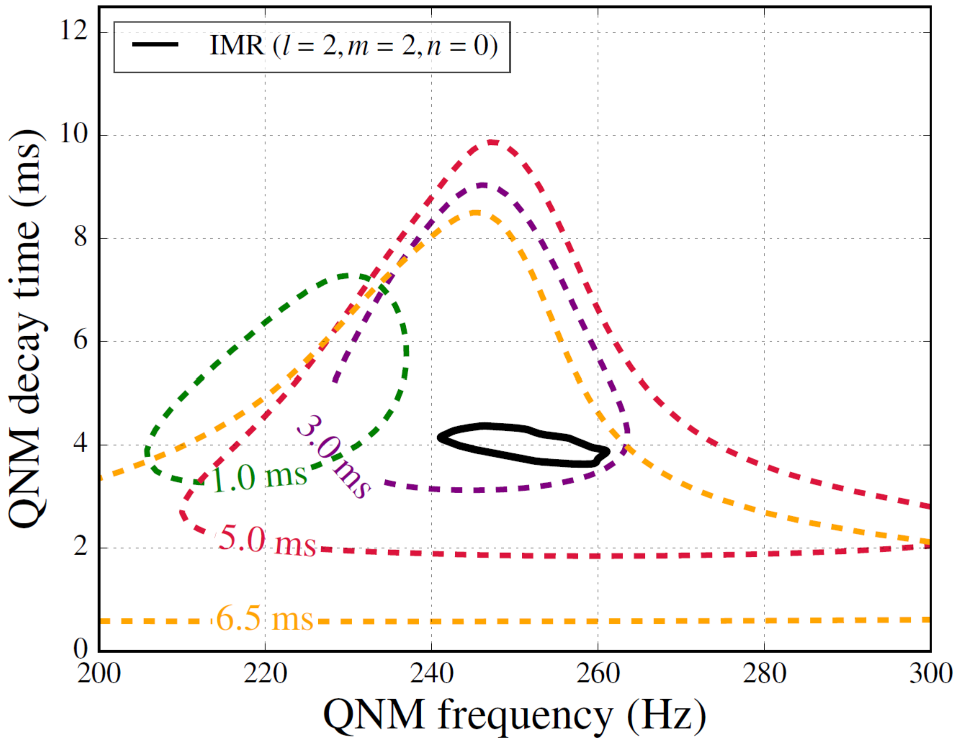
Light ring

Image: Ana Carvalho



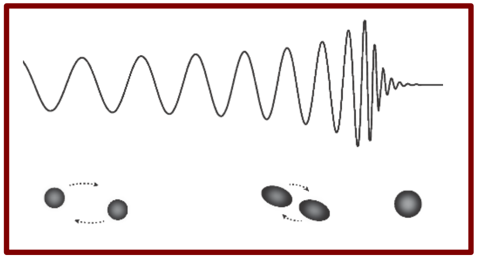
Leaver PRD34:384 (1986); Baibhav + arXiv:2302.03050; Nee+ PRD108:044032 (2023)

One and two-mode estimates: the start of spectroscopy



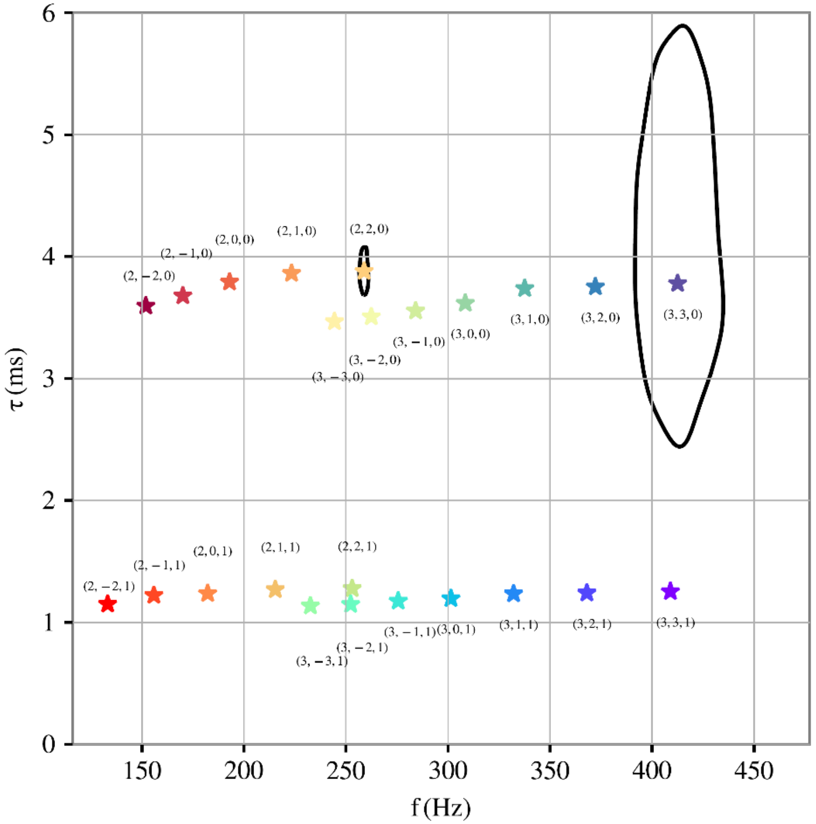
90% posterior distributions.

Black solid is 90% posterior of QNM as derived from the posterior mass and spin of remnant



LSC PRL116:221101 (2016); arXiv:2010.14529;
For future detectors, Berti+ PRL117:10102 (2016)

One and two-mode estimates: the start of spectroscopy



90% posterior distributions.

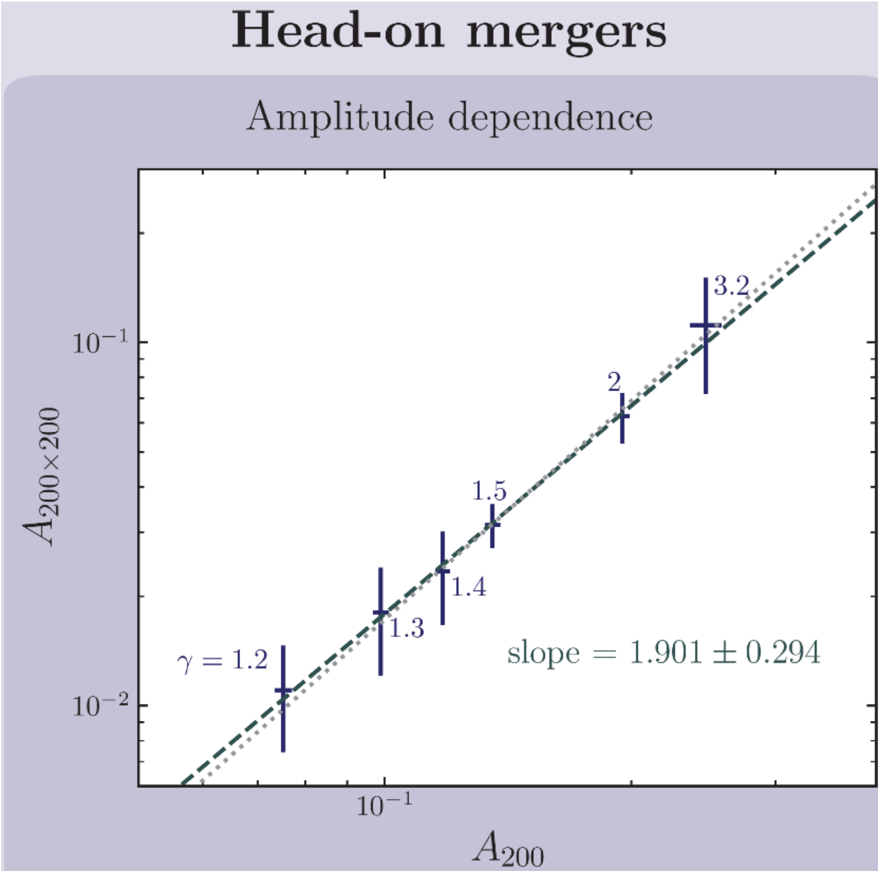
Black solid is 90% posterior of QNM from a future event with SNR=40 in ringdown.

LISA will see SNRs of thousands…

Courtesy of Gregorio Carullo
See also Berti+ PRL117:10102 (2016); Bhagwat+ arXiv:2304.02283

Nonlinearities in ringdown

$$\frac{\partial^2 \Psi^{(2)}}{\partial r_*^2} - \frac{\partial^2 \Psi^{(2)}}{\partial t^2} - V\Psi^{(2)} \sim \left(\Psi^{(1)}\right)^2$$



Cheung + PRL130:8 (2023)

Nonlinearities in ringdown

$$\frac{\partial^2 \Psi^{(2)}}{\partial r_*^2} - \frac{\partial^2 \Psi^{(2)}}{\partial t^2} - V\Psi^{(2)} \sim \left(\Psi^{(1)}\right)^2$$

$$R_{220\times 220} = \frac{A_{220\times 220}}{A_{220}^2}$$

For quasi-circular inspirals of non spinning binaries, from NR:

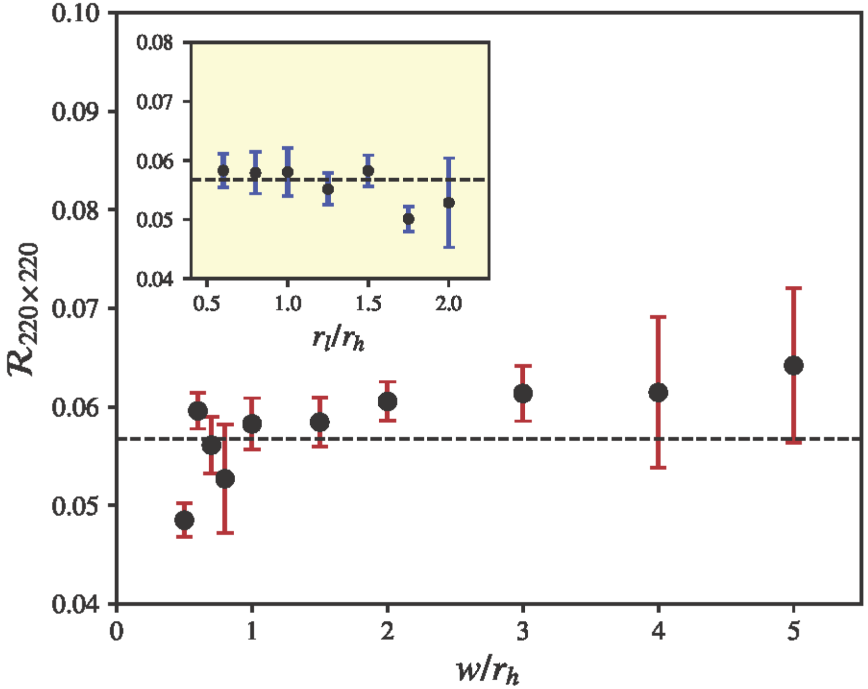
$$R_{220\times 220} = 0.16$$

It is not hopelessly small!

Similar number recovered from near-horizon symmetries (“Kerr/CFT”)

Cheung+ PRL130:8 (2023);
also Ma+ arXiv 2207.10870; Mitman+ arXiv 2208.07380; see Kehagias+ arXiv:2301.09345 for Kerr/CFT

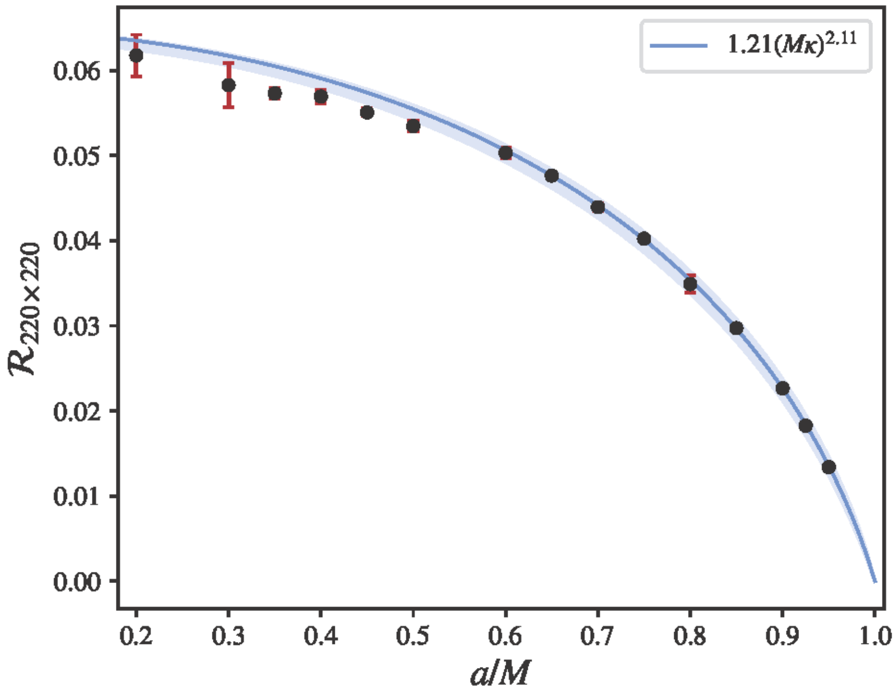
Nonlinearities in ringdown: second-order



Second order amplitude only weakly dependent on ID

Redondo-Yuste + arXiv:2308.14796

Nonlinearities in ringdown: second-order

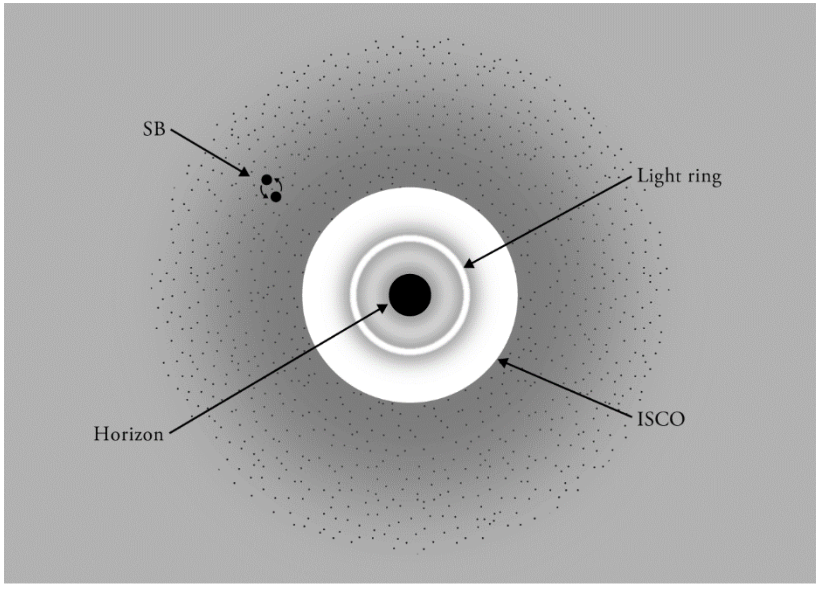


But dependent on spin…raising doubts about Kerr/CFT applications

Redondo-Yuste + arXiv:2308.14796

Spectroscopy: resonant excitation

Cardoso and Duque PRD103:L081501 (2021) see also Lynch+ CQG39:145004 (2023)



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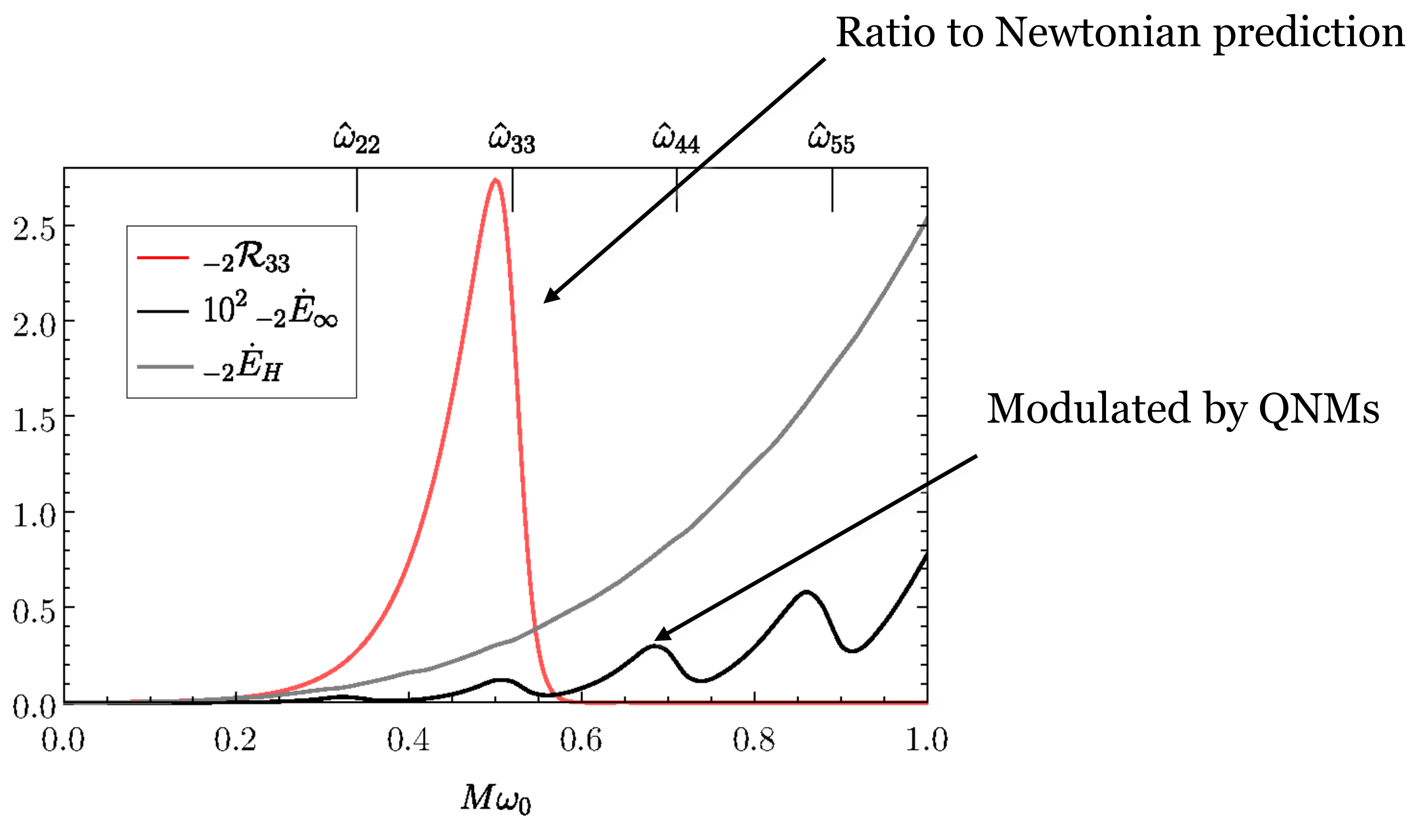
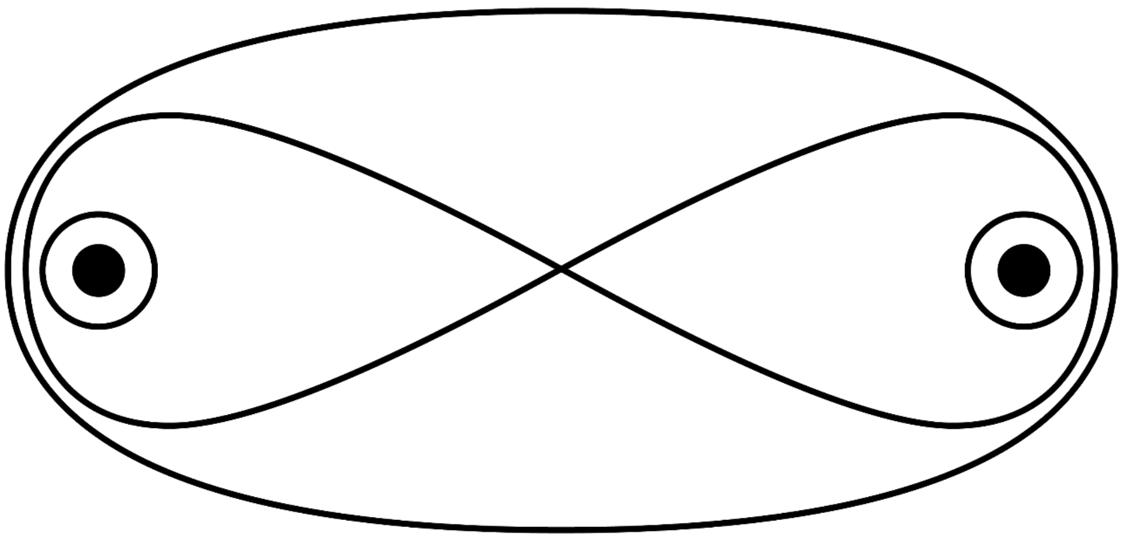


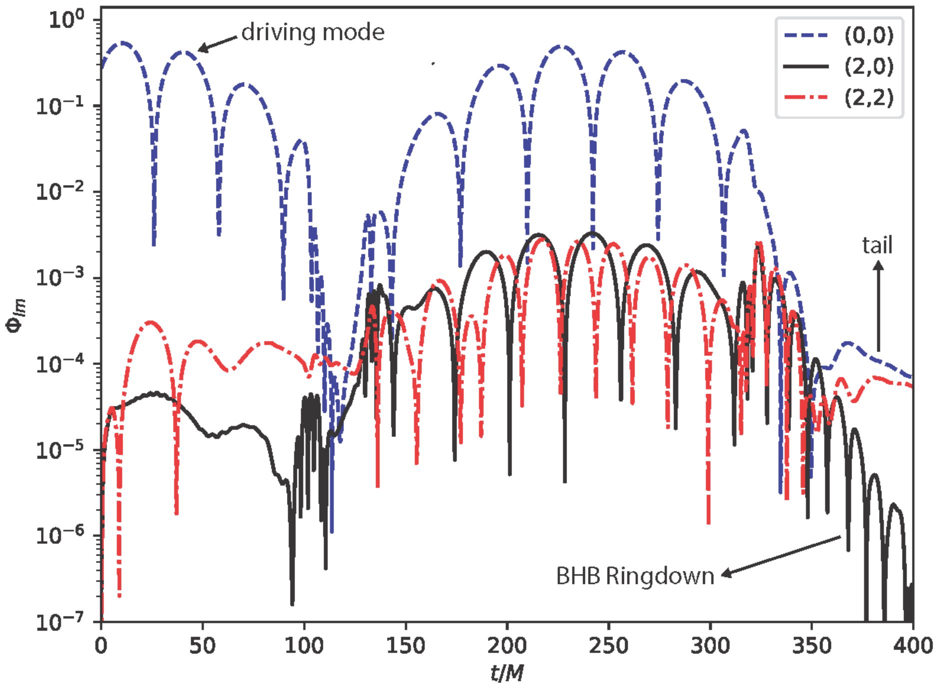
Energy output when tuning fork stands at ISCO of SMBH of spin a = 0.9M, as function of tuning fork frequency. Modal energy output ratio peaks at the lowest QNM. Also shown is flux integrated over modes: substantial component going down the SMBH horizon, total flux is modulated by QNM contributions.

Black hole chemistry?



Chandrasekhar PRSLA421:227 (1989); Assumpção+ PRD98: 064036 (2018)

Black hole chemistry?



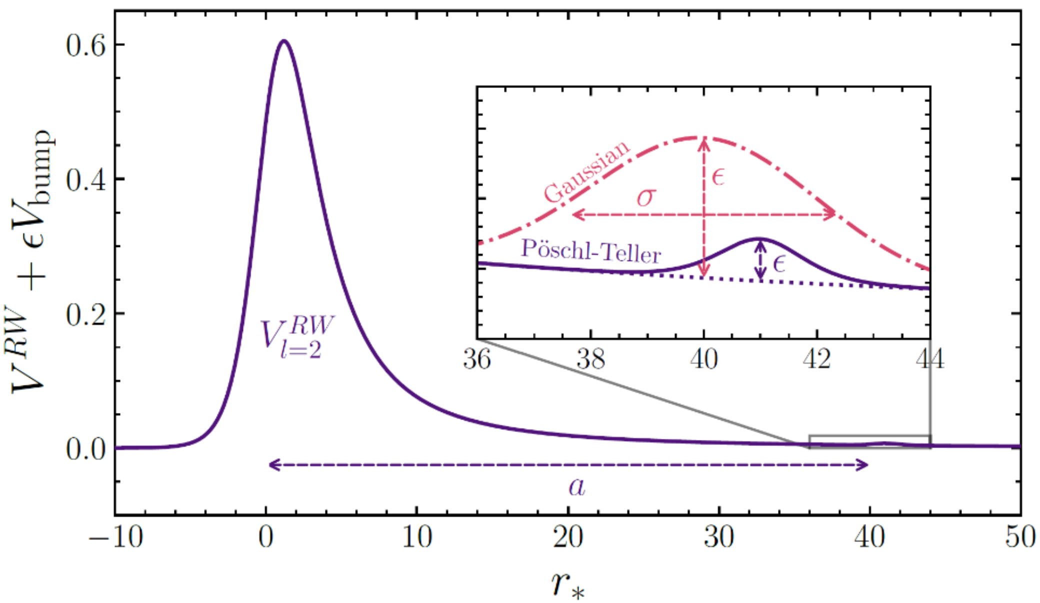
$$T = (1.03 \pm 0.04)\, L + (8 \pm 1)\, M$$

Global BHB modes may be resonantly excited?

Bernard + PRD100: 044002 (2019); arXiv:1905.05204

Spectral stability: the elephant and the flea

Spectrum is unstable: Nollert gr-qc/9602032; Barausse + PRD89:104059 (2014); Jaramillo+ PRX 11: 031003 (2021); Cheung+ PRL128:111103 (2022); PRD106:084011 (2022)



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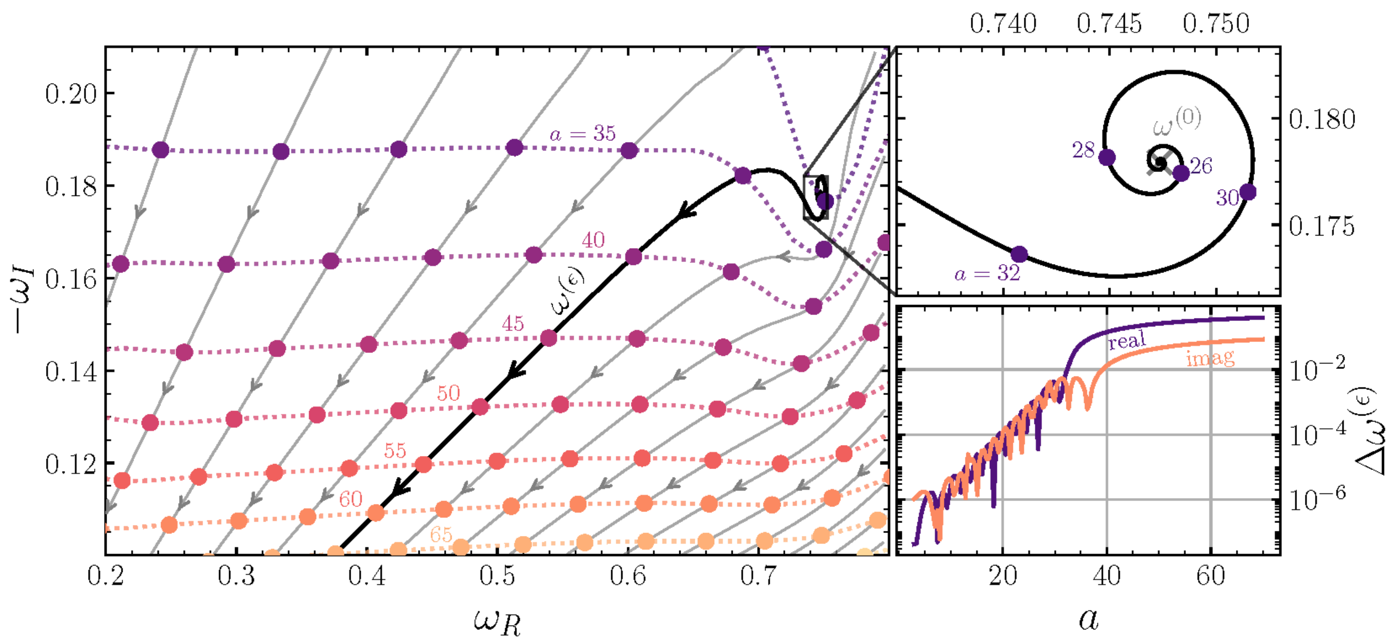

ε=10^{-6}

Spectral stability: the elephant and the flea

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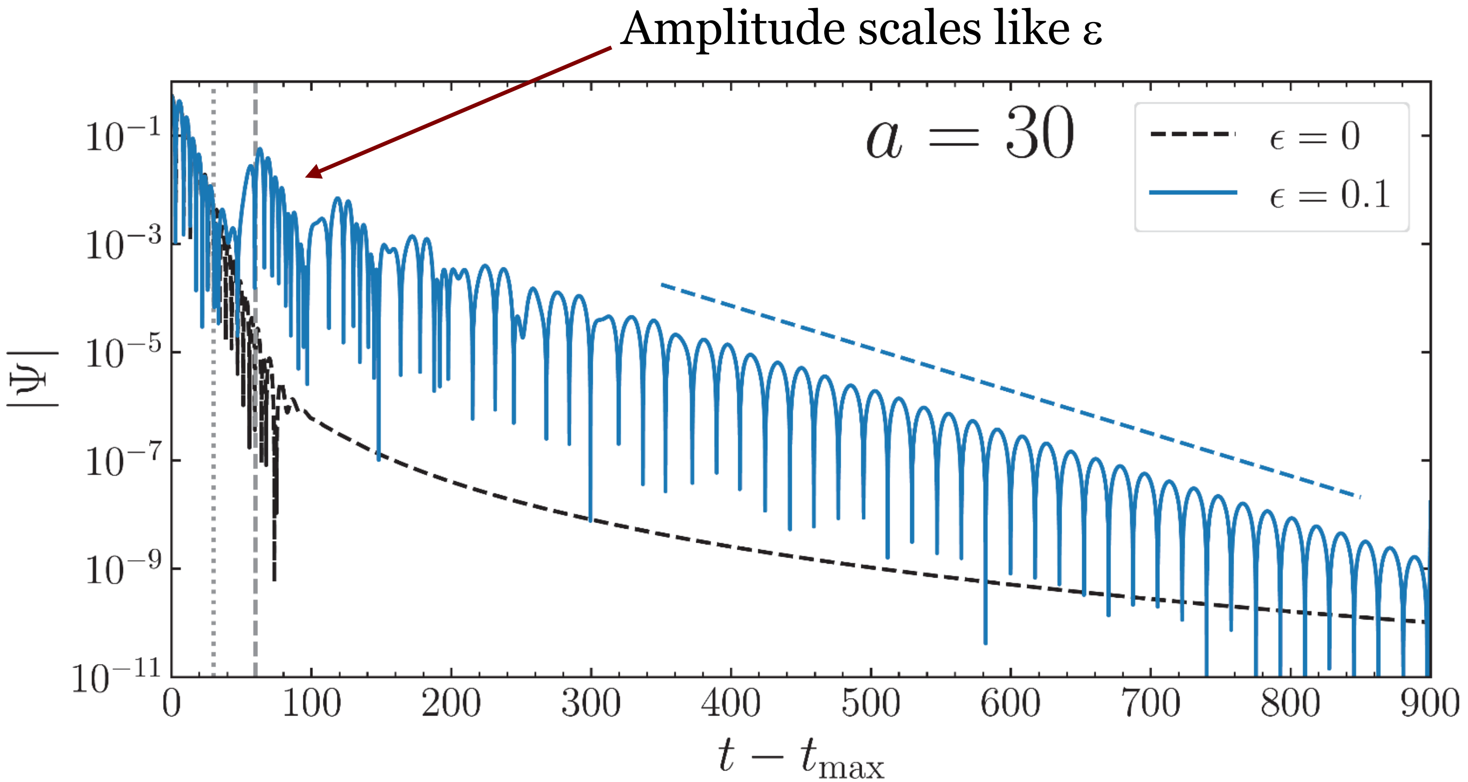


Testing black hole nature

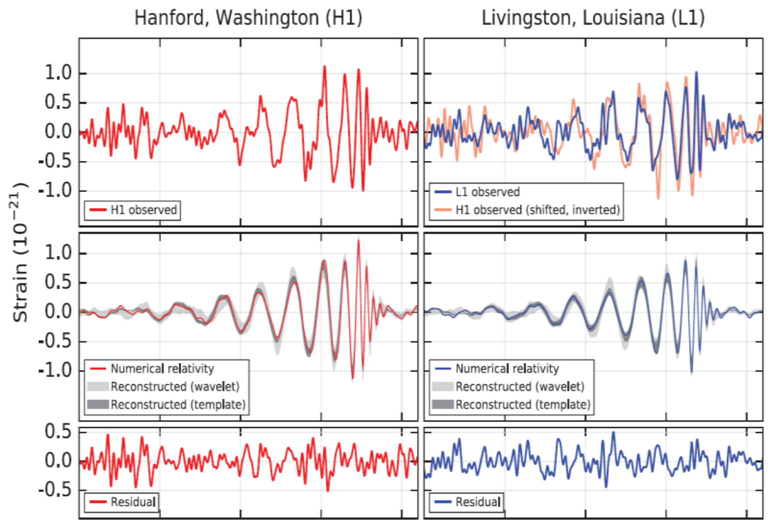
1. BH exterior is pathology-free, interior is not.

“Plus un fait est extraordinaire, plus il a besoin d'être appuyé de fortes preuves; car, ceux qui l'attestent pouvant ou tromper ou avoir été trompés, ces deux causes son d'autant plus probables que la réalité du fait l'est moins en elle-même….”
Laplace, Essai philosophique sur les probabilities 1812

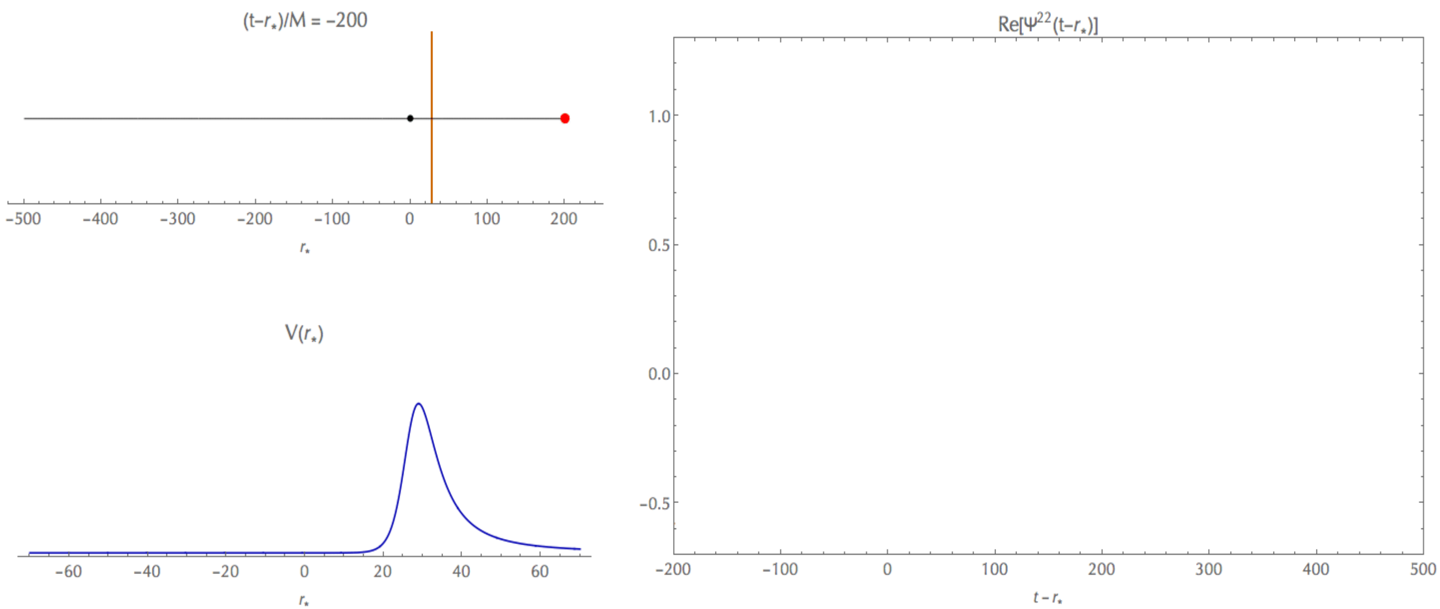
2. Dark matter exists, and interacts gravitationally. Are there compact DM clumps?

3. Physics is experimental science. We can test exterior. Aim to quantify evidence for horizons. Similar to quantifying equivalence principle.

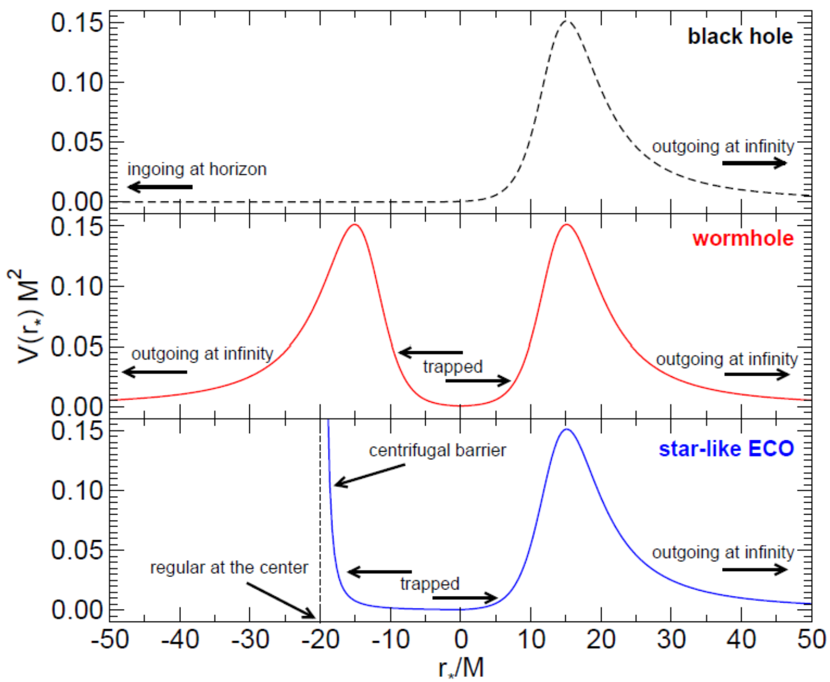
Post-merger

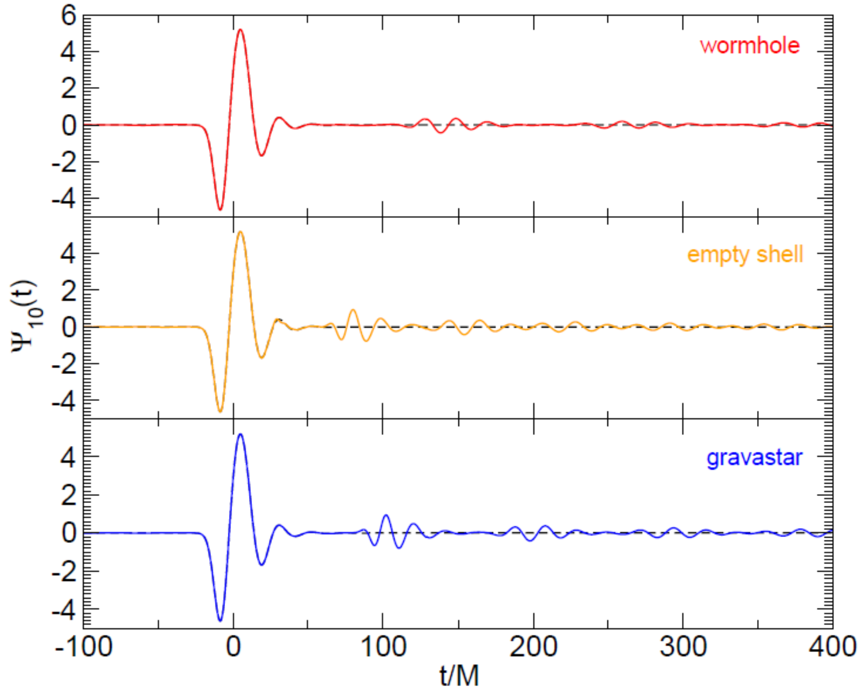


$\mathcal{E} = 1.5$, $\mathcal{J} = 0$



Echoes:
See Elisa Maggio's for further details





Cardoso + PRL116:171101 (2016); Cardoso and Pani, Nature Astronomy 1: 2017
Cardoso and Pani, Living Reviews in Relativity 22:1 (2019)
See also Abedi+ PRD96:082004 (2017);
LIGO/Virgo Collaboration arXiv:2010:14529; arXiv:2112.06861

Vacuum is an illusion: galaxy tomography?

Cardoso + PRD105:L061501 (2022); PRL129:241103 (2022)

Black holes in galaxies: an Einstein Cluster prescription (Einstein 1939)

Assume averaged stress-tensor

$$\langle T^{\mu\nu}\rangle = \frac{n}{m_p}\langle P^\mu P^\nu\rangle \Leftrightarrow T^\mu_\nu = \mathrm{diag}(-\rho, 0, P_t, P_t)$$

Impose spherical symmetry

$$ds^2 = -f dt^2 + \frac{dr^2}{1-2m(r)/r} + r^2 d\Omega^2$$

Assign mass function

Hernquist ApJ356:359 (1990)

$$m(r) = M_{\rm BH} + \frac{Mr^2}{(a_0+r)^2}\left(1-\frac{2M_{\rm BH}}{r}\right)^2$$

Solve field equations

$$f = \left(1-\frac{2M_{\rm BH}}{r}\right)e^\Upsilon$$

$$\Upsilon = -\pi\sqrt{\frac{M}{\xi}} + 2\sqrt{\frac{M}{\xi}}\arctan\frac{r+a_0-M}{\sqrt{M\xi}}$$

$$\xi = 2a_0 - M + 4M_{\rm BH}$$

Generalization to other profiles is straightforward.
see Figueiredo + arXiv:2303.08183

Environments: galaxy tomography?

Cardoso+ PRDL061501 (2022); PRL129:241103 (2022); Figueiredo + arXiv:2303.08183

Two ingredients play a role:

1. Compactness of distribution $\frac{M}{a_0}$, which dictates geometry & wave generation effects. For typical galaxies $\frac{M}{a_0}$=10^{-4}- 10^{-6}. For dark matter spikes one can pump up to 10^{-3}.

2. Density ρ which governs matter effects (dynamical friction etc)

Environments: galaxy tomography?

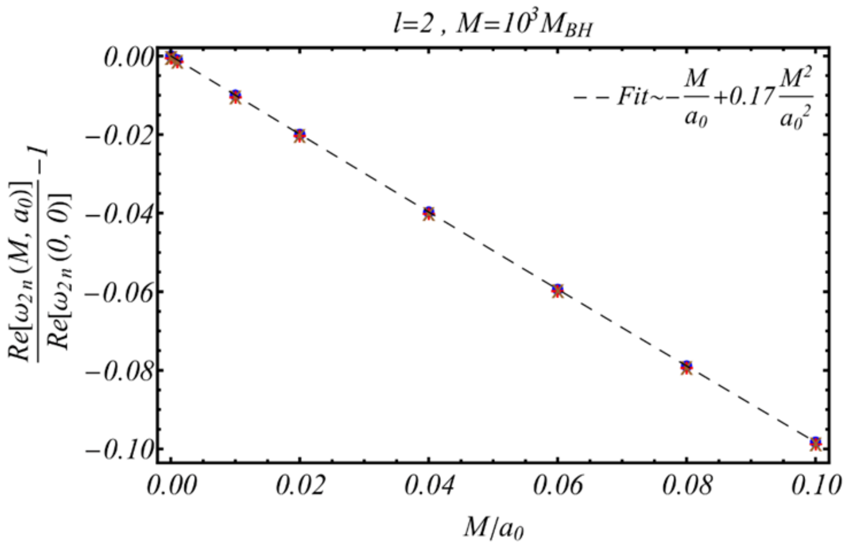
Cardoso+ PRDL061501 (2022); PRL129:241103 (2022); Figueiredo + arXiv:2303.08183

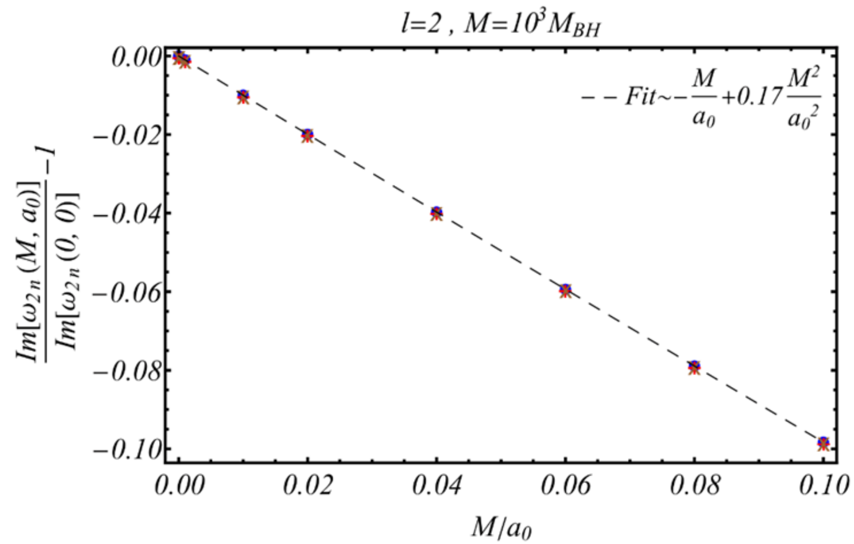
$$b_{\rm crit} = 3\sqrt{3}M_{\rm BH}\left(1 + \frac{M}{a_0} + \frac{M(5M - 18M_{\rm BH})}{6a_0^2}\right)$$

Thus EHT physics affected to levels of 10^{-8} only, *for expected parameters*
(tests on nature of compact objects can be done to very good precision)

Light-ring corrections

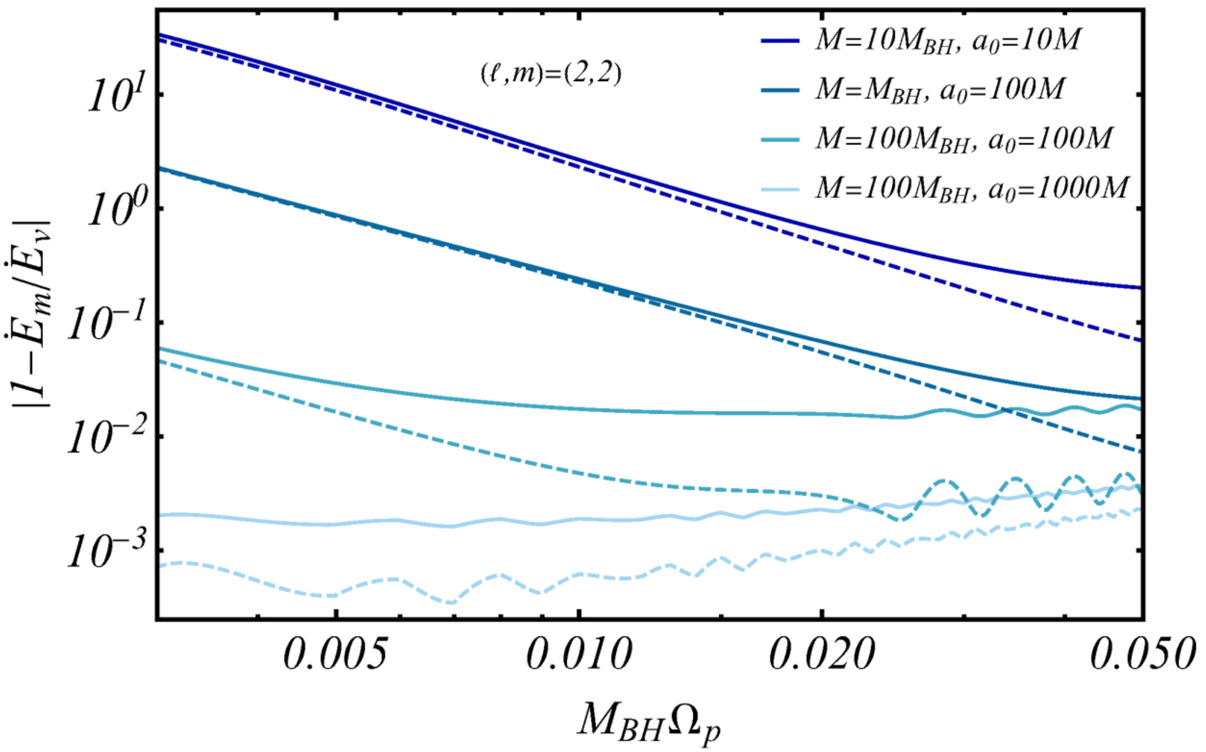
$$M_{\rm BH}\Omega_{\rm LR} \sim \frac{1}{3\sqrt{3}}\left(1-\frac{M}{a_0}-\frac{M^2}{6a_0^2}\right) \sim M_{\rm BH}\Omega_{\rm LR}^{\rm Schw}\left(1-\frac{M}{a_0}-0.17\frac{M^2}{a_0^2}\right)$$





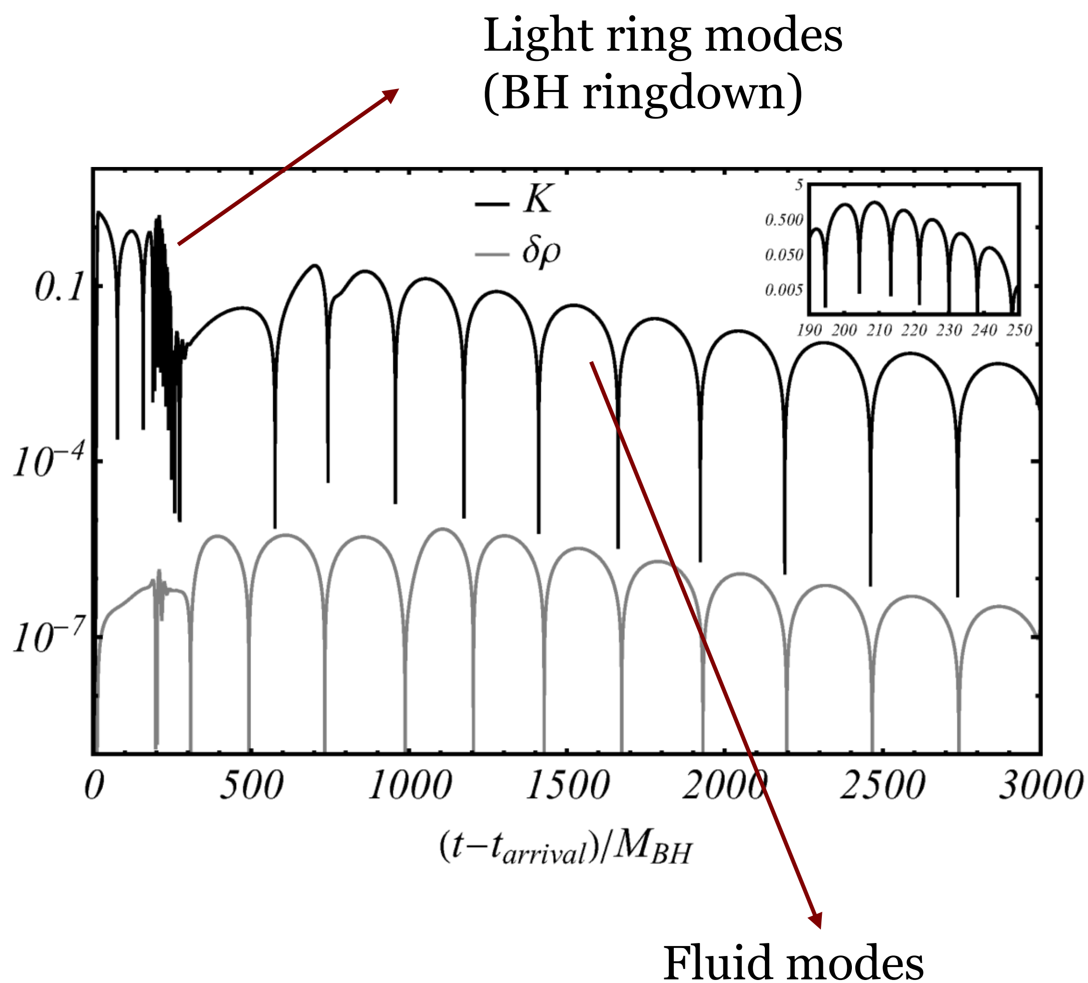
Environments: a fully relativistic analysis

Cardoso+ PRL129:241103 (2022); Figueiredo + arXiv:2303.08183



Spectral instability in action

Cardoso+ PRL129:241103 (2022)



Some questions

Spectral instability: prompt change to ringdown?

Requirements for detection?

Mode excitation by turbulent accretion disks?

Rotation, memory and tails in presence of environments

Nonlinearities and mass and spin evolution

Nonlinearities and gravitational turbulence

Are low-frequency detectors able to perform galaxy tomography?

Conclusions: exciting times!

Gravitational wave astronomy *will* become a precision discipline, mapping compact objects throughout the entire visible universe.

Strong field gravity is a fascinating topic. From precise maps of Universe to tests of Cosmic Censorship or constraints on dark matter, possibilities are endless & exciting. Black holes respond in simple way to external perturbations, and may serve as detectors for nontrivial environments.

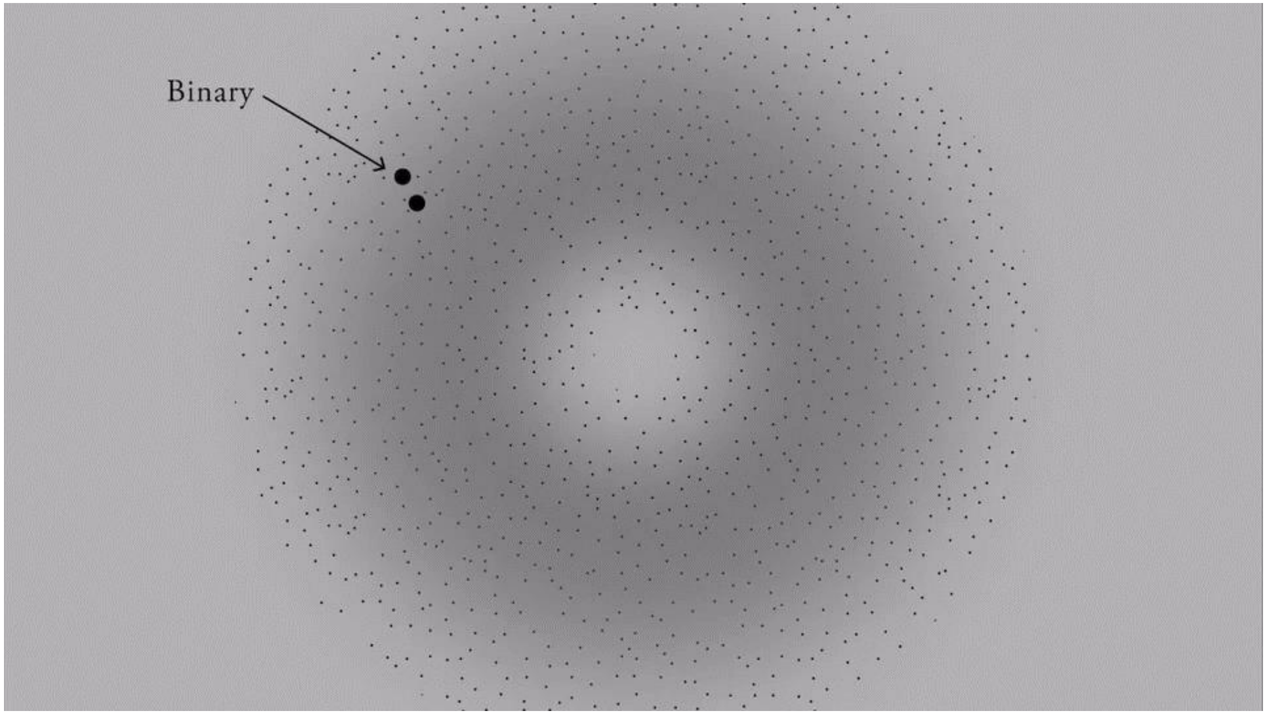
Black holes remain the most outstanding object in the universe.
Spectroscopy will allow to test GR and provide strong evidence for the presence of horizons… improved sensitivity pushes putative surface closer to horizon, like probing short-distance structure with accelerators.

Thank you

Environmental effects

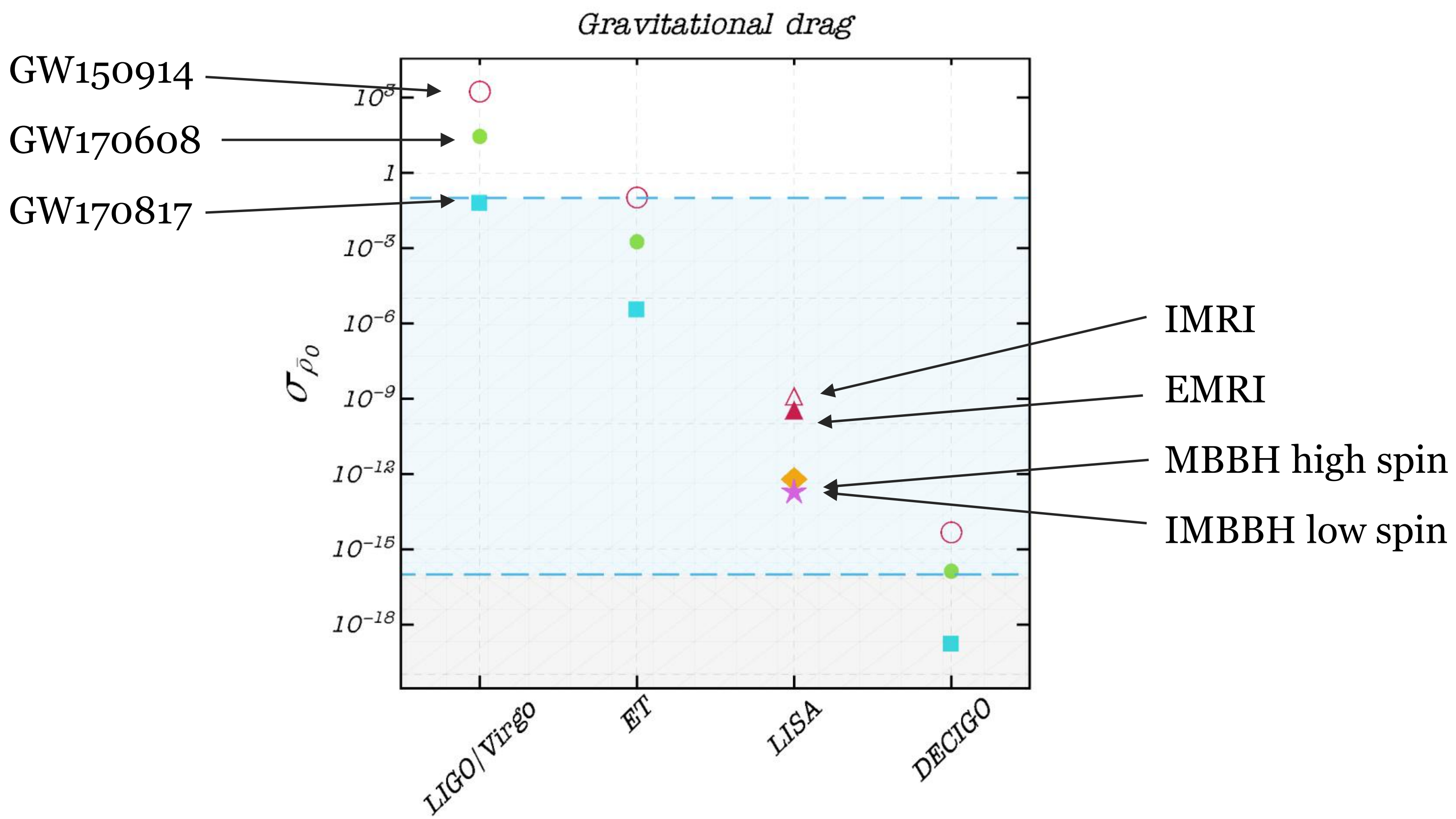
Inspiral occurs in DM-rich environment, within galaxies, and may modify the way inspiral proceeds, given dense-enough media

Eda + PRL110:221101 (2013); Macedo + ApJ774:48 (2013); Barausse+PRD89:104059 (2014); *Cardoso + AA644: A147 (2020) Kavanagh + arXiv 2002.12811; Annulli + PRD102: 063022 (2020);* Cardoso+PRDL105:104023 (2022)



Animation by Ana Carvalho

Dynamical friction: Newtonian, particle-like



Effect is -5.5 PN on GW phase

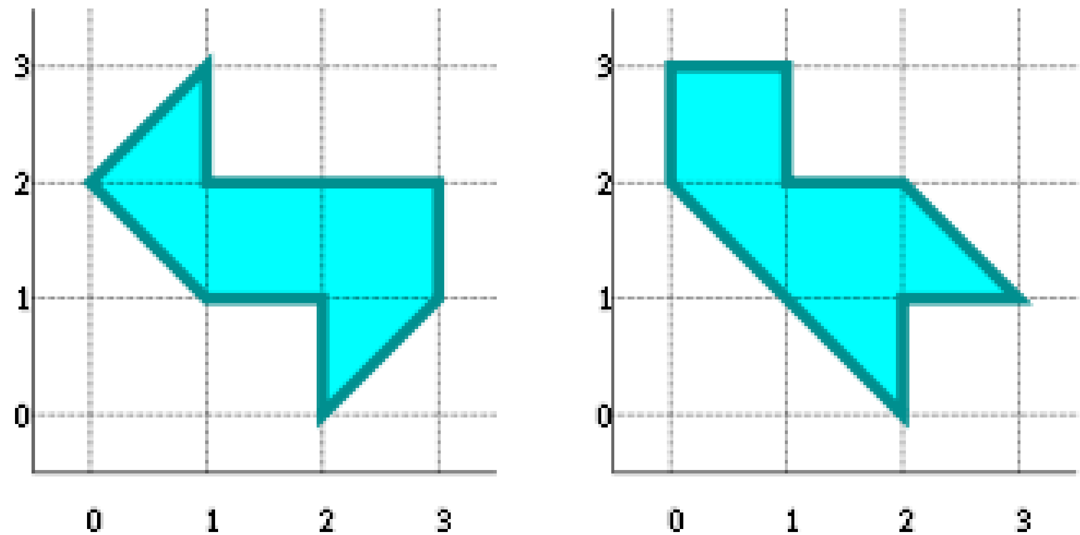
Cardoso & Maselli AA644: A147 (2020)
Also Eda + PRL 110 (2013) 221101; Macedo+ApJ774 (2013) 48; Annulli+ PRD102;063022 (2020)

"Can one hear the shape of a drum?"

Mark Kac, American Mathematical Monthly, *1966*

$$N(\lambda) = \sum_{\lambda_n < \lambda} 1 \sim \frac{|\Omega|}{2\pi}\lambda$$

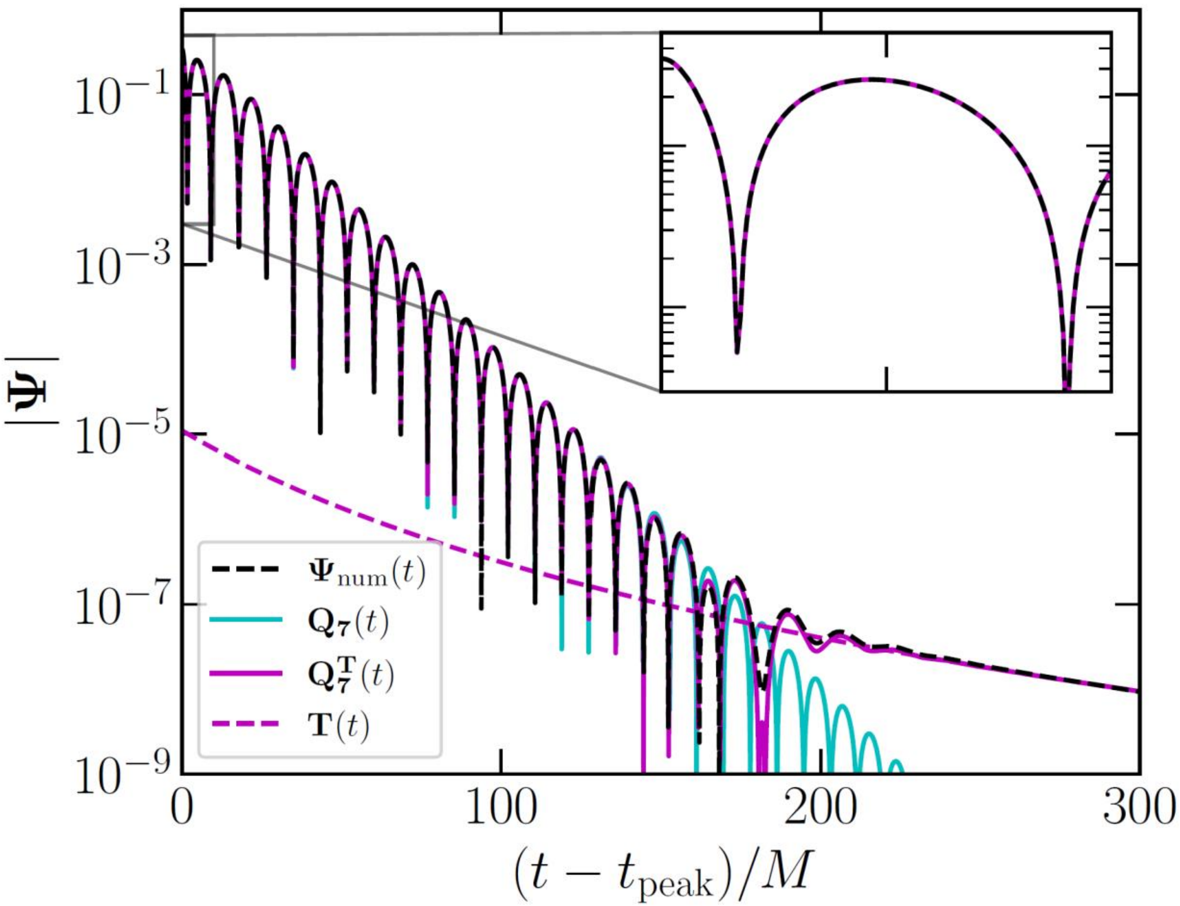
H. Weyl 1911



Gordon, Webb & Wolpert, Inventiones Mathematicae 1992

Overtones?

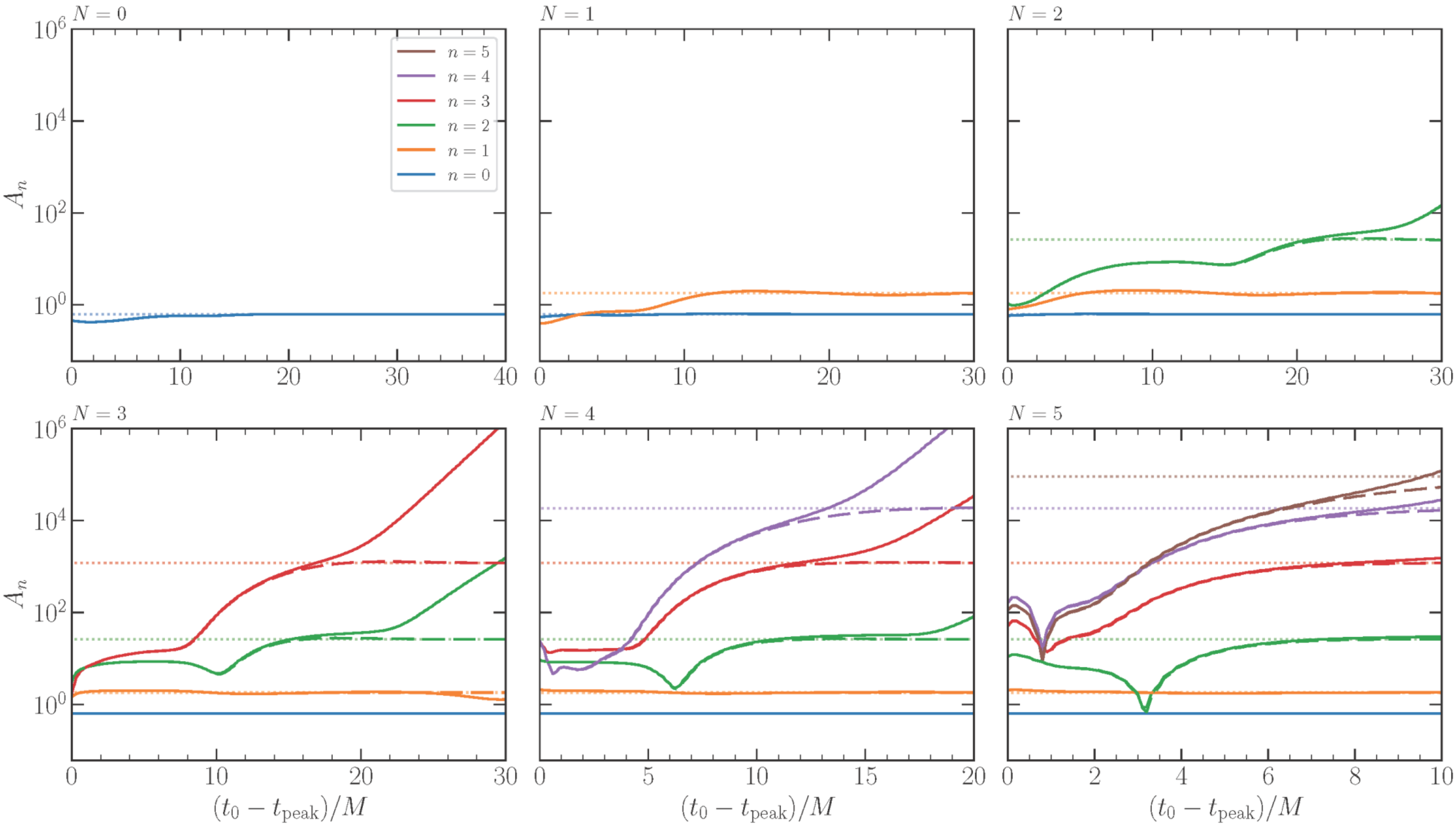
Baibhav arXiv:2302.03050 [gr-qc]



Q7: 7 overtones (plus fundamental)

Q7T: 7 overtones (plus fundamental) and tail

Overtones?



Q7: long dashed **Q7T:** solid **Injections:** faint dotted

Spectroscopy: testing black hole nature

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“Plus un fait est extraordinaire, plus il a besoin d'être appuyé de fortes preuves; car, ceux qui l'attestent pouvant ou tromper ou avoir été trompés, ces deux causes son d'autant plus probables que la réalité du fait l'est moins en elle-même....”
Laplace, Essai philosophique sur les probabilities 1812

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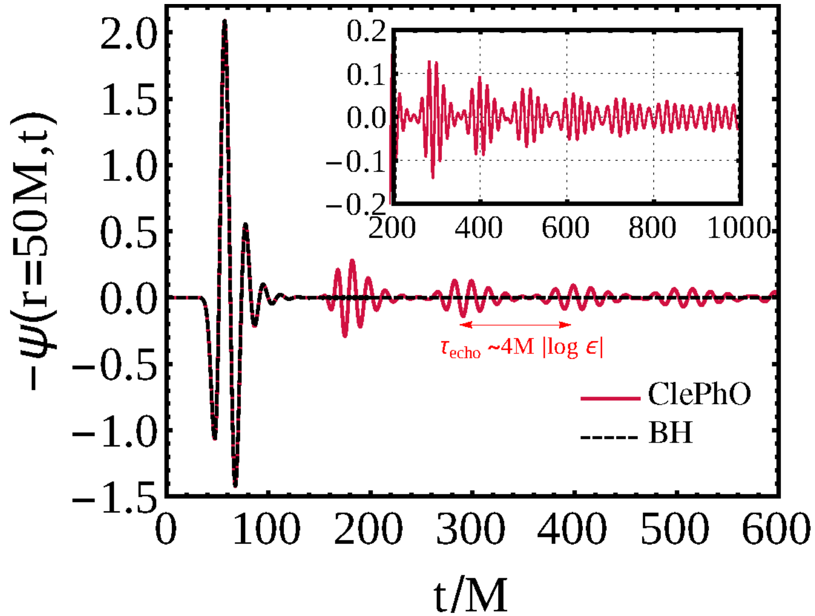
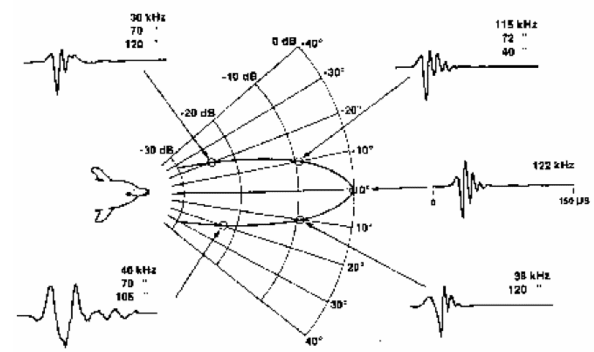
3. Physics is experimental science. We can test exterior. Aim to quantify evidence for horizons. Similar to quantifying equivalence principle.

Echoes



$$M\omega_R \sim |\log\epsilon|^{-1}$$
$$M\omega_I \sim |\log\epsilon|^{-(2l+3)}$$

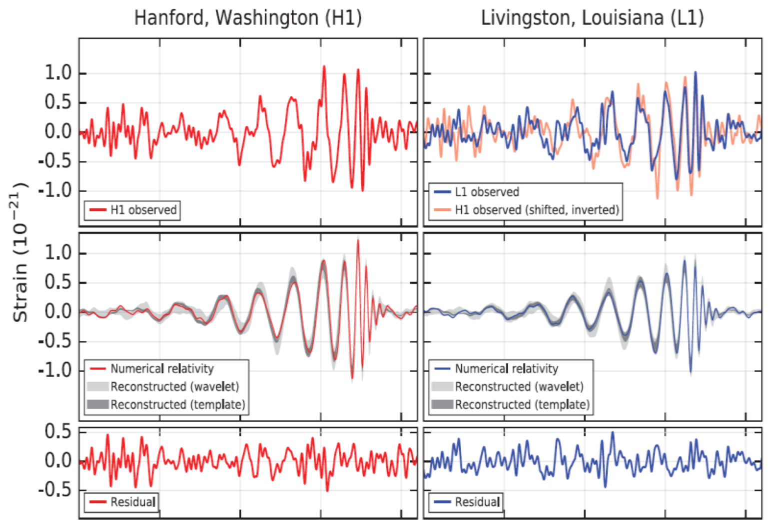


Target echo in reverberation at the dolphin's threshold of detection (from Au [8]).

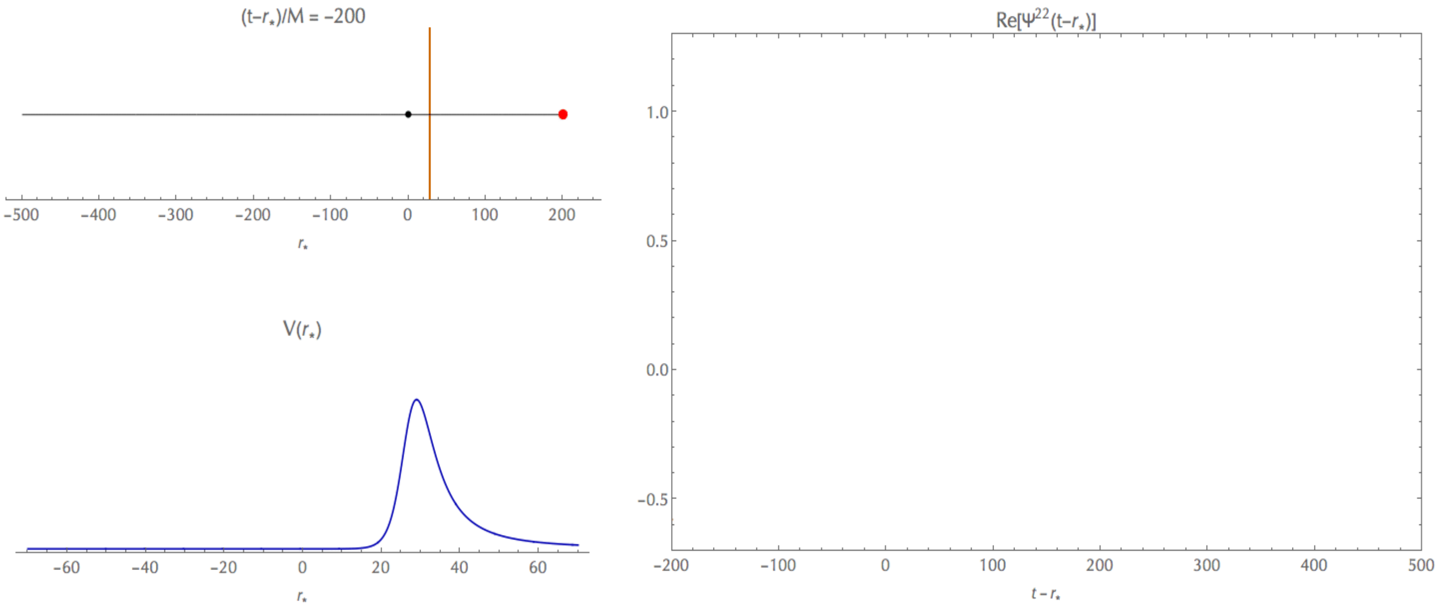
Au+ (J. Acoust. Soc. Am. 1992)

Cardoso & Pani, Nature Astronomy 1: 2017; *Living Reviews in Relativity 22:1 (2019)*
See also Abedi+ PRD96:082004 (2017);
LIGO/Virgo Collaboration arXiv:2010:14529; arXiv:2112.06861

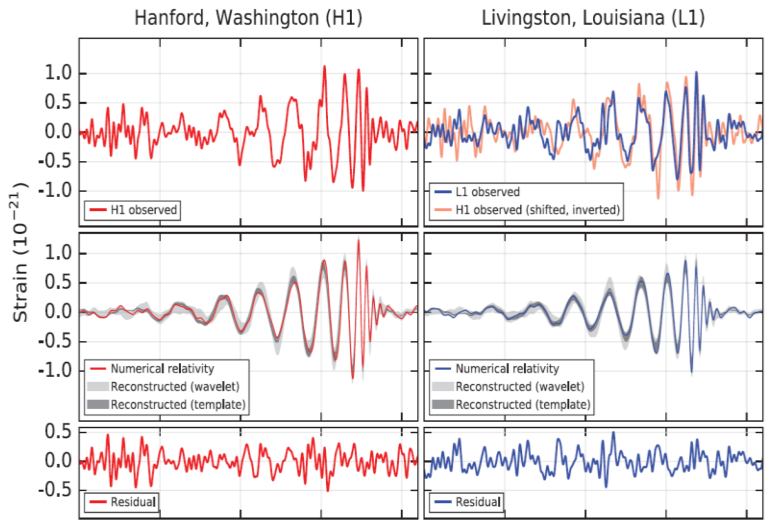
Post-merger



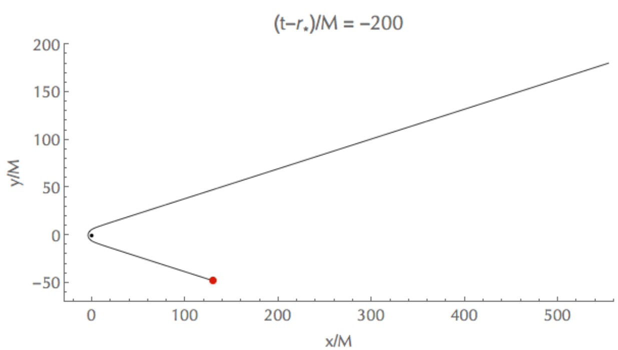
$\mathcal{E} = 1.5,\ \mathcal{J} = 0$

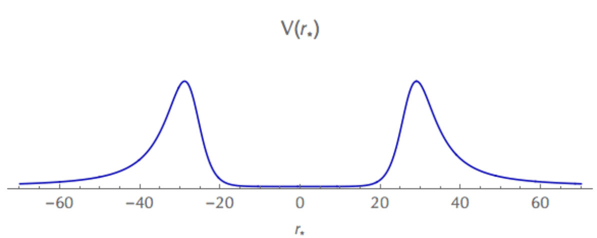


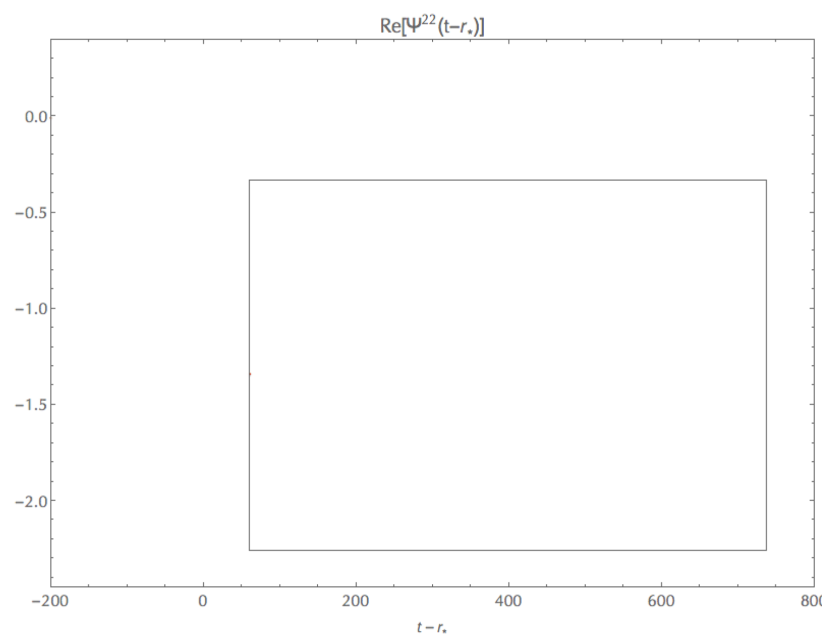
Post-merger



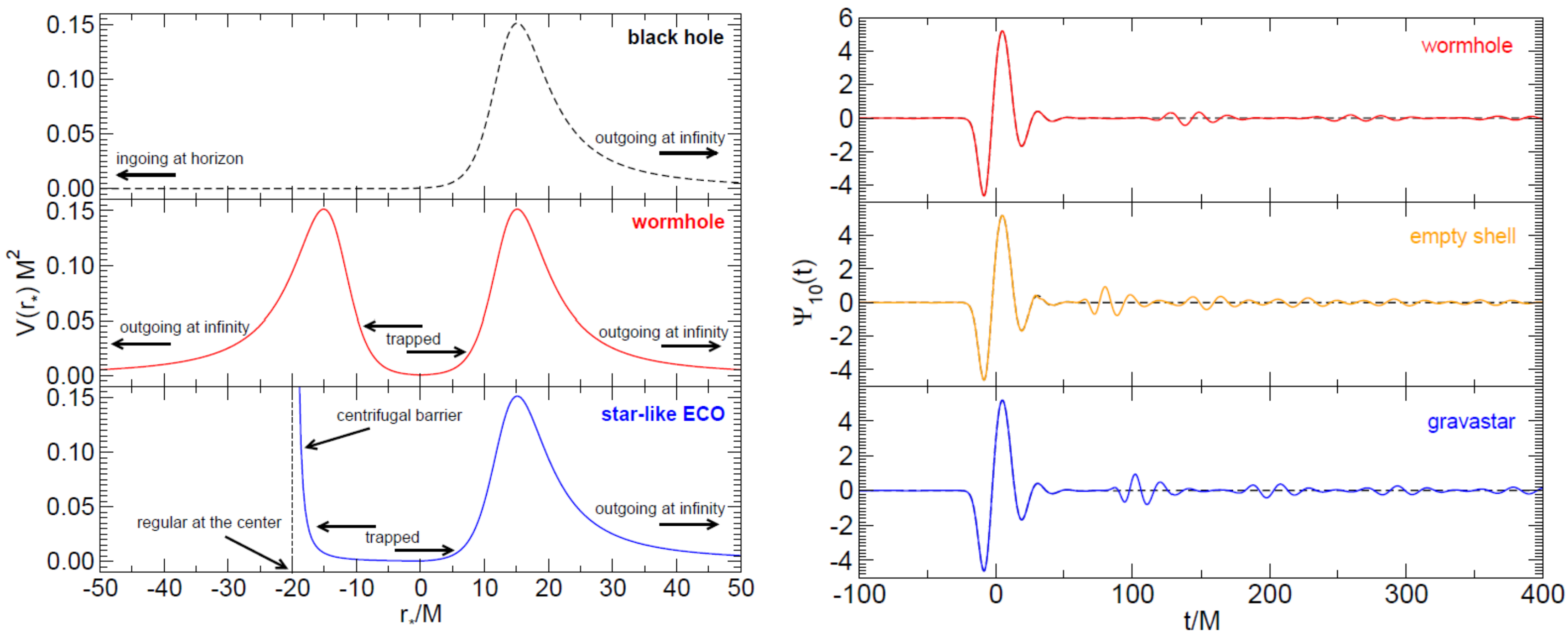
$\mathcal{E} = 1.5$, $r_{min}=4.3M$, $r_0-2M = 10^{-6}M$







Echoes



Cardoso + PRL116:171101 (2016); Cardoso and Pani, Nature Astronomy 1: 2017
Cardoso and Pani, Living Reviews in Relativity 22:1 (2019)

Some challenges

i. Are there alternatives?

ii. Do they form dynamically under reasonable conditions?

iii. Are they stable?

iv. How do they look like? Is GW or EM signal similar to BHs?

v. Observationally, how close do we get to horizons?

Answer requires understanding of theoretical framework, PDE analysis, precise modelling, observations, challenging simulations & data analysis techniques

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Gravitational-wave spectroscopy of massive black holes with the space interferometer LISA

Emanuele Berti, Vitor Cardoso, and Clifford M. Will
Phys. Rev. D **73**, 064030 – Published 24 March 2006

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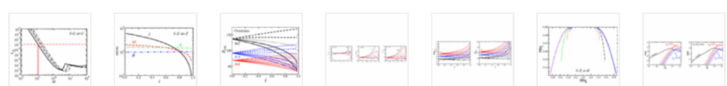
ABSTRACT

Newly formed black holes are expected to emit characteristic radiation in the form of quasinormal modes, called ringdown waves, with discrete frequencies. *LISA* should be able to detect the ringdown waves emitted by oscillating supermassive black holes throughout the observable Universe. We develop a multimode formalism, applicable to any interferometric detectors, for detecting ringdown signals, for estimating black-hole parameters from those signals, and for testing the no-hair theorem of general relativity. Focusing on *LISA*, we use current models of its sensitivity to compute the expected signal-to-noise ratio for ringdown events, the relative parameter estimation accuracy, and the resolvability of different modes. We also discuss the extent to which uncertainties on physical parameters, such as the black-hole spin and the energy emitted in each mode, will affect our ability to do black-hole spectroscopy.



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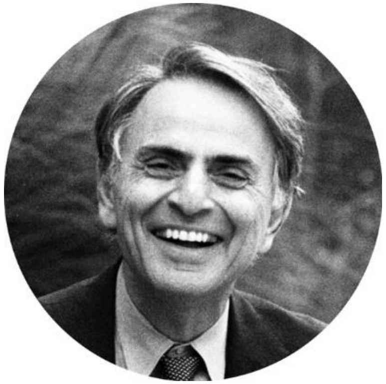
Testing BH nature

1. BH exterior is pathology-free, interior is not.



"*Extraordinary claims require extraordinary evidence.*"
Carl Sagan

2. Quantum effects not fully understood. Non-locality to solve information paradox?

3. Tacitly assumed quantum effects at Planck scales. Planck scale could be significantly lower *(Arkani-Hamed+ 1998; Giddings & Thomas 2002)*. Even if not, many orders of magnitude standing, surprises can hide (Bekenstein & Mukhanov 1995).

4. Dark matter exists, and interacts gravitationally. Are there compact DM clumps?

5. Physics is experimental science. We can test exterior. Aim to quantify evidence for horizons. Similar to quantifying equivalence principle.