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Fundamental decoherence from quantum spacetime

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Vittorio D'Esposito

✉ vittorio.desposito@unina.it

🏛️ Università degli studi di Napoli "Federico II"

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Gravitational field (or spacetime) as an **omnipresent** environment [A. Bassi *et al*, *Class. Quant. Grav.* 34, 193002 (2017).].

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Several frameworks: Quantum clocks [R. Gambini et al, *Phys.Rev.Lett.* 93 (2004)], metric fluctuations [H.P. Breuer et al, *Class. Quant. Grav.* 26, 105012 (2009)], fluctuating minimal length and GUP models [L. Petrucciello and F. Illuminati, *Nat.Comm.* 12, 4449 (2021)], ...

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Phenomenology: Neutron interferometry and neutral kaons [J. Ellis et al, *Nucl.Phys.B* 241, 381 (1984)], neutrino oscillations [V. D'Esposito and G. Gubitosi, *arXiv:2306.14778*], optomechanical cavities [C. Pfister et al, *Nature Communications* 7, 13022 (2016)].

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Deformed symmetries are described with **Hopf algebras**

Non-linear algebraic sector

$$[X_i, X_j] = f(\mathbf{X})$$

Coalgebra becomes relevant

$$\Delta : \mathbb{H} \rightarrow \mathbb{H} \otimes \mathbb{H} \text{ (coproduct)}$$

$$S : \mathbb{H} \rightarrow \mathbb{H} \text{ (antipode)}$$

Deformed symmetries in QM

κ -Galilei algebra: **undeformed algebra and deformed coalgebra.**

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Undeformed coalgebra

$$\Delta P_\mu \left(|p\rangle \otimes |q\rangle \right) = (p_\mu + q_\mu) |p\rangle \otimes |q\rangle \quad (1)$$

$$P_\mu \langle q| := \langle q| S(P_\mu) = -q_\mu \langle q| \quad (2)$$

Deformed coalgebra

$$\Delta P_\mu \left(|p\rangle \otimes |q\rangle \right) = (p_\mu \oplus q_\mu) |p\rangle \otimes |q\rangle \quad (3)$$

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The algebra becomes the standard Galilei algebra. Non trivial structures on the coalgebra are

$$\Delta P_0 = P_0 \otimes \mathbb{1} + \mathbb{1} \otimes P_0 + \frac{1}{\kappa} P_n \otimes P^n, \quad S(P_0) = -P_0 + \frac{1}{\kappa} P^2 \quad (5)$$

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If coalgebra structures are trivial

$$\text{ad}_A(B) = (id \otimes S) \underbrace{(A \otimes \mathbb{1} + \mathbb{1} \otimes A)}_{\Delta A} \diamond B = (A \otimes \mathbb{1} - \underbrace{\mathbb{1} \otimes A}_{S(A)=-A}) \diamond B = [A, B] \quad (7)$$

Density operator time evolution

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These requirements are met with

$$i \partial_t \rho = \frac{1}{2} \left\{ \text{ad}_{P_0}(\rho) - [\text{ad}_{P_0}(\rho)]^\dagger \right\} \quad (8)$$

leading to

$$\partial_t \rho = -i [P_0, \rho] - \frac{1}{2\kappa} \left(\rho \mathbf{P}^2 + \mathbf{P}^2 \rho - 2 P_n \rho P^n \right) \quad (9)$$

Decoherence time and fundamental constraint on the mass

Free particle solution in momentum basis is

$$\langle \mathbf{p} | \rho | \mathbf{q} \rangle := \rho_{pq}(t) = \rho_{pq}(0) \exp \left\{ -it[E(p) - E(q)] - \frac{t}{2\kappa} (\mathbf{p} - \mathbf{q})^2 \right\} \quad (10)$$

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With $E = (2m)^{-1}p^2$, from (13) and for **momentum localized** states, $\frac{\delta p}{p} \ll 1$

$$m \lesssim \kappa \quad (14)$$

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- From the decoherence time, a **fundamental constraint** on the mass of quantum systems arises.
- **Phenomenology**: neutrino oscillation, table-top experiments with optomechanical cavities.
- Can this framework be extended to field theories? **Primordial perturbations decoherence...**



Thank you!

Interaction with the Environment

von Neumann measurement:

$$|o_n\rangle \otimes |R\rangle \mapsto |o_n\rangle \otimes |a_n\rangle \Rightarrow |\psi\rangle \otimes |R\rangle \mapsto \sum_n c_n |o_n\rangle \otimes |a_n\rangle \quad (15)$$

$|a_n\rangle \in \mathcal{H}_A, |o_n\rangle \in \mathcal{H}_S.$

Two-level system: $|\psi_i\rangle \otimes |E_0\rangle \mapsto |\psi_i\rangle \otimes |E_i\rangle, i = 1, 2.$ Environment-system entanglement emerges dynamically

$$|\psi\rangle \otimes |E_0\rangle = \frac{1}{\sqrt{2}} \left(|\psi_1\rangle + |\psi_2\rangle \right) \otimes |E_0\rangle \mapsto \frac{1}{\sqrt{2}} \left(|\psi_1\rangle \otimes |E_1\rangle + |\psi_2\rangle \otimes |E_2\rangle \right) \quad (16)$$

The density operator of the system is

$$\rho_S = \text{tr}_E \{ \rho \} = \frac{1}{2} \left(|\psi_1\rangle \langle \psi_1| + |\psi_2\rangle \langle \psi_2| + |\psi_1\rangle \langle \psi_2| \langle E_1| E_2 + |\psi_2\rangle \langle \psi_1| \langle E_2| E_1 \right) \quad (17)$$

For macroscopic systems $|E_i\rangle = \prod_{\alpha=1}^N |e_{\alpha}^{(i)}\rangle, \langle e_{\alpha}^{(1)}| e_{\alpha}^{(2)}\rangle = \varepsilon \lesssim 1,$ thus $\langle E_1| E_2 \sim \varepsilon^N \sim 0$

Example of system-environment interaction

Two-level system $\{|0\rangle, |1\rangle\}$; environment composed of N two-level systems $\{|\uparrow\rangle_i, |\downarrow\rangle_i\}$.

$$\hat{H}_{\text{int}} = \frac{1}{2} \hat{\sigma}_z \otimes \hat{E}, \quad \hat{E} := \sum_{i=1}^N g_i \hat{\sigma}_z^{(i)} \quad (18)$$

$2^N - 1$ energy levels for E given by $|n\rangle = |\uparrow\rangle_i |\downarrow\rangle_i \dots |\uparrow\rangle_i$ with $\epsilon_n = \sum_{i=1}^N (-1)^{n_i} g_i$.

$$e^{-it\hat{H}_{\text{int}}} |\psi\rangle = e^{-it\hat{H}_{\text{int}}} (a|0\rangle + b|1\rangle) \otimes \sum_{i=1}^{2^N-1} c_n |n\rangle = a|0\rangle |\epsilon_0(t)\rangle + b|1\rangle |\epsilon_1(t)\rangle \quad (19)$$

with $|\epsilon_0(t)\rangle = |\epsilon_1(-t)\rangle = \sum_{i=1}^{2^N-1} c_n e^{-it\frac{\epsilon_n}{2}}$.

The decoherence parameter

$$r(t) = \langle \epsilon_1(t) | \epsilon_0(t) \rangle = \sum_n |c_n|^2 e^{-i\epsilon_n t} \Rightarrow \boxed{\langle |r(t)| \rangle \sim 2^{-N}} \quad (20)$$

Recurrence time τ_{rec} exists since N is always finite. $g_i = g \forall i$ and $\sum_{i=1}^{2^N-1} c_n |n\rangle = \otimes_{i=1}^N \frac{1}{\sqrt{2}} (|\downarrow\rangle_i + |\uparrow\rangle_i)$

give $r(t) = [\cos(gt)]^N$ with $\tau_{\text{rec}} = \frac{\pi}{g}$. Highly improbable, typically $\tau_{\text{rec}} \propto N!$

Environmental superselection

Interaction with the environment selects the preferred basis.

$$|\psi_{\pm}\rangle \otimes |E_0\rangle \mapsto \frac{1}{\sqrt{2}} \left(|\psi_1\rangle \otimes |E_1\rangle \pm |\psi_2\rangle \otimes |E_2\rangle \right) \quad (21)$$

$|\psi_{\pm}\rangle$ get entangled with the environment.

Superselected states are the ones that get **least entangled** with the environment ($|\psi_i\rangle$).

Typically $\hat{H} = \hat{H}_S + \hat{H}_E + \hat{H}_{\text{int}}$.

Quantum measurement limit: $\hat{H} \approx \hat{H}_{\text{int}}$. Typically $\hat{H}_{\text{int}} = \hat{S} \otimes \hat{E}$, thus eigenstates of \hat{S} get selected

$$e^{-it\hat{S} \otimes \hat{E}} |s_i\rangle \otimes |E_0\rangle = |s_i\rangle \otimes e^{-it\lambda_i \hat{E}} |E_0\rangle := |s_i\rangle \otimes |E_i(t)\rangle \quad (22)$$

Quantum limit: $\hat{H} \approx \hat{H}_S$. Constants of motion of S (energy) get selected.

Lindblad equation

In general $\hat{\rho}_S(t) = \text{tr}_E \left\{ \hat{U}(t) \hat{\rho}_{S,E}(0) \hat{U}^\dagger(t) \right\}$. Master equations give approximations

$$\partial_t \hat{\rho}_S(t) = -i[\hat{H}_S, \hat{\rho}_S] + \hat{D}[\hat{\rho}_S(t)] \quad (23)$$

Lindblad equation: most general master equation preserving the **positivity** of $\hat{\rho}_S$.

$$\partial_t \hat{\rho} = -i[\hat{H}_S, \hat{\rho}_S] - \frac{1}{2} g^{mn} \left(\hat{Q}_m \hat{Q}_n \hat{\rho} + \hat{\rho} \hat{Q}_m \hat{Q}_n - 2 \hat{Q}_m \hat{\rho} \hat{Q}_n \right) \quad (24)$$

with $g^{mn} \in \mathbb{R}$ and $u_m u_n g^{mn} \geq 0$, $\forall u$.

Details of the contraction

The contraction

$$\mathbf{N} \mapsto c^{-1}\mathbf{N} \ , \ \mathbf{P} \mapsto c^{-1}\mathbf{P} \ , \ \mathbf{K} \mapsto c^{-2}\mathbf{K} \quad (25)$$

gives

$$\Pi_0 = \frac{1}{c^2\mathbf{K}}P_0 + \sqrt{\mathbb{1} - \frac{1}{c^4\mathbf{K}^2}(P_0^2 - c^2\mathbf{P}^2)} \rightarrow \mathbb{1} \ , \ \Pi_0^{-1} = \frac{-\frac{1}{c^2\mathbf{K}}P_0 + \sqrt{\mathbb{1} - \frac{1}{c^4\mathbf{K}^2}(P_0^2 - c^2\mathbf{P}^2)}}{\mathbb{1} - \frac{1}{c^4\mathbf{K}^2}(P_0^2 - c^2\mathbf{P}^2)} \rightarrow \mathbb{1} \quad (26)$$

thus

$$\begin{aligned} \Delta P_0 &= P_0 \otimes \Pi_0 + \Pi_0^{-1} \otimes P_0 + \frac{1}{c^2\mathbf{K}}cP_n \Pi_0^{-1} \otimes cP^n \rightarrow P_0 \otimes \mathbb{1} + \mathbb{1} \otimes P_0 + \frac{1}{\mathbf{K}}P_n \otimes P^n \\ S(P_0) &= -P_0 + \frac{1}{c^2\mathbf{K}}c^2\mathbf{P}^2 \Pi_0^{-1} \rightarrow -P_0 + \frac{1}{\mathbf{K}}\mathbf{P}^2 \ , \ cS(P_i) = -cP_i \Pi_0^{-1} \Rightarrow S(P_i) = -P_i \end{aligned} \quad (27)$$

Representation on Hilbert Space

To study the effect of these deformations we need to represent the algebra on the algebra of operators of quantum systems $\mathcal{A}_{\mathcal{H}}$.

This is done with the standard $G \mapsto -iR(G)$ procedure.

For the coalgebra this means

$$\Delta P_0 = P_0 \otimes \mathbb{1} + \mathbb{1} \otimes P_0 - \frac{i}{\kappa} P_n \otimes P^n, \quad S(P_0) = -P_0 - \frac{i}{\kappa} \mathbf{P}^2 \quad (28)$$