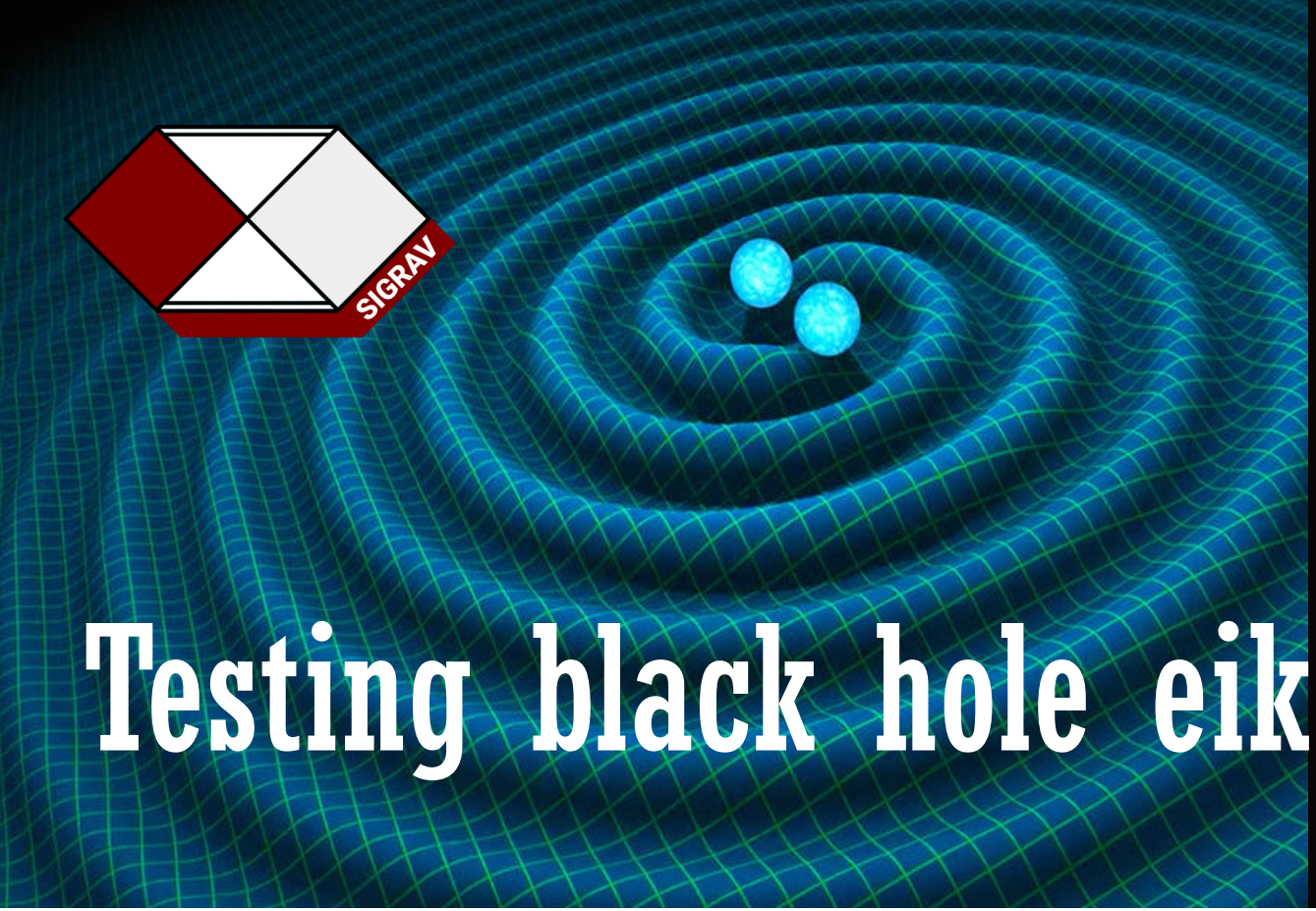
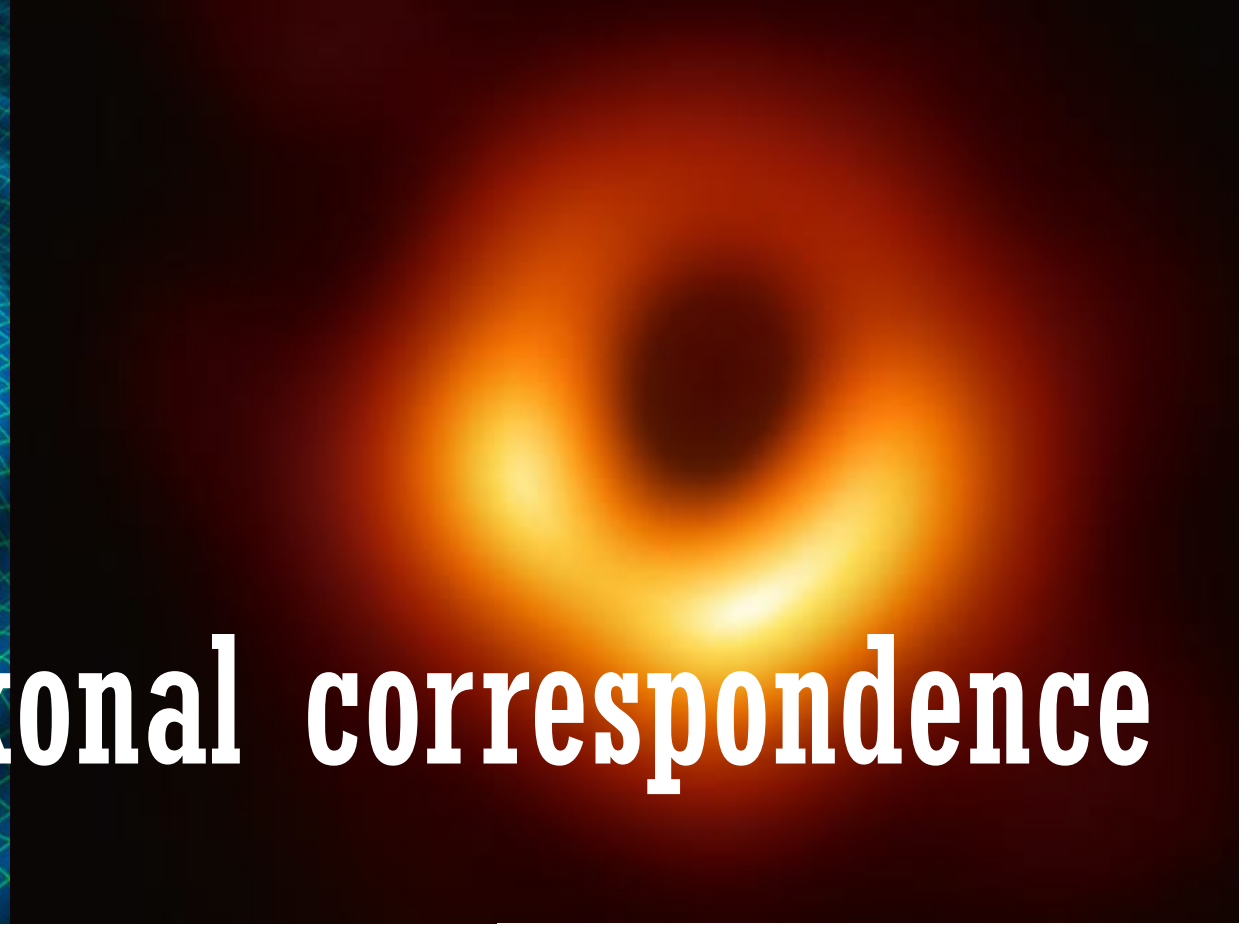




Testing black hole eikonal correspondence

Che-Yu Chen

iTHEMS (SPDR), RIKEN, JAPAN

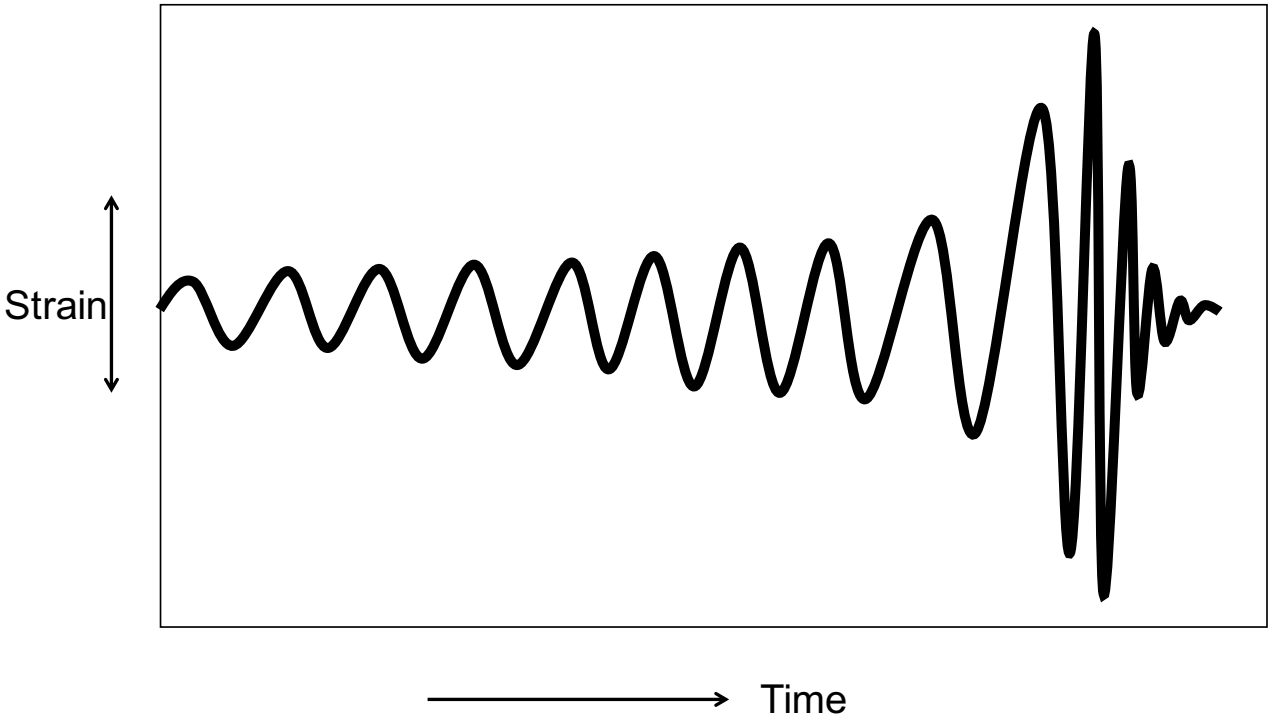
- **CYC**, Yu-Jui Chen (NTU), Meng-Yuan Ho (NTU), Yung-Hsuan Tseng (NTHU)

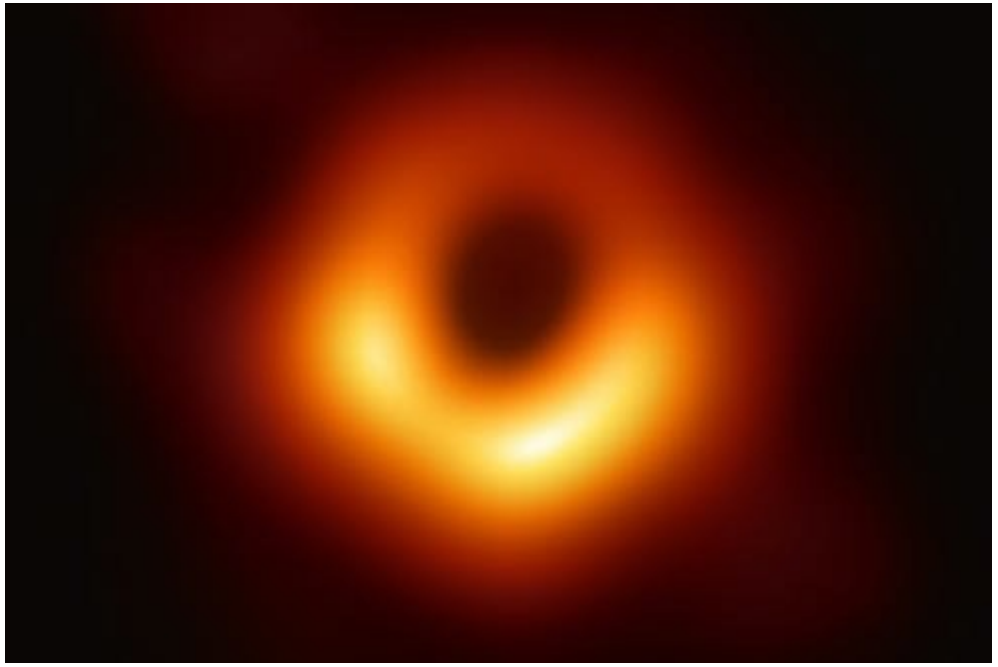
  Phys. Lett. B 845, 138153 (2023)

RIKEN's Programs for Junior Scientists

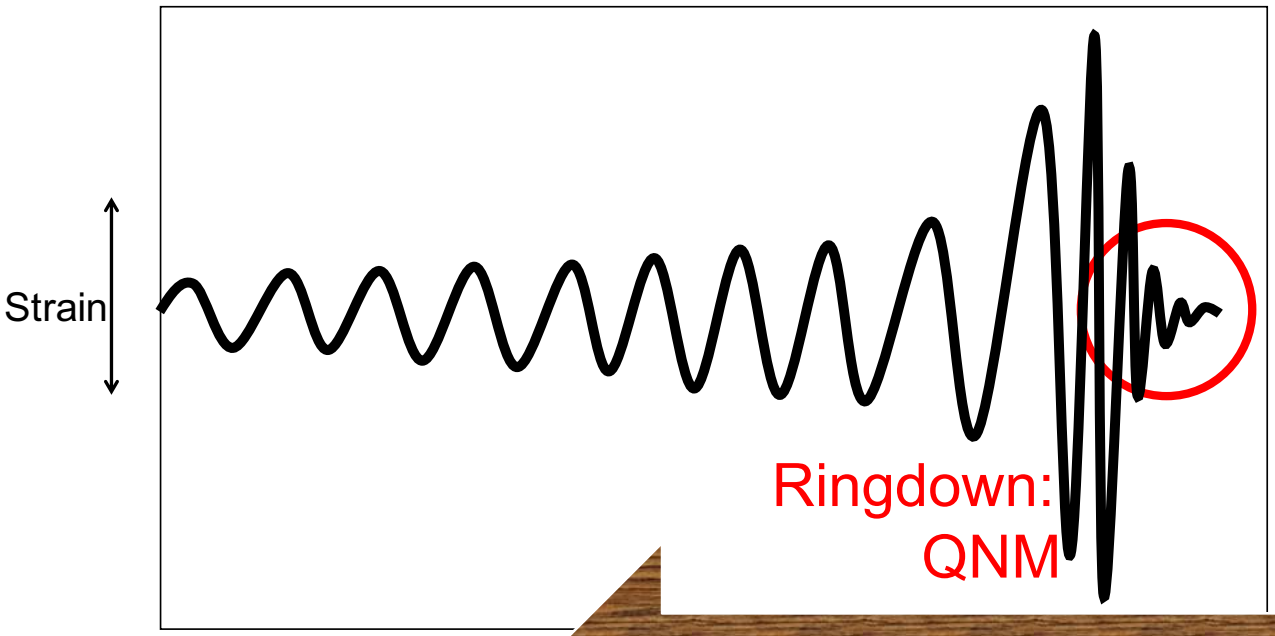
iTHEMS RIKEN Interdisciplinary Theoretical and Mathematical Sciences Program

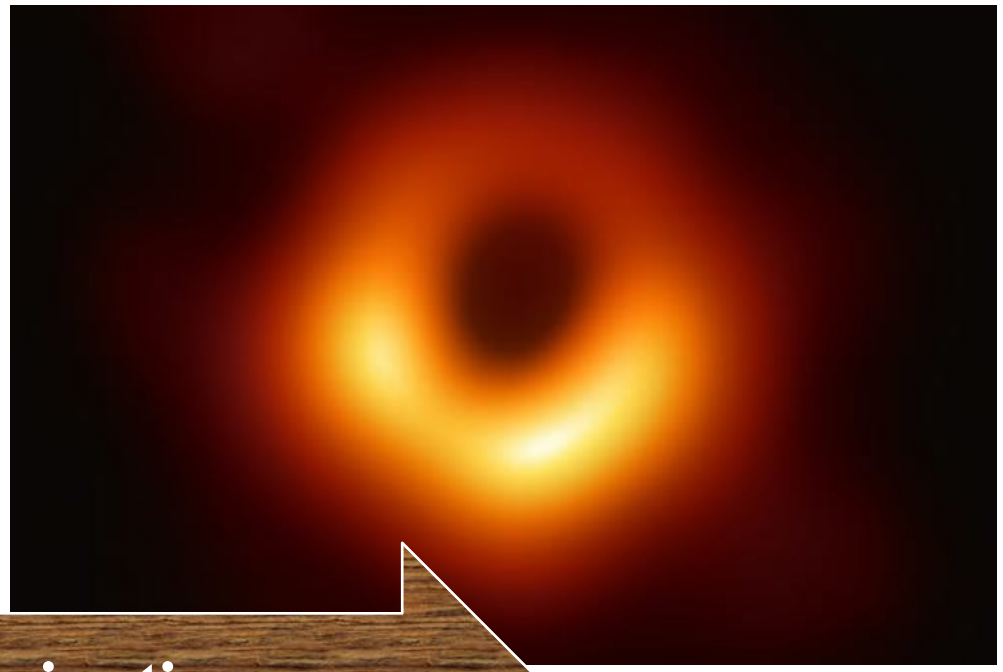
Black Hole Observations





Eikonal QNMs Correspondence





Geometric-optics approximations

ω_R ↔ Angular frequency Ω_c on photon sphere ↔ Size of shadow image

ω_I ↔ Lyapunov exponent λ_c on photon sphere ↔ Higher-order ring structures

Talks of Cardoso and Maggio

Cardoso, *et al.* (2009), Jusufi (2020), Cuadros-Melgar *et al.* (2020)
Jusufi (2020), Yang (2021) **CYC**, Chiang, Tsao (2023)

Eikonal Correspondence Violation

- The eikonal correspondence may be violated
  - Nonlinear electrodynamics
  - Nonminimal coupling between matter and curvature
  - String-inspired models

Nomura, Yoshida (2022)

CYC, Bouhmadi-López, Chen (2019) (2021) **CYC**, Chen (2020)

Cardoso, Gualtieri (2010) Konoplya, Stuchlik (2017) Moura, Rodrigues (2021)

- A preliminary proposal (i.e., nonrotating BH) for testing eikonal correspondence based on ringdown and image observations of black holes with similar masses

- The violation of the correspondence could be a smoking gun of physics beyond GR

QNM Observables

$$\gamma_l^{QNM} \equiv 2l \frac{|\omega_I|}{\omega_R}$$

If the eikonal correspondence is satisfied:

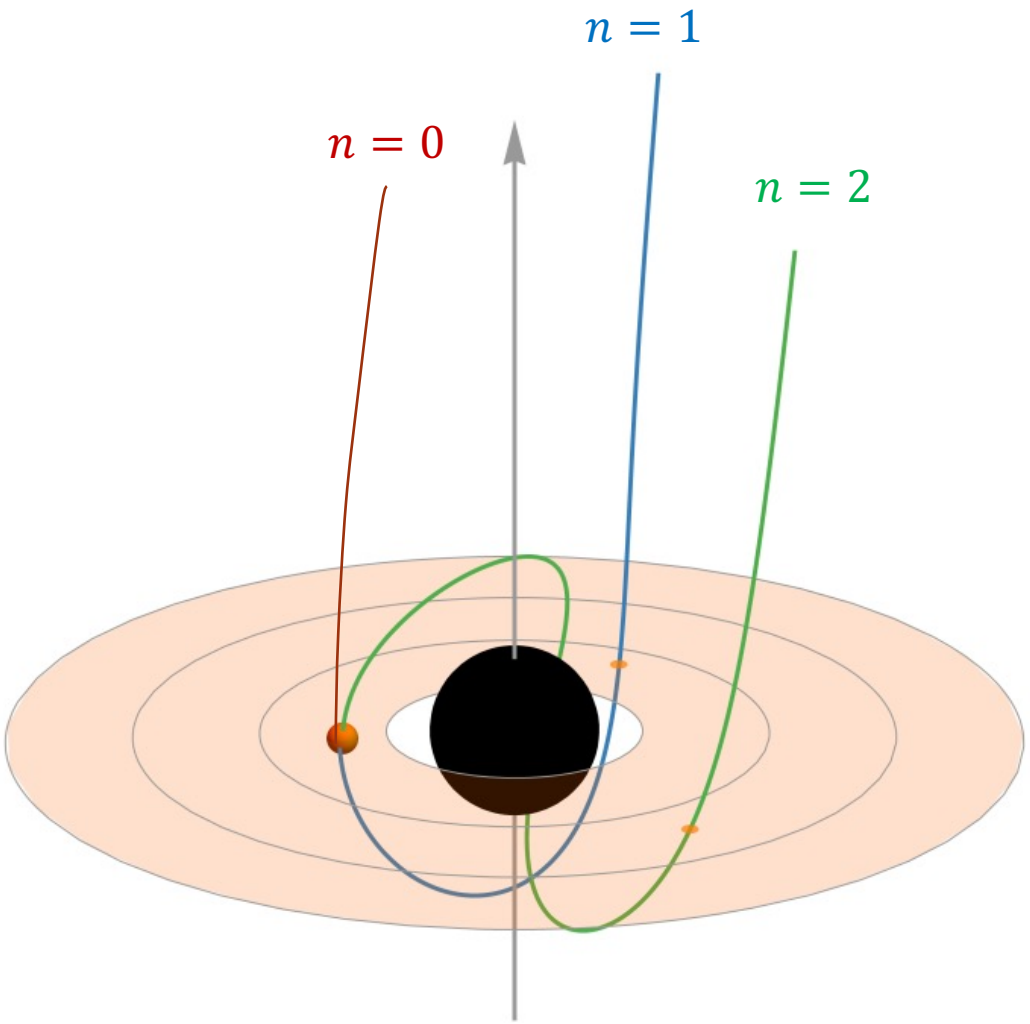
$$\gamma_l^{QNM} = \left(1 - \frac{1}{2l}\right)\gamma + O(l^{-2})$$

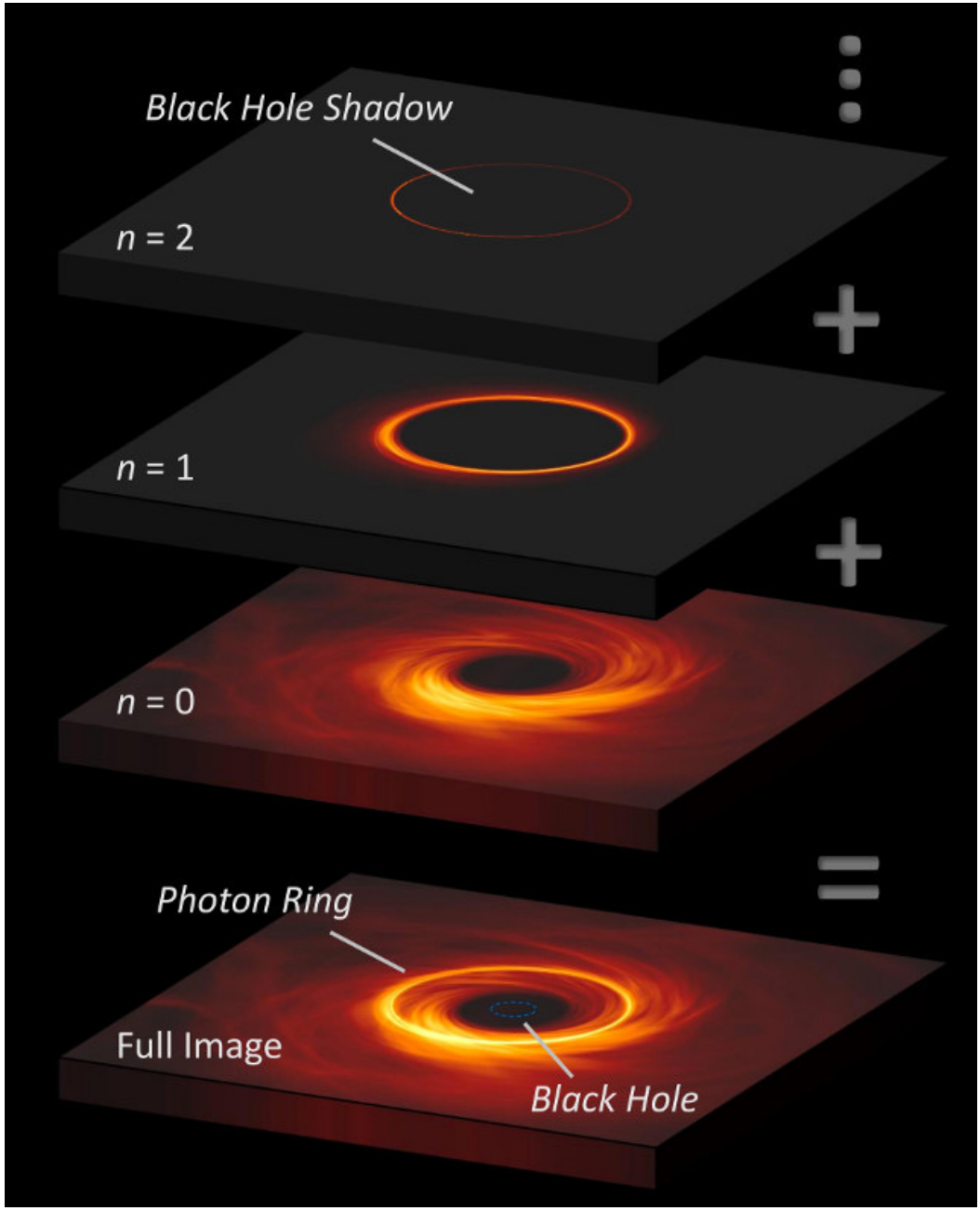
$\gamma \equiv \lambda_c/\Omega_c$ (critical exponent)

γ_l^{QNM} converges to γ from below when $l \to \infty$

CYC *et al*. (2023)

Photon Ring Observables





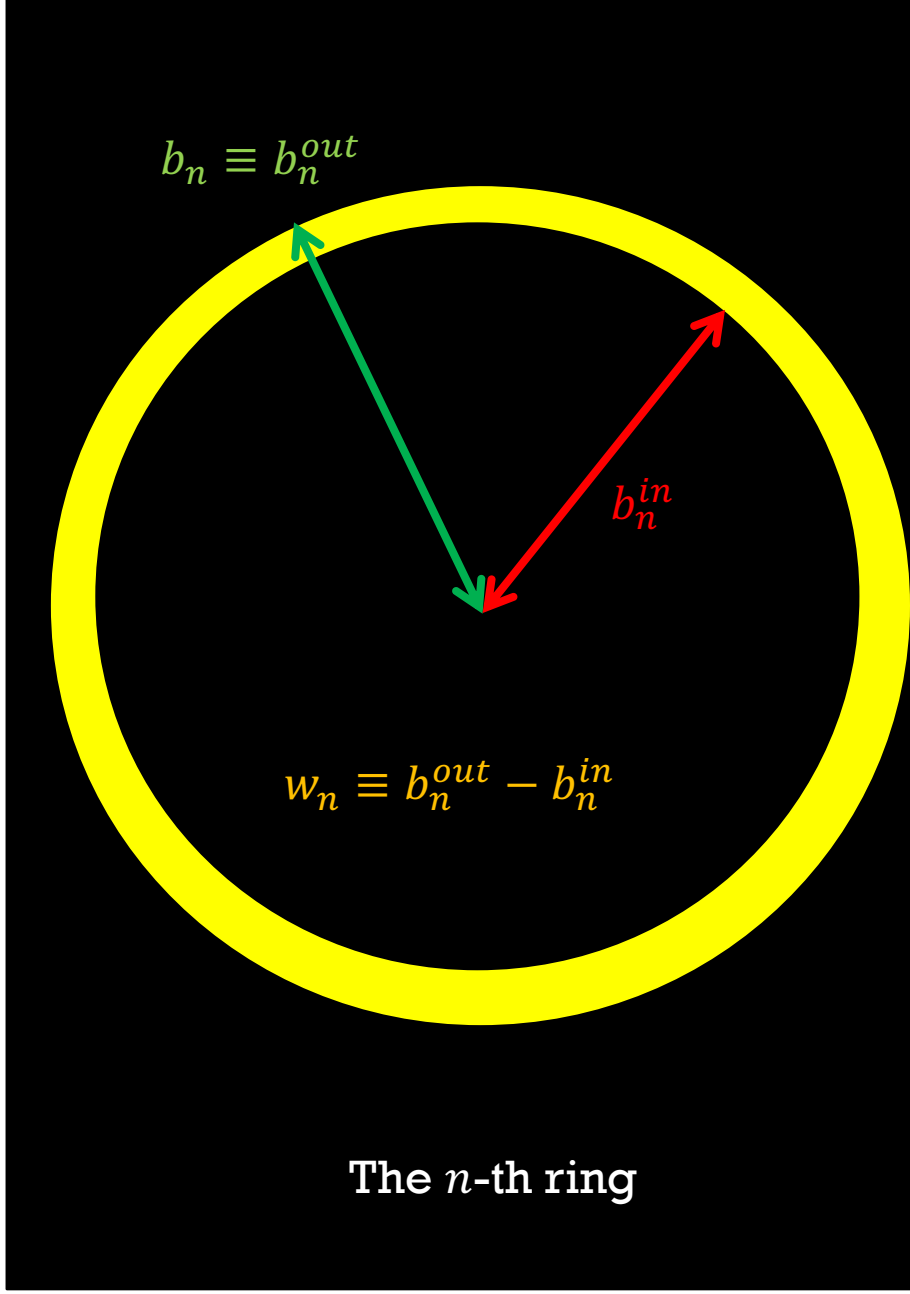
Gralla, et al. (2020)

Photon Ring Observables

$$\gamma_n^w \equiv \frac{1}{\pi} \ln \frac{w_n}{w_{n+1}}$$

$$\gamma_n^b \equiv \frac{1}{\pi} \ln \frac{b_n - b_{n+1}}{b_{n+1} - b_{n+2}}$$

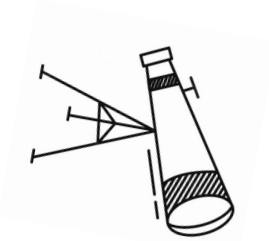
Kocherlakota, Rezzolla, Roy, Wielgus (2023)

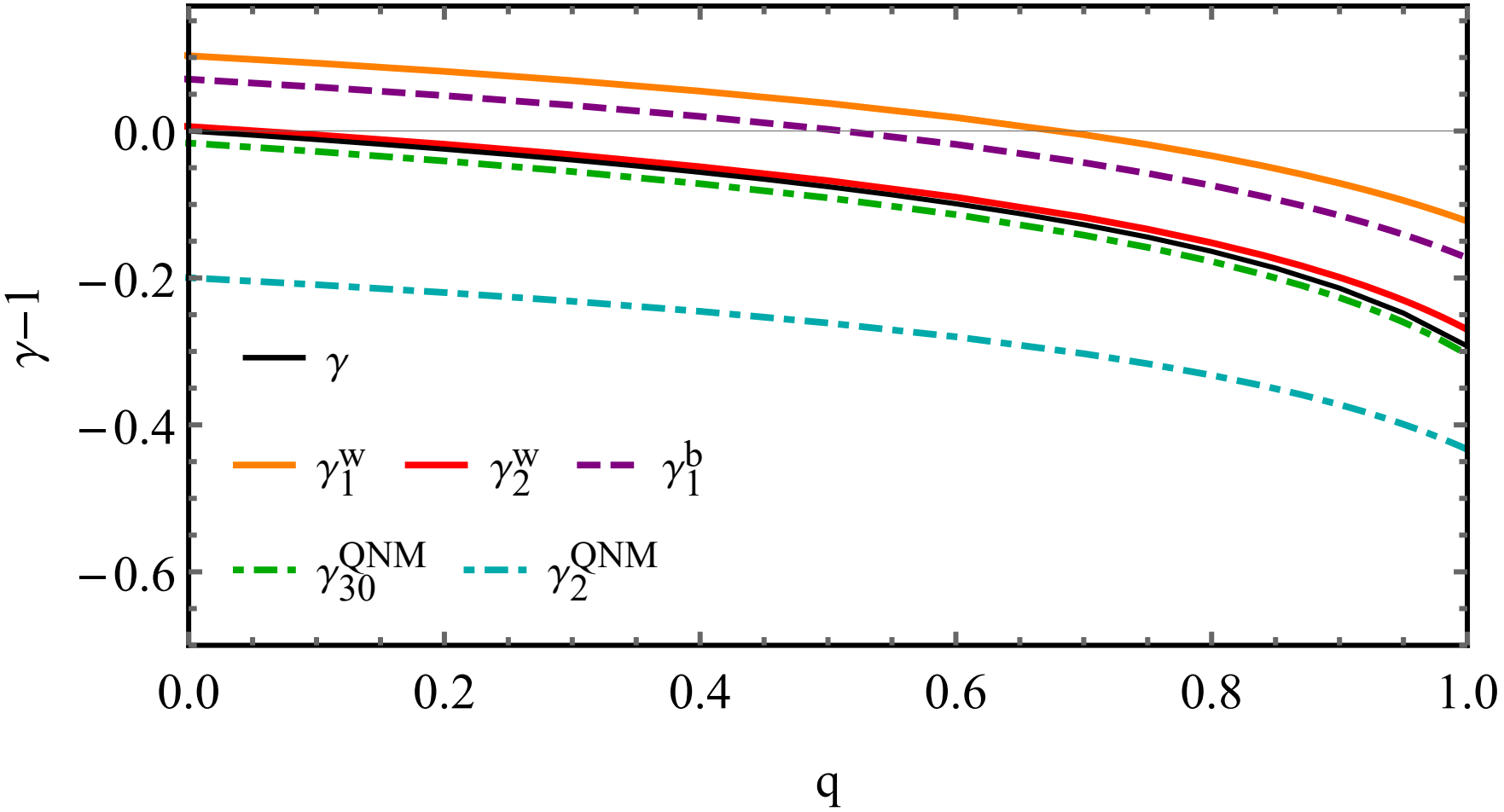


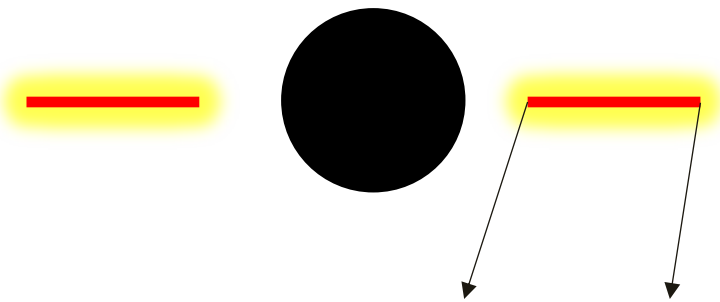
- Two ring observables converge to γ <u>from above</u> when $n \to \infty$

$\gamma \equiv \lambda_c / \Omega_c$ (critical exponent)

Example: Reissner-Nordström Black Holes

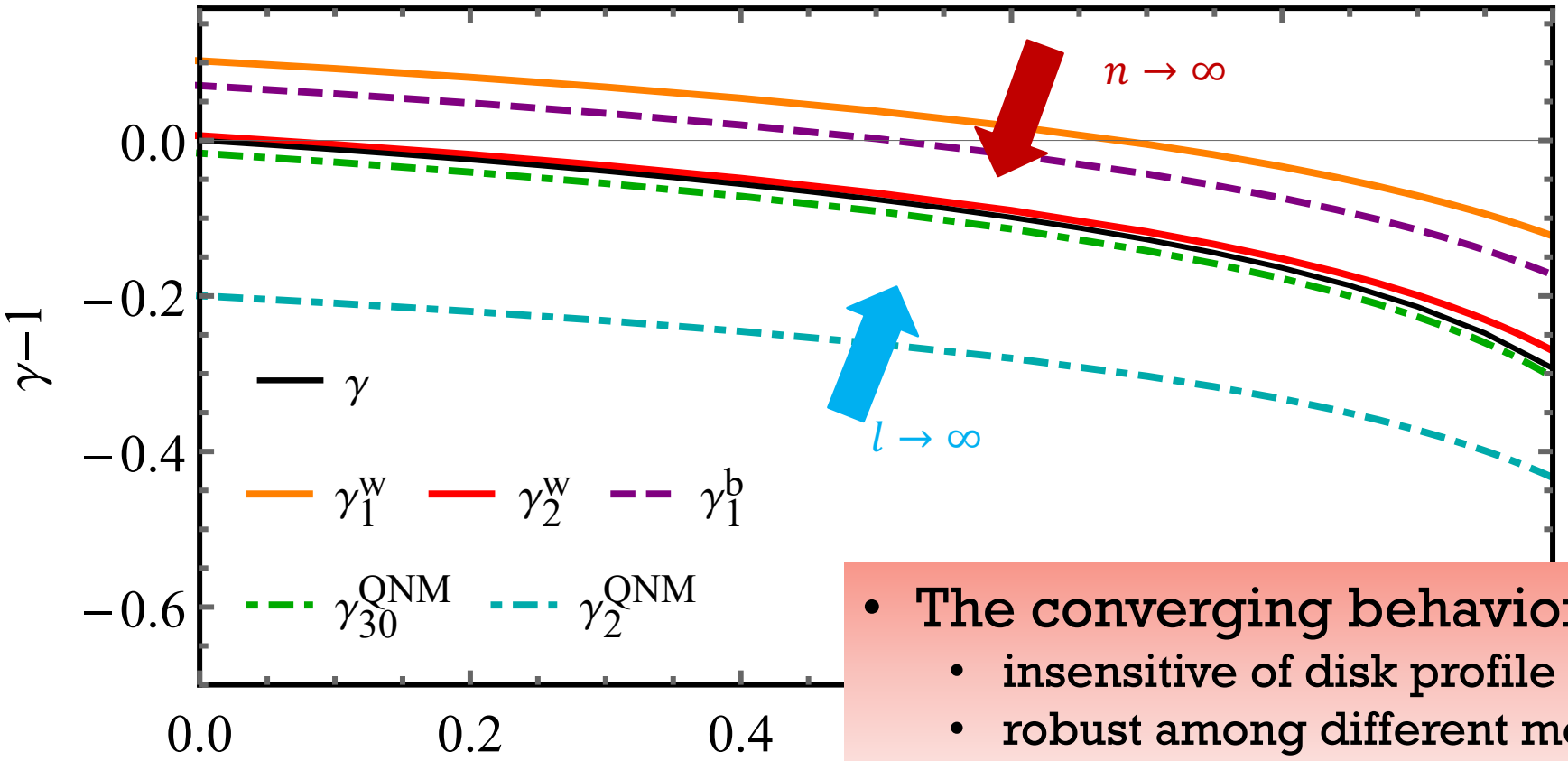


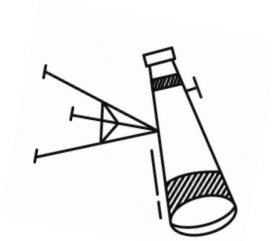


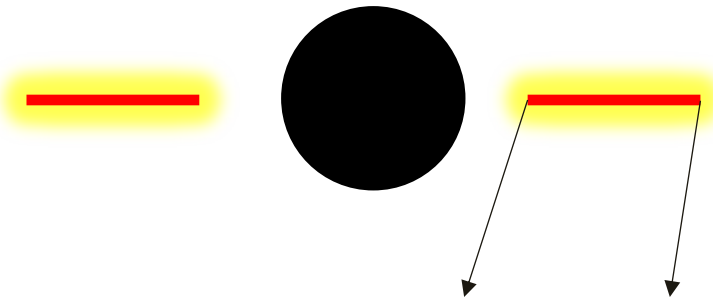


Disk size: $r_{ISCO} \sim 100M$

Example: Reissner-Nordström Black Holes







Disk size: $r_{ISCO} \sim 100M$

- The converging behavior is
  - insensitive of disk profile
  - robust among different metrics

Example: Dynamical Chern-Simons Gravity

$$S = \int d^4x\sqrt{-g}\left(\kappa R + \frac{\alpha}{4}\vartheta RR^*\right) - \frac{\beta}{2}\int d^4x\sqrt{-g}(\partial\vartheta)^2$$

CS correction dynamical scalar field

- Parity-violating term from the CS correction

Jackiw, Pi (2003) Alexander, Yunes (2009)

- Motivated from string theory

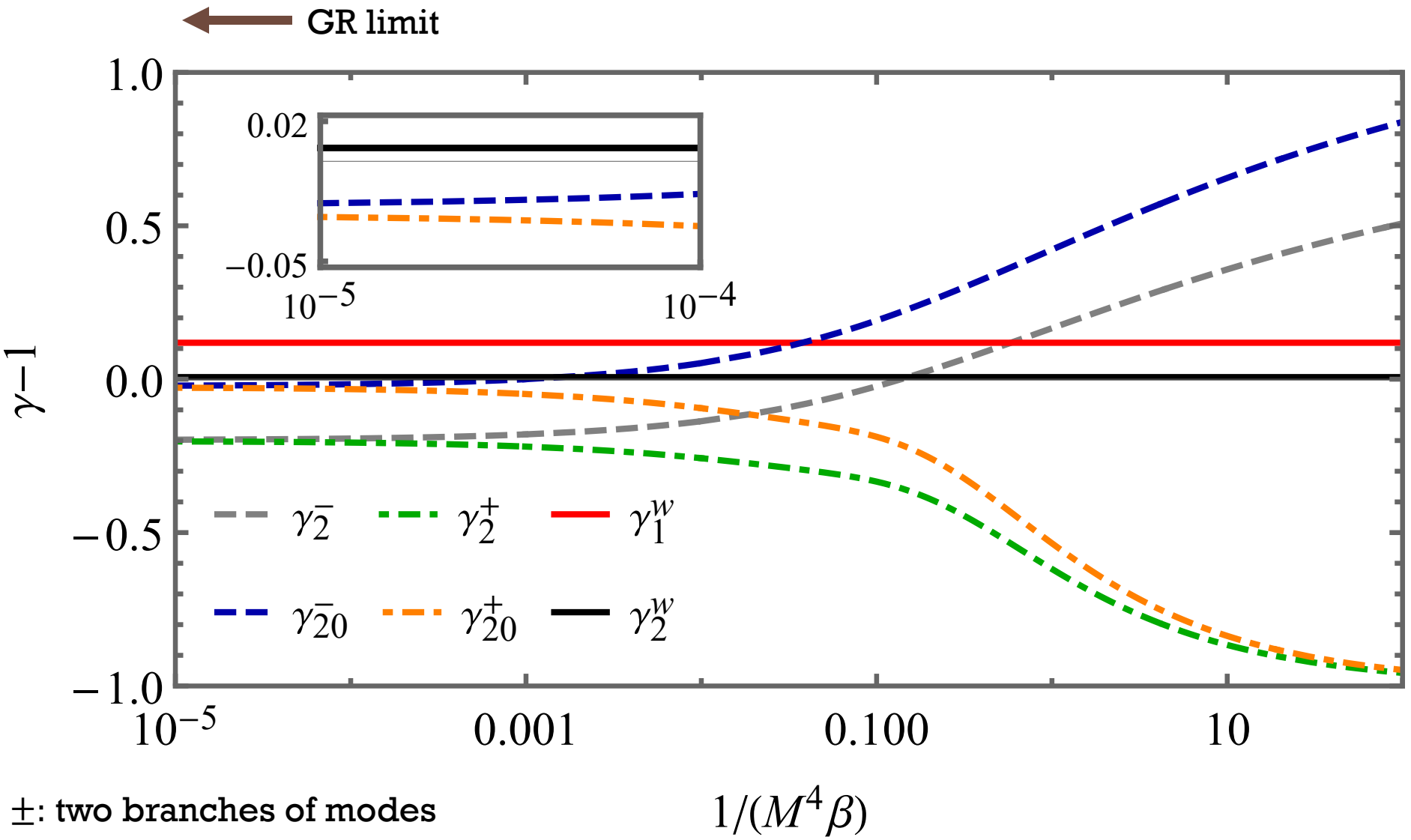
Campbell, Kaloper, Madden, Olive (1993) Moura, Schiappa (2006)

- Schwarzschild metric: an exact vacuum solution
- Schwarzschild perturbations: Axial mode coupled to scalar modes

Cardoso, Gualtieri (2010) Molina, Pani, Cardoso, Gualtieri (2010) Motohashi, Suyama (2011)(2012) Kimura (2018)

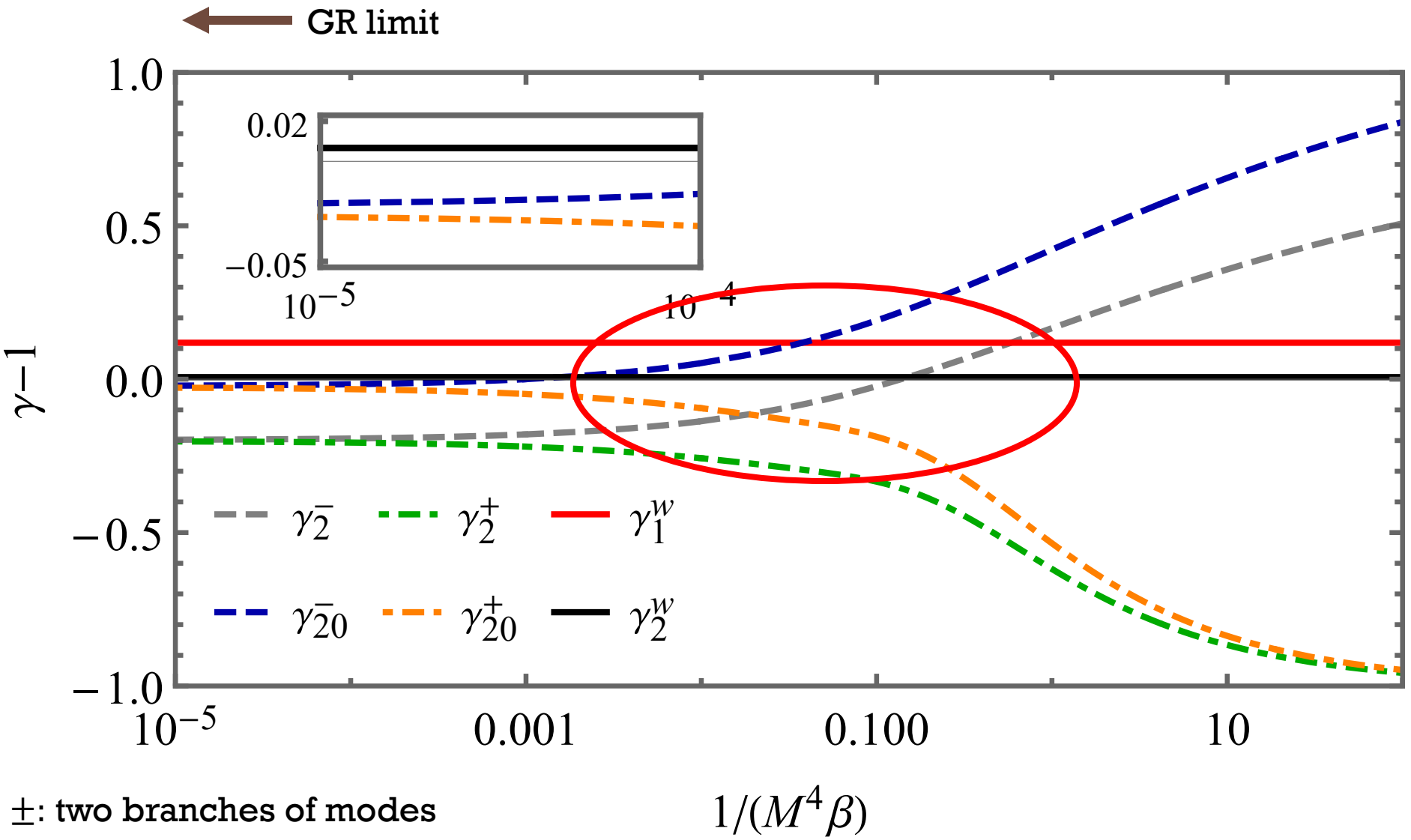
- The modes violate eikonal correspondence

Example: Dynamical Chern-Simons Gravity



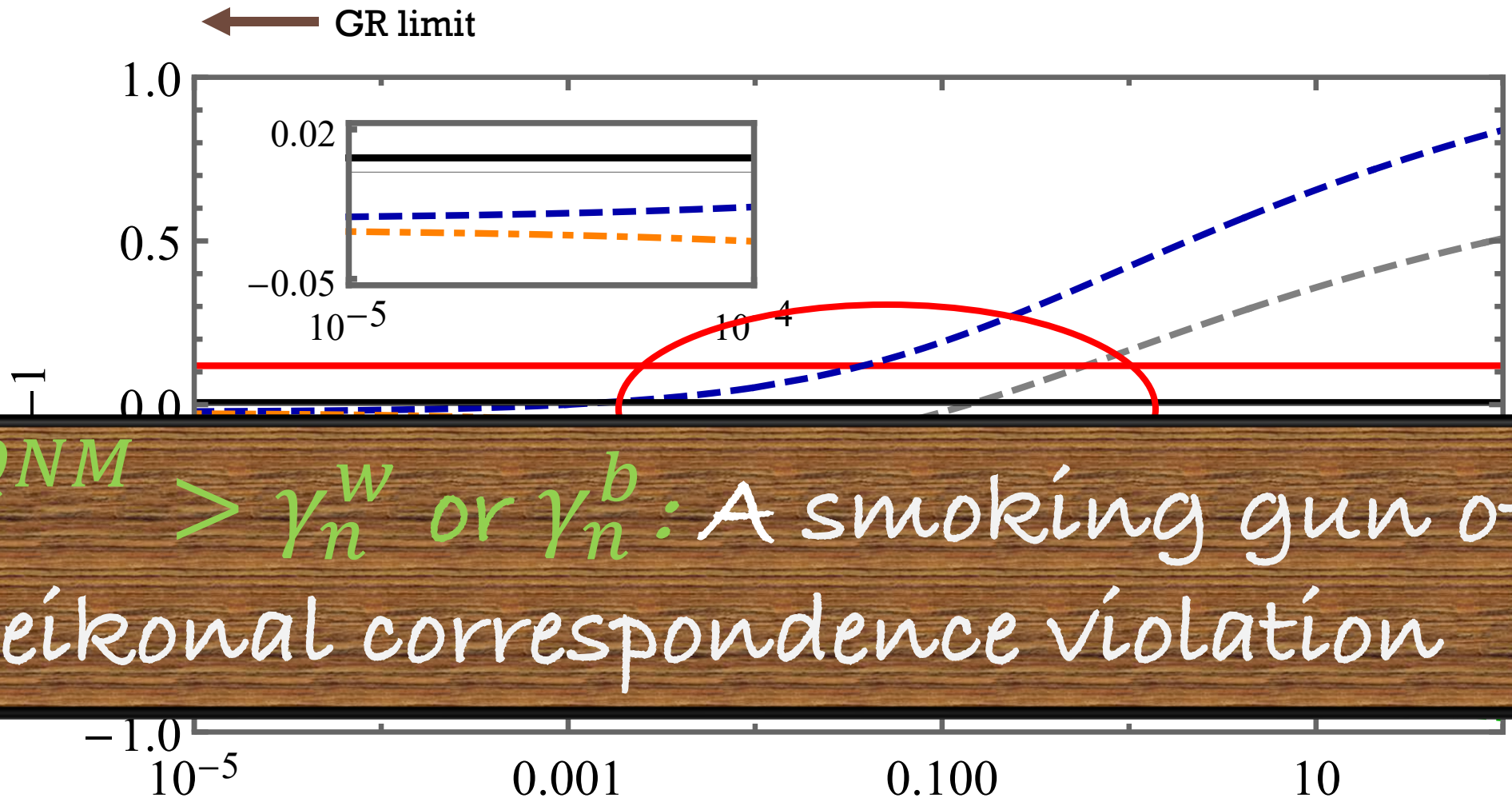
±: two branches of modes

Example: Dynamical Chern-Simons Gravity



±: two branches of modes

Example: Dynamical Chern-Simons Gravity



$\gamma_l^{QNM} > \gamma_n^w$ or γ_n^b: A smoking gun of eikonal correspondence violation

±: two branches of modes

Conclusions

- Correspondence between eikonal QNMs and BH images
- Testing the correspondence
  - QNM observables and photon ring observables
  - They converge to critical exponent γ from opposite directions
  - Smoking gun of eikonal correspondence violation, place constraints… etc
- Future:
  - The issues of different target masses
  - Rotating cases