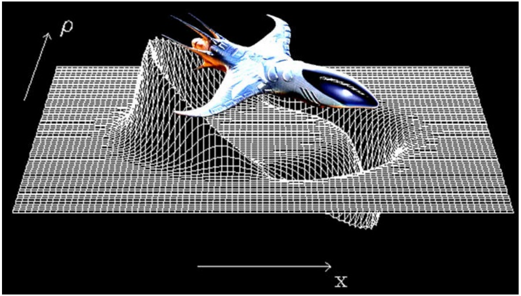
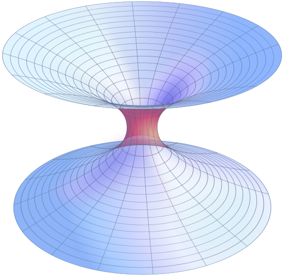
On the Wormhole - Warp Drive Correspondence





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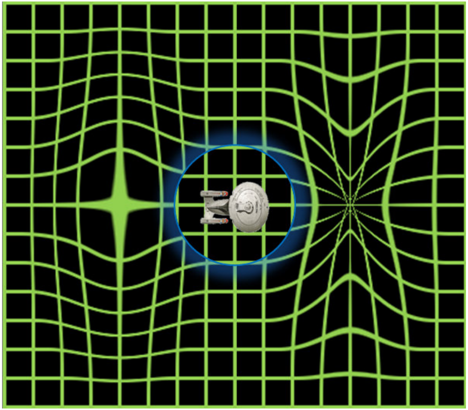
What is a warp drive?



A "bubble" propagating through some background spacetime. The observers inside the bubble are locally at rest (they move *with* the spacetime, not *through* it), so no external energy source required!

Natario (2001):

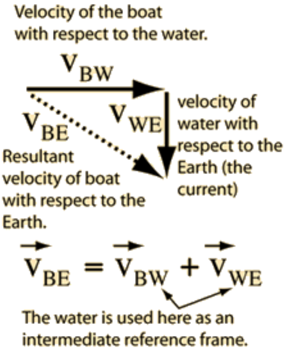
- A metric of the form

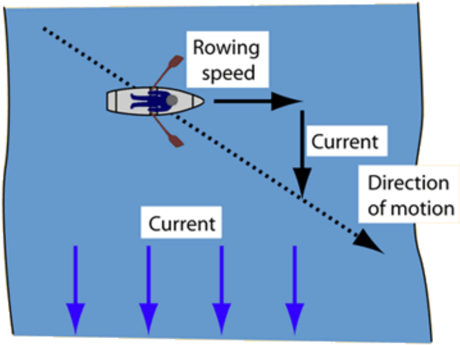
$$-dt^2 \; + \; (d\vec{x} - \vec{\beta}dt)^2 \tag{1}$$

- The vector $\vec{\beta}$ is given by

$$\vec{\beta} \; = \; (1-f)\vec{v}_{out}(\vec{r}) \; + \; f\vec{v}_{in}(t) \; , \tag{2}$$

where $f(|\vec{r} - \vec{r}_s(t)|)$ is a (usually step-type) function, equal to 1 inside the bubble ($|\vec{r} - \vec{r}_s(t)| < R$), and to 0 outside; $\vec{v}_{in} = \frac{d\vec{r}_s}{dt}$.





The warp bubble can be thought of as a boat in a current: v_{out} is the velocity of the current, and v_{in} is the sum of the boat's rowing speed and the current velocity. If $v_{out} = v_{in}$, it means the boat is not rowing, just flowing along the current (normal gravitational freefall).

H. G. Ellis (2004):

A black hole is described by the Schwarzschild metric:

$$-(1-\frac{2GM}{r})dt^2+\frac{dr^2}{1-\frac{2GM}{r}}+r^2d\Omega^2\ . \tag{3}$$

We can switch to Gullstrand–Painlevé coordinates:

$$t\ =\ T\ +\ \int\ dr\ \frac{\sqrt{\frac{2GM}{r}}}{1-\frac{2GM}{r}}\ , \tag{4}$$

resulting in

$$-dT^2\ +\ (dr+\sqrt{\frac{2GM}{r}}dT)^2\ +\ r^2d\Omega^2 \tag{5}$$

- This corresponds to a warp drive–type metric with

$$\vec{\beta} \;=\; \sqrt{\frac{2GM}{r}}\frac{\vec{r}}{r} \tag{6}$$

- Now, if we introduce an actual warp drive that navigates in the background Schwarzschild field, we would have:

$$\vec{\beta} \;=\; (1-f)\sqrt{\frac{2GM}{r}}\frac{\vec{r}}{r} \;+\; f\vec{v}_{in}(t) \tag{7}$$

The Morris–Thorne metric

$$-e^{2\Phi(r)}dt^2 \; + \; \frac{dr^2}{1-\frac{b(r)}{r}} \; + \; \vec{r}^2 d\Omega^2 \tag{8}$$

can be transformed via

$$t \; = \; T \; + \; \int \; dr \; \frac{\sqrt{e^{-2\Phi(r)}-1}}{\sqrt{1-\frac{b(r)}{r}}} \tag{9}$$

to obtain

$$-dT^2 + (g(r)dr - \beta dT)^2 + r^2 d\Omega^2 \tag{10}$$

or

$$-dT^2 + h_{ij}(dx^i + \beta^i dT)(dx^j + \beta^j dT) \tag{11}$$

Generalization to wormholes

Here

$$\beta^i = \frac{x_i}{r}\beta \, , \, \beta = \sqrt{1-e^{2\Phi}} \, , \, g(r) = \frac{e^{\Phi}}{\sqrt{1-\frac{b}{r}}} \, , \qquad (12)$$

$$h_{ij} = \delta_{ij} + (g^2-1)\frac{x_i x_j}{r^2} \qquad (13)$$

This is a class of metrics more generic than Natario! (Natario corresponds to $g = 1$)
Nonetheless, we can follow the same procedure to embed a warp drive:

$$\beta^i = (1-f)\frac{x_i}{r}\beta + f\dot{r}_s^i(t) \, , \qquad (14)$$

Traversability Conditions

1. Flare–out:

$$b'(r) < \frac{b(r)}{r} \Leftrightarrow \left(\frac{1-\beta^2}{g^2}\right)' > 0 \tag{15}$$

2. Throat condition:

$$b(a) = a \tag{16}$$

3. Absence of horizon:

$$e^{2\Phi} > 0 \Leftrightarrow \beta < 1 \tag{17}$$

2+3 means that $g(r)$ has a singularity at a!

- To see if the singularity is a physical one or just a coordinate one, we can compute the Ricci scalar.
- Taking $(\vec{v}\vec{r}) = vr\cos\theta$, we may write the metric in spherical coordinates:

$$-(1-\Phi^2-\Psi^2)dt^2 + 2g\Phi dtdr + 2\Psi rdtd\theta + g^2dr^2 + r^2d\Omega^2 \ , \tag{18}$$

with

$$\Phi \ = \ (1-f)\beta + fgv_s\cos\theta \ , \ \Psi \ = \ -fv_s\sin\theta \ . \tag{19}$$

Ricci Scalar

Assuming ϕ-independence, the Ricci scalar for this metric is given by:

$$\mathcal{R} = \frac{1}{2r^2g^3}\Big(Ag' + B(g'^2 - gg'') + Cg + Dgg' + Eg^4 + \tag{20}$$

$$+Fg^5 + Gg^2 + Hg^2g' + Ig^2g'' + Jg^3 + K\Big) \tag{21}$$

with

$$B = -4r^2 f(1-f)\beta v\cos\theta \,, \tag{22}$$

$$D = 4r^2 v\cos\theta\partial_r\left((1-f)f\beta\right) \,, \tag{23}$$

$$E = -2\beta\partial_\theta\left(f\cos\theta\right) \,, \tag{24}$$

$$F = \left(\partial_\theta\left(f\cos\theta\right)\right)^2 \,, \tag{25}$$

$$H = 4fv^2r\cos\theta\left(f\cos\theta + 3r\partial_r f\cos\theta - 2\partial_\theta f\sin\theta\right) \,, \tag{26}$$

$$I = 4r^2f^2v^2\cos^2\theta \,. \tag{27}$$

For nonzero f, it's not possible to set to zero all the divergent terms' coefficients, so the singularity is real! There's also a horizon!

What about a different approach? Let's also interpolate the inner metric:

$$\beta^i = (1-f)\frac{x_i}{r}\beta + f\dot{r}_s^i(t) \,, \tag{28}$$

$$h_{ij} = \delta_{ij} + (1-f)(g^2-1)\frac{x_i x_j}{r^2} \tag{29}$$

The metric then has the form:

$$-(1 - G\frac{\Phi^2}{g^2} - \Psi^2)dt^2 + 2G\frac{\Phi}{g}dtdr + 2\Psi r dt d\theta + Gdr^2 + r^2 d\Omega^2 \ , \quad (30)$$

with

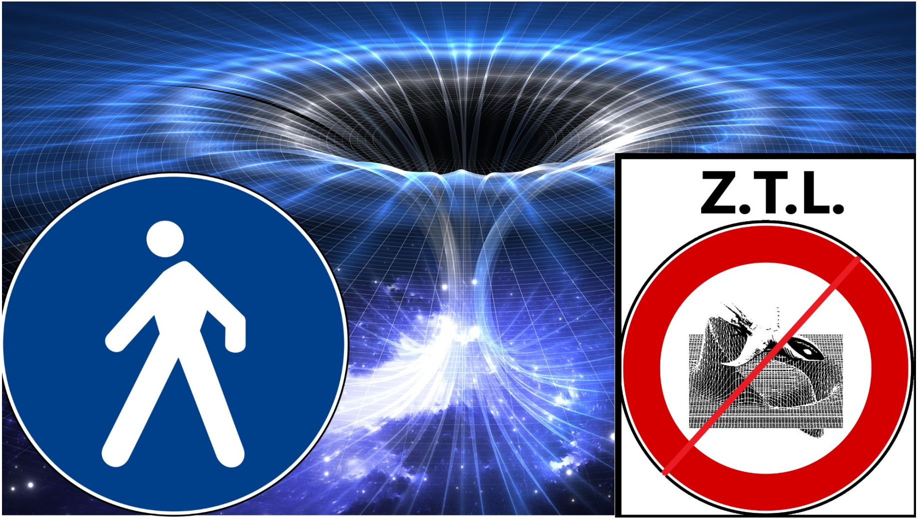
$$\begin{aligned} \Phi &= (1-f)\beta + fgv_s\cos\theta \ , && (31) \\ \Psi &= -fv_s\sin\theta \ , && (32) \\ G &= f + (1-f)g^2 && (33) \end{aligned}$$

Outside the bubble, it gives Morris–Thorne, and inside it yields just an Alcubierre–type metric, but we also have the transition region where $0 < f < 1$! When the warp drive "shell" approaches the wormhole, it would still "feel" both a singularity and a horizon, due to the term

$$g^2(1-f)f^2v_s^2\cos^2\theta + 2g(1-f)^2 f\beta v_s\cos\theta \quad (34)$$

in g_{00}!

Paradoxical conclusion: warp drives can traverse wormholes with horizons, but encounter problems with humanly traversable ones!


Z.T.L.

We recently observed a black hole, but we still haven't seen wormholes and warp drives!

What are the consequences for "real world" physics?

Analogue Gravity

William Unruh (1981): Fluid systems can simulate gravitational systems, the flow of the fluid imitates gravitational force!

The fundamental equations of hydrodynamics are

$$\partial_t \rho + \vec{\nabla}(\rho \vec{v}) = 0 \tag{35}$$

$$\partial_t \vec{v} = \vec{v} \times (\vec{\nabla} \times \vec{v}) - \frac{1}{\rho} \vec{\nabla} p - \vec{\nabla}(\frac{1}{2} v^2) \tag{36}$$

Assuming irrotationality ($\vec{v} = \vec{\nabla}\phi$) and barotropy ($\rho$ depends only on p), we can do perturbation theory:

$$\rho = \rho_0 + \delta\rho \ , \ \phi = \phi_0 + \delta\phi \tag{37}$$

to obtain

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \delta\phi) = 0 \tag{38}$$

This is exactly the equation for a scalar field in the effective metric

$$g_{\mu\nu} dx^\mu dx^\nu \; = \; \frac{\rho}{c} \left[-(c^2 - v^2) dt^2 + 2 v_i dx^i dt + (dx^i)^2 \right] \, , \qquad (39)$$

with the sound speed

$$c^2 \; = \; (\frac{\partial \rho}{\partial p})^{-1} \, . \qquad (40)$$

It looks exactly like the warp drive metric!

By substituting

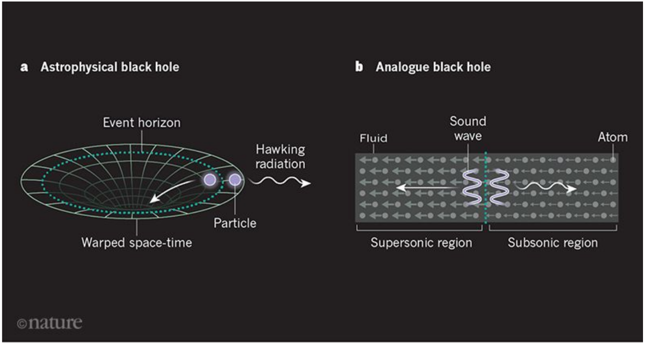
$$dt \;\to\; dt + v dr \;, \tag{41}$$

we obtain

$$\frac{\rho}{c}\left[-(c^2 - v^2)dt^2 + \frac{c^2}{c^2 - v^2}dr^2 + r^2 d\Omega^2\right] \;, \tag{42}$$

which is the Schwarzschild metric for the radial velocity profile

$$v_r = c\sqrt{\frac{2GM}{r}} \;. \tag{43}$$



- Experiments with analogue BHs were done in 1+1D, with a tube of Bose-Einstein condensate of Rb atoms (Steinhauer et al.)
- In principle, one can simulate the warp drive propagation by a wave propagating with velocity w on a BH background:

$$v(r,t) \;=\; c\sqrt{\frac{2GM}{r}} + \delta v(r-wt)\;. \tag{44}$$

What about Analogue Wormholes?

- The **topology** of the wormhole cannot be simulated, but one can try to simulate the near–throat physics.
- Additional degrees of freedom needed $\Rightarrow$ vary the speed of sound c. External electromagnetic fields (Feshbach resonance, etc.)?
- By making identifications

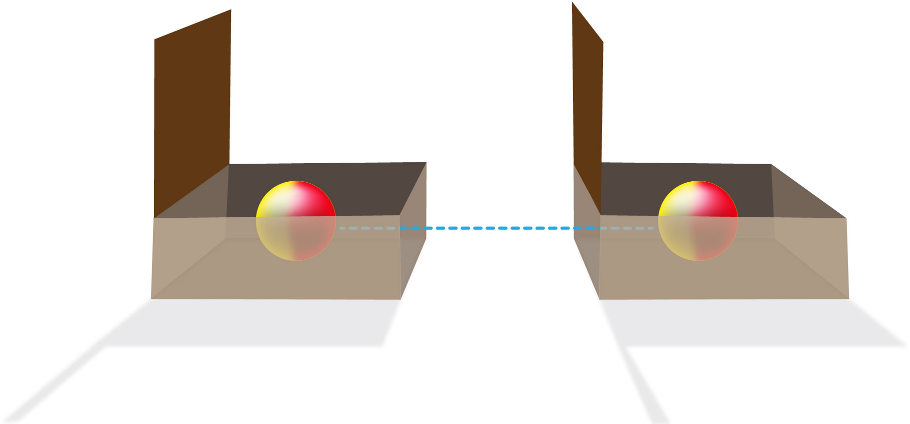
$$c_s(r) \;=\; g(r) \;=\; \frac{e^{\Phi(r)}}{\sqrt{1-\frac{b(r)}{r}}}\,,\; v(r) \;=\; g(r)\sqrt{\frac{b(r)}{r}}\,, \tag{45}$$

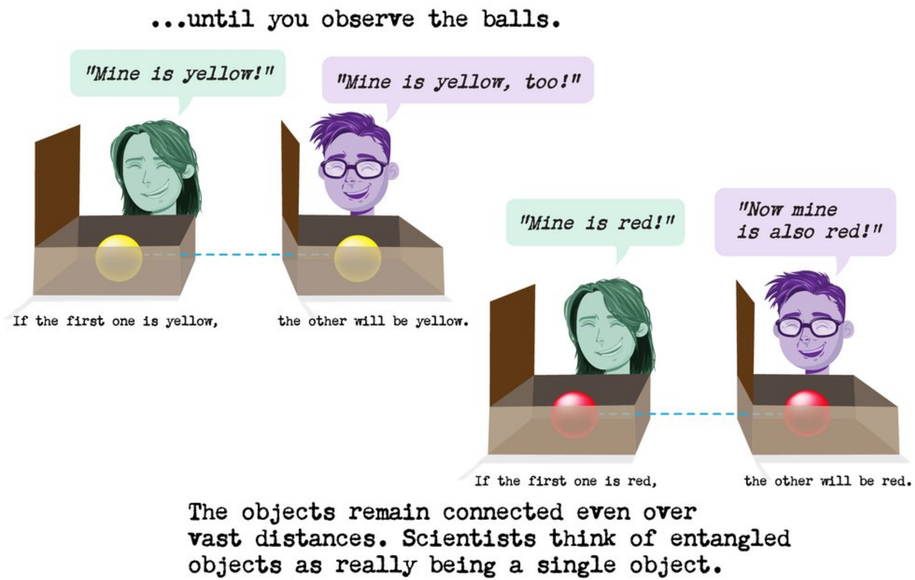
we get

$$\frac{\rho}{c}\left(-e^{2\Phi(r)}dt^2+\frac{dr^2}{1-\frac{b(r)}{r}}+r^2d\Omega^2\right) \tag{46}$$

- Conformal prefactor $\frac{\rho}{c}$ may not be a problem (*Hossenfelder, Zingg, 2017*), but one can also try to avoid it by choosing $\rho \propto p^{1/3}$

Let's say you have two entangled balls, each in its own box. In this metaphor, the balls can be either yellow or red once observed. For now, they are in a state of superposition, or both yellow and red at the same time . . .




...until you observe the balls.
"Mine is yellow!"
"Mine is yellow, too!"
If the first one is yellow,
the other will be yellow.
"Mine is red!"
"Now mine is also red!"
If the first one is red,
the other will be red.
The objects remain connected even over vast distances. Scientists think of entangled objects as really being a single object.

ER=EPR

- Einstein-Podolsky-Rosen paradox: quantum particles can affect each other at great distances ("quantum entanglement"), implying superluminal communication!
- *Juan Maldacena, Leonard Susskind (2013)*: two entangled black holes may be connected by a wormhole. But black holes have horizons, how is communication possible?
- If warp drives can traverse horizons, tiny warp drives could communicate between entangled particles!

Conclusions

- We found a generalization of the Gullstrand–Painlevé coordinates to wormholes, allowing the embedding of a warp drive.
- Surprisingly, warp drives can easily traverse wormholes with horizons, but have issues with humanly traversable ones!
- Possible relation to the ER=EPR correspondence: tiny warp drives may serve as the mechanism for superluminal communication. Need to take into account quantum effects and Hawking radiation!
- Analogue gravity could be a way to probe the physics of warp drives traversing BHs and WHs.