

Beyond the Casimir Wormholes

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The traversable wormhole metric

M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).

$$ds^2 = -\exp(-2\phi(r))dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2$$

Condition

$b(r)$ is the shape function

$$r \in [r_0, +\infty)$$

$$b_{\pm}(r_0) = r_0$$

$\phi(r)$ is the redshift function

$$b_{\pm}(r) < r$$

Proper radial distance

$$l(r) = \pm \int_{r_0}^r \frac{dr'}{\sqrt{1 - b_{\pm}(r')/r'}}$$

$$\lim_{r \rightarrow \infty} b_{\pm}(r) = b_{\pm} \quad \text{Appropriate asymptotic}$$

$$\lim_{r \rightarrow \infty} \phi_{\pm}(r) = \phi_{\pm} \quad \text{limits}$$

Einstein Field Equations

Orthonormal frame

$$b'(r) = 8\pi G \rho c^2 r^2$$

$$\phi'(r) = \frac{b + 8\pi G p_r r^3}{2r^2 (1 - b(r)/r)} \quad \tau(r) = -p_r$$

$$p'_r(r) = \frac{2}{r} (p_t(r) - p_r(r)) - (\rho(r) + p_r(r)) \phi'(r)$$

Exotic Energy

$$\rho(r) + p_r(r) < 0 \quad r \in [r_0, r_0 + \varepsilon] \quad \longleftrightarrow \quad b'(r) < b(r)/r \quad r \in [r_0, r_0 + \varepsilon]$$

Flare-Out Condition

Candidate  Casimir Energy

Take seriously the Casimir Energy \rightarrow State of the Art

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \longrightarrow \quad G_{\mu\nu} = \frac{8\pi G}{c^4} \langle T_{\mu\nu} \rangle^{Ren}$$

See also

[M.S. Morris, K.S. Thorne, U. Yurtsever \(Caltech\)](#). 1988. 4 pp. Published in Phys.Rev.Lett. 61 (1988) 1446-1449

M. Visser, Lorentzian Wormholes: From Einstein to Hawking (American Institute of Physics, New York), 1995.

$$\rho(a) = -\frac{\hbar c \pi^2}{720 a^4} \quad p_r(a) = -3 \frac{\hbar c \pi^2}{720 a^4} \quad p_t(a) = \frac{\hbar c \pi^2}{720 a^4} \quad \longrightarrow \quad b(r) = r_0 - \frac{\pi^3}{270 a^4} \left(\frac{\hbar G}{c^3} \right) (r^3 - r_0^3),$$

This is not a TW because there is no A. Flatness.

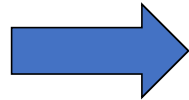
It is Asymptotically de Sitter

It can be transformed into a TW with the junction condition method matching the solution with the Schwarzschild metric at some point $r=c$

Take seriously the Casimir Energy \rightarrow State of the Art

R.G. Eur.Phys.J.C 79 (2019) 11, 951 ArXiv: 1907.03623 [gr-qc].

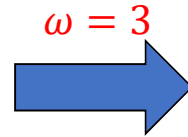
When $\omega = \frac{r_0^2}{r_1^2}$



Traversable Wormhole

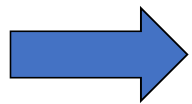
$$r_1^2 = \frac{\pi^3 l_p^2}{90}$$

$$\phi(r) = \frac{1}{2}(\omega - 1) \ln \left(\frac{r(\omega + 1)}{(\omega r + r_0)} \right)$$

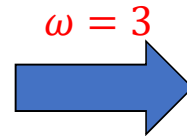


$$\phi(r) = \ln \left(\frac{4r}{3r + r_0} \right)$$

Planckian



$$b(r) = \left(1 - \frac{1}{\omega} \right) r_0 + \frac{r_0^2}{\omega r}$$



$$b(r) = \frac{2}{3} r_0 + \frac{r_0^2}{3r}$$

$$SET \quad T_{\mu\nu} = \left(\frac{r_0^2}{3kr^4} \right) \left[\text{diag}(-1, -3, 1, 1) + \left(\frac{6r}{3r + r_0} \right) \text{diag}(0, 0, 1, 1) \right]$$

$$\rho(a) = -\frac{\hbar c \pi^2}{720 a^4}$$

$$p_r(a) = -3 \frac{\hbar c \pi^2}{720 a^4}$$

$$p_t(a) = \frac{\hbar c \pi^2}{720 a^4}$$

a is replaced by the radial coordinate r

Other Profile \rightarrow Generalized Absurdly Benign TW

R.G. *Eur.Phys.J.C* 80 (2020) 12, 1172 ArXiv: 2008.05901 [gr-qc]

Identify the
Casimir Energy
Density

$$b(r) = \frac{1}{r_0^{\alpha-1}} \left[r_0 - \frac{\rho_0 \kappa}{3\alpha} (r^3 - r_0^3) \right]^\alpha, \quad \alpha > 1; \quad \Phi(r) = 0; \quad r_0 \leq r \leq \bar{r}$$

$$b(r) = 0, \quad \Phi(r) = 0; \quad r \geq \bar{r},$$

Close to the throat $b(r) \simeq r_0 \left(1 - \frac{r_0 l_P^2 \pi^3}{90 a^4} (r - r_0) \right)$

Plate separation nm



$$r_0 = \frac{3}{\pi} \sqrt{\frac{10}{\pi}} \frac{a^2}{l_P} \simeq 1.7 \times 10^{17} m$$

Absurdly Benign
Traversable Wormhole



$$b(r) = r_0 \left(1 - \left(\frac{r - r_0}{a} \right) \right)^2, \quad \Phi(r) = 0; \quad r_0 \leq r \leq r_0 + a$$

$$b(r) = 0, \quad \Phi(r) = 0; \quad r \geq r_0 + a.$$

Plate separation pm



$$r_0 \simeq 10^{11} m.$$

Superposing Yukawa-Casimir Wormholes

Eur.Phys.J.C 81, Article number: 824 (2021)

arXiv:2107.09276 [gr-qc]

$$b(r) = r_0 \left(\alpha \exp(-\mu(r-r_0)) + (1-\alpha) \left(\frac{r_0}{r} \right)^c \exp(-\nu(r-r_0)) \right)$$

$$r_0 \left(\alpha \exp(-\mu(r-r_0)) + (1-\alpha) \left(\frac{r_0}{r} \right)^c \exp(-\nu(r-r_0)) \right)$$

$$\omega(r) = \frac{r_0 \left(\alpha \exp(-\mu(r-r_0)) + (1-\alpha) \left(\frac{r_0}{r} \right)^c \exp(-\nu(r-r_0)) \right)}{r \left(r_0 \mu \alpha \exp(-\mu(r-r_0)) + (1-\alpha) \left(\frac{r_0}{r} \right)^c \exp(-\nu(r-r_0)) \left(\nu r_0 + c \frac{r_0}{r} \right) \right)}$$

Zero Tidal Forces

$$\Phi(r) = 0$$

$$r_0 = \frac{3}{\pi} \sqrt{\frac{10}{\pi}} \frac{a^2}{l_P} \simeq 1.7 \times 10^{17} m$$

Plate separation pm



$$r_0 \simeq 10^{11} m.$$

Superposing Yukawa-Casimir Wormholes

Reverse Procedure

$$\rho(r) = \frac{r_0 \rho}{r} \left(\alpha \exp(-\mu(r-r_0)) - (1-\alpha) \exp(-\nu(r-r_0)) \right)$$

$$\kappa = \frac{8\pi G}{c^4}$$

On the throat

$$\omega(r_0) = \frac{(1 + \nu r_0) \mu^2 + (1 + \mu r_0) \nu^2}{2\nu^2 r_0^2 \mu^2 + r_0^3 \rho \kappa \nu \mu (\nu - \mu) + r_0^2 \rho \kappa (\nu^2 - \mu^2)}$$

Zero Tidal Forces

$$\Phi(r) = 0$$

$$\mu = \frac{m}{r_0}; \quad \nu = \frac{n}{r_0} \quad \text{and} \quad r_0 = \frac{x}{\sqrt{\rho \kappa}}; \quad m, n \in \mathbb{R}_+$$

$$\omega(r_0) = 1$$



Solution



$$x = \frac{\sqrt{(2n^2 - n - 1)m^2 - n^2m - n^2}}{\sqrt{((n+1)m+n)(m-n)}}$$


$$r_0 \simeq x \times 10^{17} m.$$

$$\frac{(1+n)m^2 + (1+m)n^2}{(2n^2m^2 + x^2nm(n-m) + x^2(n^2 - m^2))} = 1,$$

Combining different Sources

Eur.Phys.J.C **83**, 824 (2023)
arXiv:2302.04043 [gr-qc]

Partially supported by
Limiteless Space Institute

Electrovacuum source=Casimir+Electromagnetic  $\rho(r) + p_r(r) = -\frac{4\hbar c\pi^2}{720r^4} < 0$

$$\rho(r) = \rho_C(r) + \rho_E(r) = -\frac{\hbar c\pi^2}{720r^4} + \frac{Q^2}{2(4\pi)^2 \epsilon_0 r^4} = -\frac{r_1^2}{\kappa r^4} + \frac{r_2^2}{\kappa r^4}$$

$$r_1^2 = \frac{\pi^3 l_p^2}{90} \quad r_2^2 = \frac{GQ^2}{4\pi c^4 \epsilon_0} \quad \text{blue arrow} \quad \rho(r) < 0 \quad \text{when} \quad r_1 > r_2 \quad \omega = \frac{p_r(r)}{\rho(r)} = \frac{3r_1^2 + r_2^2}{r_1^2 - r_2^2}$$

$$r_0 = \sqrt{3r_1^2 + r_2^2} \quad x = \frac{r_2}{r_1} = \frac{90GQ^2}{2\pi c^4 \epsilon_0 \pi^3 l_p^2} = \frac{180}{\pi^3} n^2 \frac{e^2}{4\pi \epsilon_0 \hbar c} = 180 \frac{n^2}{\pi^3} \alpha \quad \alpha = \frac{1}{137}$$

Combining different Sources

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*Electrovacuum source=Casimir+Electromagnetic
with the plates separation d constant*



$$\rho(r) + p_r(r) = -\frac{4\hbar c\pi^2}{720d^4} < 0$$

$$\rho(r) = \rho_C(d) + \rho_E(r) = -\frac{\hbar c\pi^2}{720d^4} + \frac{Q^2}{2(4\pi)^2 \epsilon_0 r^4}$$

$$b(r) = r_0 + \frac{r_2^2}{r_0} - \frac{r_2^2}{r} - \frac{r_1^2}{3d^4} (r^3 - r_0^3)$$



Not A.Flat

However $\Phi'(r) = 0$ on r_0

$$d = \sqrt[4]{3}\sqrt{2r_1r_2} \quad \longrightarrow \quad r_0 = \sqrt{2}r_2 \simeq 10^{13}m$$

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$$b(r) = r_0 + \frac{r_2^2}{r_0} - \frac{r_2^2}{r} - \frac{r_1^2}{3d^4} (r^3 - r_0^3)$$



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However $\Phi'(r) = 0$ on r_0

$$d = \sqrt[4]{3}\sqrt{2r_1r_2} \quad \longrightarrow \quad r_0 = \sqrt{2}r_2 \simeq 10^{13}m$$

$$\begin{cases} b(r) = \frac{14}{9}r_0 - \frac{r_0^2}{2r} - \frac{r^3}{18r_0^2} & r_0 \leq r \leq \tilde{r} \\ \Phi(r) = 0. & \end{cases} \quad b(r) = 0 \quad r \geq \tilde{r}$$

$$\tilde{r} = 2.9208r_0 \quad \text{where} \quad b(\tilde{r}) = 0.$$

Conclusions and Perspectives

- Casimir energy is the only source of exotic matter that can be generated in laboratory.
- Traversable wormholes can be sustained by Casimir Energy.
- The Wormhole is traversable in principle but not in practice.
- Generalized Absurdly Benign Traversable Wormholes seem to have the right properties for traversability together with Yukawa–Casimir wormholes. Need to be carefully investigated
- Adding Extra Sources: e.g. Electromagnetic Field \longrightarrow Promising!!!
- Adding other Extra Sources: e.g. Scalar Fields (see talk of A.Tzikas).
- Connection with Warp Drive (see talk of K. Zatrimeylov).
- Including quantum fluctuations.
- TW relevant for GW as BH mimickers.

Thank You for Your Attention

Outlook

