LIMITLESS SPACE INSTITUTE

CQRC

XXV SIGRAV Conference
Trieste 4-9 September 2023

Beyond the Casimir Wormholes

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The traversable wormhole metric

M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).

$$ds^2 = -\exp\left(-2\phi(r)\right)dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2 d\theta^2 + r^2\sin^2\theta\, d\varphi^2$$

Condition

$\boldsymbol{b(r)}$ is the shape function

$\phi(\boldsymbol{r})$ is the redshift function

$$r \in [r_0, +\infty)$$

$$b_\pm(r_0) = r_0$$

$$b_\pm(r) < r$$

Proper radial distance

$$l(r) = \pm\int_{r_0}^{r} \frac{dr'}{\sqrt{1 - b_\pm(r')/r'}}$$

$$\lim_{r\to\infty} b_\pm(r) = b_\pm \qquad \text{Appropriate asymptotic}$$

$$\lim_{r\to\infty} \phi_\pm(r) = \phi_\pm \qquad \text{limits}$$

Einstein Field Equations

Orthonormal frame

$$b'(r) = 8\pi G \rho c^2 r^2$$

$$\phi'(r) = \frac{b + 8\pi G p_r r^3}{2r^2\left(1 - b(r)/r\right)} \qquad \tau(r) = -p_r$$

$$p'_r(r) = \frac{2}{r}\left(p_t(r) - p_r(r)\right) - \left(\rho(r) + p_r(r)\right)\phi'(r)$$

Exotic Energy

$\rho(r) + p_r(r) < 0 \quad r \in [r_0, r_0 + \varepsilon]$ ⟺ $b'(r) < b(r)/r \quad r \in [r_0, r_0 + \varepsilon]$

Flare-Out Condition

Candidate ⟹ Casimir Energy

Take seriously the Casimir Energy → State of the Art

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \Longrightarrow \quad G_{\mu\nu} = \frac{8\pi G}{c^4} \langle T_{\mu\nu} \rangle^{Ren}$$

See also

M.S. Morris, K.S. Thorne, U. Yurtsever (Caltech). 1988. 4 pp. Published in Phys.Rev.Lett. 61 (1988) 1446-1449

M. Visser, Lorentzian Wormholes: From Einstein to Hawking (American Institute of Physics, New York), 1995.

$$\rho(a) = -\frac{\hbar c \pi^2}{720 a^4} \qquad p_r(a) = -3\frac{\hbar c \pi^2}{720 a^4} \qquad p_t(a) = \frac{\hbar c \pi^2}{720 a^4} \quad \Longrightarrow \quad b(r) = r_0 - \frac{\pi^3}{270 a^4}\left(\frac{\hbar G}{c^3}\right)\left(r^3 - r_0^3\right),$$

This is not a TW because there is no A. Flatness.

It is Asymptotically de Sitter

It can be transformed into a TW with the junction condition method matching the solution with the Schwarzschild metric at some point r=c

Take seriously the Casimir Energy → State of the Art

R.G. Eur.Phys.J.C 79 (2019) 11, 951 ArXiV: 1907.03623 [gr-qc].

When $\omega = \frac{r_0^2}{r_1^2}$ → Traversable Wormhole

$$r_1^2 = \frac{\pi^3 l_p^2}{90}$$

$$\phi(r) = \frac{1}{2}(\omega - 1)\ln\left(\frac{r(\omega+1)}{(\omega r + r_0)}\right) \xrightarrow{\omega = 3} \phi(r) = \ln\left(\frac{4r}{3r + r_0}\right)$$

Planckian

$$b(r) = \left(1 - \frac{1}{\omega}\right) r_0 + \frac{r_0^2}{\omega r} \xrightarrow{\omega = 3} b(r) = \frac{2}{3} r_0 + \frac{r_0^2}{3r}$$

$$SET \quad T_{\mu\nu} = \left(\frac{r_0^2}{3\kappa r^4}\right)\left[diag(-1,-3,1,1) + \left(\frac{6r}{3r + r_0}\right) diag(0,0,1,1)\right]$$

$$\rho(a) = -\frac{\hbar c \pi^2}{720 a^4} \qquad p_r(a) = -3\frac{\hbar c \pi^2}{720 a^4} \qquad p_t(a) = \frac{\hbar c \pi^2}{720 a^4}$$

a is replaced by the radial coordinate r

Other Profile → Generalized Absurdly Benign TW

R.G. *Eur.Phys.J.C* 80 (2020) 12, 1172ArXiV: 2008.05901 [gr-qc]

Identify the Casimir Energy Density

$$b(r) = \frac{1}{r_0^{\alpha-1}}\left[r_0 - \frac{\rho_0\kappa}{3\alpha}\left(r^3 - r_0^3\right)\right]^\alpha, \qquad \alpha > 1; \qquad \Phi(r) = 0; \qquad r_0 \le r \le \bar{r}$$

$$b(r) = 0, \qquad \Phi(r) = 0; \qquad r \ge \bar{r},$$

Close to the throat $b\left(r\right) \simeq r_0\left(1 - \frac{r_0 l_P^2 \pi^3}{90a^4}\left(r - r_0\right)\right)$

Plate separation *nm* →

$$r_0 = \frac{3}{\pi}\sqrt{\frac{10}{\pi}}\frac{a^2}{l_P} \simeq 1.7 \times 10^{17} m$$

Absurdly Benign Traversable Wormhole →

$$b(r) = r_0\left(1 - \left(\frac{r - r_0}{a}\right)\right)^2, \qquad \Phi(r) = 0; \qquad r_0 \le r \le r_0 + a$$

$$b(r) = 0, \qquad \Phi(r) = 0; \qquad r \ge r_0 + a.$$

Plate separation *pm* →

$$r_0 \simeq 10^{11} m.$$

Superposing Yukawa-Casimir Wormholes

Eur.Phys.J.C 81, Article number: 824 (2021)   arXiv:2107.09276 [gr-qc]

$$b(r) = r_0\left(\alpha \exp\left(-\mu\left(r-r_0\right)\right) + \left(1-\alpha\right)\left(\frac{r_0}{r}\right)^c \exp\left(-\nu\left(r-r_0\right)\right)\right)$$

$$\omega(r) = \frac{r_0\left(\alpha \exp\left(-\mu\left(r-r_0\right)\right) + \left(1-\alpha\right)\left(\frac{r_0}{r}\right)^c \exp\left(-\nu\left(r-r_0\right)\right)\right)}{r\left(r_0\mu\alpha \exp\left(-\mu\left(r-r_0\right)\right) + \left(1-\alpha\right)\left(\frac{r_0}{r}\right)^c \exp\left(-\nu\left(r-r_0\right)\right)\left(\nu r_0 + c\frac{r_0}{r}\right)\right)}$$

Zero Tidal Forces

$$\Phi(r) = 0$$

$$r_0 = \frac{3}{\pi}\sqrt{\frac{10}{\pi}}\frac{a^2}{l_P} \simeq 1.7 \times 10^{17} m$$

Plate separation *pm* → $r_0 \simeq 10^{11} m.$

Superposing Yukawa-Casimir Wormholes

Reverse Procedure

$$\rho(r) = \frac{r_0 \rho}{r}\left(\alpha \exp\left(-\mu\left(r - r_0\right)\right) - \left(1 - \alpha\right)\exp\left(-\nu\left(r - r_0\right)\right)\right) \qquad \kappa = \frac{8\pi G}{c^4}$$

On the throat

$$\omega\left(r_0\right) = \frac{\left(1 + \nu r_0\right)\mu^2 + \left(1 + \mu r_0\right)\nu^2}{2\nu^2 r_0^2 \mu^2 + r_0^3 \rho\kappa\nu\mu\left(\nu - \mu\right) + r_0^2 \rho\kappa\left(\nu^2 - \mu^2\right)}.$$

Zero Tidal Forces

$$\Phi(r) = 0$$

$$\mu = \frac{m}{r_0}; \qquad \nu = \frac{n}{r_0} \qquad \text{and} \qquad r_0 = \frac{x}{\sqrt{\rho\kappa}}; \qquad m, n \in \mathbb{R}_+$$

$$\omega\left(r_0\right) = 1$$

Solution

$$x = \frac{\sqrt{\left(2n^2 - n - 1\right)m^2 - n^2 m - n^2}}{\sqrt{\left(\left(n + 1\right)m + n\right)\left(m - n\right)}}.$$

$$\frac{\left(1 + n\right)m^2 + \left(1 + m\right)n^2}{\left(2n^2 m^2 + x^2 nm\left(n - m\right) + x^2\left(n^2 - m^2\right)\right)} = 1,$$

$$r_0 \simeq x \times 10^{17} m.$$

Combining different Sources

Eur.Phys.J.C 83, 824 (2023)
arXiv:2302.04043 [gr-qc]

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Electrovacuum source=Casimir+Electromagnetic → $\rho(r) + p_r(r) = -\frac{4\hbar c\pi^2}{720 r^4} < 0$

$$\rho\left(r\right) = \rho_C\left(r\right) + \rho_E\left(r\right) = -\frac{\hbar c\pi^2}{720 r^4} + \frac{Q^2}{2\left(4\pi\right)^2 \varepsilon_0 r^4} = -\frac{r_1^2}{\kappa r^4} + \frac{r_2^2}{\kappa r^4}$$

$$r_1^2 = \frac{\pi^3 l_p^2}{90} \quad r_2^2 = \frac{GQ^2}{4\pi c^4 \varepsilon_0} \quad \Rightarrow \quad \rho\left(r\right) < 0 \quad when \quad r_1 > r_2 \quad \omega = \frac{p_r\left(r\right)}{\rho\left(r\right)} = \frac{3r_1^2 + r_2^2}{r_1^2 - r_2^2}$$

$$r_0 = \sqrt{3r_1^2 + r_2^2} \qquad x = \frac{r_2}{r_1} = \frac{90GQ^2}{2\pi c^4 \varepsilon_0 \pi^3 l_p^2} = \frac{180}{\pi^3} n^2 \frac{e^2}{4\pi\varepsilon_0 \hbar c} = 180 \frac{n^2}{\pi^3} \alpha \qquad \alpha = \frac{1}{137}$$

Combining different Sources

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Electrovacuum source=Casimir+Electromagnetic with the plates separation d constant ⇒ $\rho(r) + p_r(r) = -\frac{4\hbar c\pi^2}{720 d^4} < 0$

$$\rho(r) = \rho_C(d) + \rho_E(r) = -\frac{\hbar c\pi^2}{720 d^4} + \frac{Q^2}{2(4\pi)^2 \varepsilon_0 r^4}$$

$b(r) = r_0 + \frac{r_2^2}{r_0} - \frac{r_2^2}{r} - \frac{r_1^2}{3d^4}\left(r^3 - r_0^3\right)$ ⇒ Not A.Flat However $\Phi'(r) = 0$ $on\ r_0$

$d = \sqrt[4]{3}\sqrt{2 r_1 r_2}$ ⇒ $r_0 = \sqrt{2} r_2 \simeq 10^{13} m$

Combining different Sources

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Electrovacuum source=Casimir+Electromagnetic with the plates separation d constant ⇒ $\rho(r) + p_r(r) = -\frac{4\hbar c\pi^2}{720 d^4} < 0$

$$\rho(r) = \rho_C(d) + \rho_E(r) = -\frac{\hbar c\pi^2}{720 d^4} + \frac{Q^2}{2(4\pi)^2 \varepsilon_0 r^4}$$

$b(r) = r_0 + \frac{r_2^2}{r_0} - \frac{r_2^2}{r} - \frac{r_1^2}{3d^4}\left(r^3 - r_0^3\right)$ ⇒ Not A.Flat

However $\Phi'(r) = 0$ *on* r_0

$d = \sqrt[4]{3}\sqrt{2 r_1 r_2}$ ⇒ $r_0 = \sqrt{2} r_2 \simeq 10^{13} m$

$$\begin{cases} b(r) = \frac{14}{9} r_0 - \frac{r_0^2}{2r} - \frac{r^3}{18 r_0^2} & r_0 \le r \le \tilde{r} \\ \Phi(r) = 0. \end{cases}$$

$b(r) = 0 \qquad r \ge \tilde{r}$

$\tilde{r} = 2.9208 r_0 \qquad where \qquad b(\tilde{r}) = 0.$

Conclusions and Perspectives

- Casimir energy is the only source of exotic matter that can be generated in laboratory.
- Traversable wormholes can be sustained by Casimir Energy.
- The Wormhole is traversable in principle but not in practice.
- Generalized Absurdly Benign Traversable Wormholes seem to have the right properties for traversability together with Yukawa–Casimir wormholes. Need to be carefully investigated
- Adding Extra Sources: e.g. Electromagnetic Field ⟶ Promising!!!
- Adding other Extra Sources: e.g. Scalar Fields (see talk of A.Tzikas).
- Connection with Warp Drive (see talk of K. Zatrimaylov).
- Including quantum fluctuations.
- TW relevant for GW as BH mimickers.

Thank You for Your Attention

Outlook