Constraining modifications of black hole perturbation potentials near the light ring with quasi-normal modes

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Based on:
Sebastian H. Völkel, Nicola Franchini, Enrico Barausse and Emanuele Berti
PRD 106, 124036 (2022), arXiv:2209.10564
Bonus material:
Peter James Nee, Sebastian H. Völkel, Harald P. Pfeiffer,
PRD 108 (2023) 4, 044032, arxiv:2302.06634

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WHY QUASI-NORMAL MODES?

Motivation:

- quasi-normal modes (QNMs) are a central piece of **black hole spectroscopy**
- allow one to test the **no-hair theorems hypotheses** and general relativity (GR)
- black hole mergers measured by LVK[1] become **more frequent and precise**
- **in principle clean test**, less challenging astrophysics (e.g. compared to EHT[2])
- multi disciplinary: **perturbation theory**, **numerical relativity** and **data analysis**

[1] LIGO-Virgo-KAGRA (LVK)
[2] Event Horizon Telescope (EHT)

ARE THE RELEVANT PROBLEMS NOT ALREADY SOLVED?

Much activity in recent years on[3]:

- robust extraction of QNM overtones from simulations
- non-linear modes and spherical mode coupling
- environmental effects and spectral stability
- horizonless compact objects and "echoes"
- QNMs for rotating black holes beyond GR very challenging, much less known

[3] Giesler et al. PRX 9 041060 (2019), Mitman et al. PRL 130 081402 (2023), Baibhav et al. arxiv: 2302.03050, Nee et al., PRD 108, 044032, (2023), . . .

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If one would measure deviations tomorrow, what would one actually learn? (inverse problem)

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SOME COMMENTS FOR GR AND BEYOND

Qualitative features of GR (vacuum):

- perturbation equations are separable
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"Theory specific" approach:

- start from a theory, compute background and perturbations
- clear connections between observations, metric and theory

MOTIVATION FOR OUR STUDIES

Assume a few QNMs would be known from observations [4]:

- Can one recover the effective potential?
- Can one recover the presence of coupling functions to additional fields?
- Could one probe the underlying black hole metric?
- How do measurement uncertainties impact the reconstruction?
- **What are robust "features"?** PN-like expansion: $\sim r_{\rm H}/r$?

[4] Other related works:
SV and Barausse, PRD 102 084025 (2020)
SV, Franchini and Barausse, PRD 105 084046 (2022)

METHODS OVERVIEW I

We adopt the parameterized framework to study non-rotating black holes[5, 6]:

- coupled equations are a prototype for rotation and alternative theories

$$f\frac{\mathrm{d}}{\mathrm{d}r}\left(f\frac{\mathrm{d}\mathbf{\Phi}}{\mathrm{d}r}\right)+\left[\omega^2-f\mathbf{V}\right]\mathbf{\Phi}=0, \tag{1}$$

- with $f(r)=1-r_0/r$, with r_0 being the location of the event horizon and

$$\mathbf{\Phi}=\left[\Phi^{\mathrm{scalar}},\Phi^{\mathrm{polar}},\Phi^{\mathrm{axial}},\ldots\right]. \tag{2}$$

[5]Cardoso et al., PRD 99 104077 (2019)
[6]McManus et al., PRD 100 044061 (2019)

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- it is assumed the potential can be written as ("natural extension" to GR)

$$V_{ij}=V_{ij}^{\text{GR}}+\delta V_{ij}, \qquad \delta V_{ij}=\frac{1}{r_H^2}\sum_{k=0}^{\infty}\alpha_{ij}^{(k)}\left(\frac{r_H}{r}\right)^k \quad (3)$$

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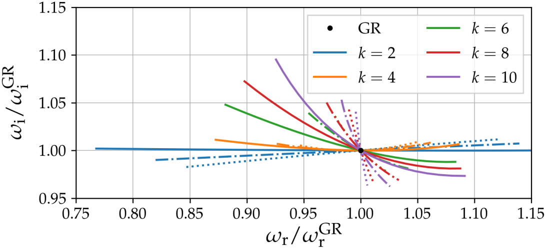
METHODS OVERVIEW II

- approximate expression up to quadratic order in $\alpha^{(k)}$

$$\omega = \omega^0 + \alpha_{ij}^{(k)} d_{(k)}^{ij} + \alpha_{ij}^{(k)} \partial_\omega \alpha_{pq}^{(s)} d_{(k)}^{ij} d_{(s)}^{pq} + \frac{1}{2} \alpha_{ij}^{(k)} \alpha_{pq}^{(s)} e_{(ks)}^{ijpq} \quad (4)$$

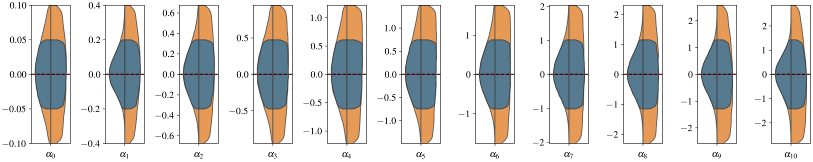
- coefficients $d_{(k)}$, $e_{(ks)}$ are universal, $\alpha^{(k)}$ theory dependent or free parameters
- coefficients of $n = 1,2$ overtones obtained recently[7]



Relative change of the QNM spectrum along small $\alpha^{(k)}$ range.

[7] SV, Franchini and Barausse, PRD 105 084046 (2022)

RESULTS I



GR: $n = 0,1$ QNMs ω with relative errors of $1.0\,\%, 4.7\,\%, 3.4\,\%, 8.2\,\%$. We then sample r_H and $\alpha^{(k)}$ for $k = 0 \ldots 10$. Left violins show r_H with one specific $\alpha^{(k)}$ varied at a time, right violins show r_H varied with all $\alpha^{(k)}$ at the same time. Different colors correspond to different prior ranges for $\alpha^{(k)}$.

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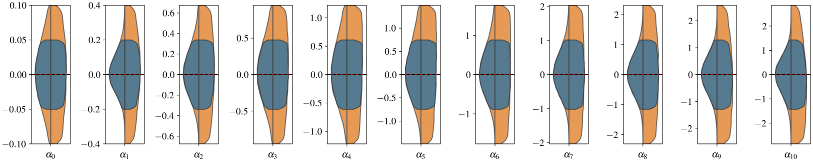

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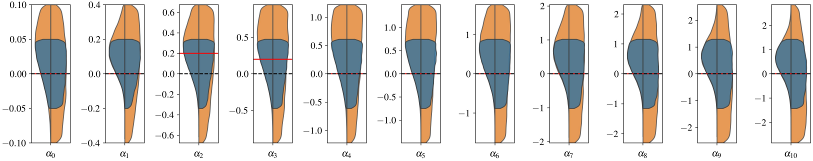

Non-GR: $n = 0,1$ QNMs $\vec{\omega}$ with non-zero $\alpha^{(k)}$ for $k = 2$ and $k = 3$ (shown in red), but other assumptions the same as in the GR case.

CAN ONE DO BETTER?

Previous results are not encouraging:

- single α_k cannot be robustly constrained, agnostic posteriors **strongly correlated**
- not surprising, $(r_{\rm H}/r)^k$ for weak fields, QNMs are related to the strong field

[8] Schutz and Will 1985, Iyer Will 1987, Konoplya 2003, ...

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Do strongly correlated, agnostic $\alpha^{(k)}$ posteriors robustly constrain the potential peak?

- WKB method connects derivatives of **potential peak** with **QNMs**[8]

$$\omega_n^2 = V^{(0)} + i\sqrt{-2V^{(2)}}\left(n + \frac{1}{2}\right) + \sum_i \tilde{\Lambda}_i(n) \tag{5}$$

- but, method is approximate, validity for strongly correlated $\alpha^{(k)}$ hard to quantify

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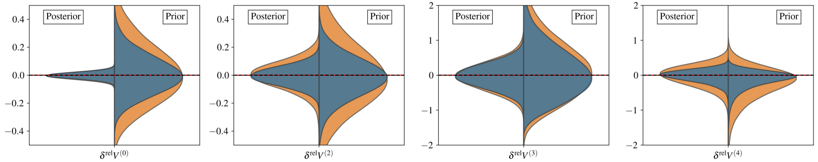
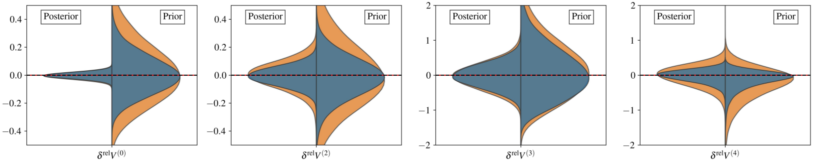
RESULTS II



GR: Sampling derivatives of the effective potential with respect to tortoise coordinate from the "all $\alpha^{(k)}$" posterior distributions (left sides) versus sampling from the priors of $\alpha^{(k)}$ (right sides).

RESULTS II



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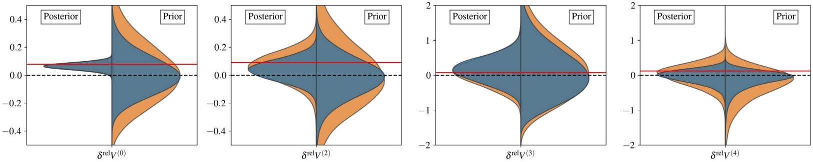

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Summary:

- potential/coupling corrections like $\alpha_k \left(r_\mathrm{H}/r\right)^k$ quite common beyond GR
- **useful** parametrization for **direct problem**, **tricky for agnostic inverse problem**
- MCMC analysis yields **strong correlations**, single α_k cannot be constrained
- **QNMs are very informative, robust feature is the local behavior around peak**

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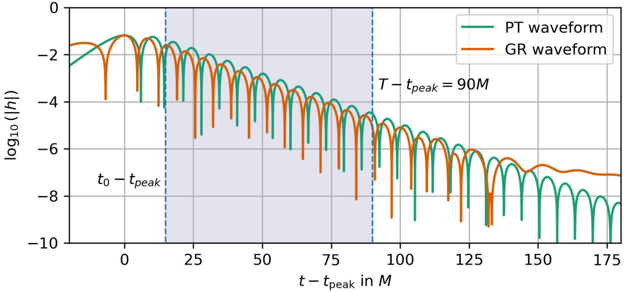
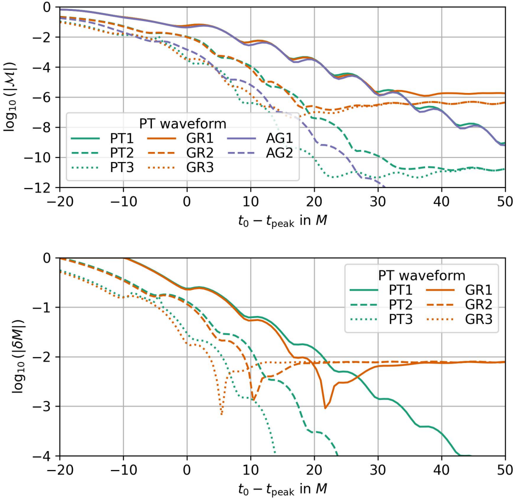
The elephant in the room, can one robustly extract QNMs from signals?
(comprehensive study: Vishal Baibhav+, arxiv:2302.03050)

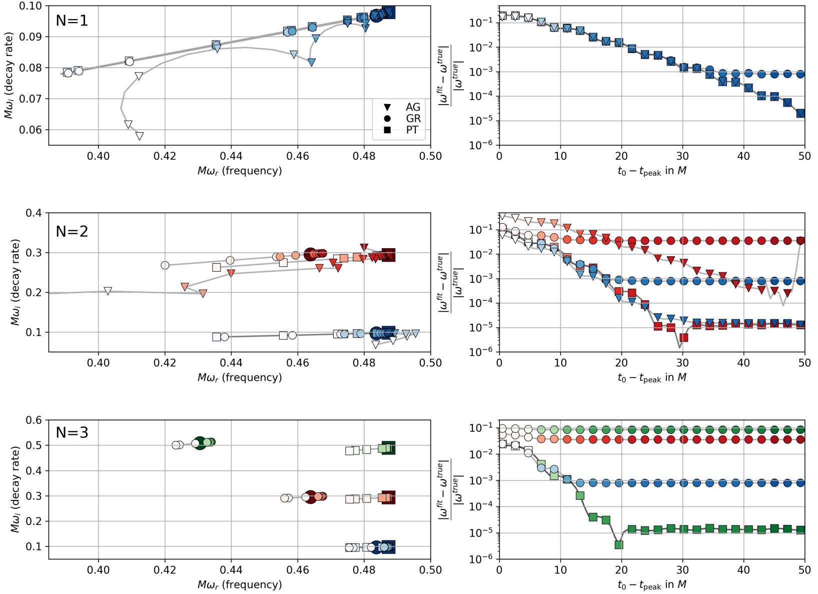
Peter James Nee, Sebastian H. Völkel, Harald P. Pfeiffer, PRD 108 (2023) 4, 044032, arxiv:2302.06634
title now: *Role of black hole quasinormal mode overtones for ringdown analysis*



Waveforms generated from the same Gaussian initial data evolved in the Pöschl-Teller potential (green) and GR potential (orange). The shaded area indicates a typical fit interval, starting at $t_0 - t_{\rm peak}$, and ending at $T - t_{\rm peak}$. Note that the time has been shifted to align both waveforms at their respective peak time $t_{\rm peak}$, which introduced a relative shift with respect to the simulation time (not shown).



Results for a waveform generated using the Pöschl-Teller potential. We apply the PT and GR model with $N = 1 \ldots 3$ modes, as well as the agnostic model with $N = 1$ or $N = 2$ modes. **Top panel**: mismatch $\mathscr{M}$ as a function of starting time of the fit. **Bottom panel:** relative error δM of the recovered black hole mass as a function of starting time of the fit.


N=1
N=2
N=3
AG
GR
PT
$M\omega_i$ (decay rate)
$M\omega_r$ (frequency)
$\frac{|\omega^{fit} - \omega^{true}|}{|\omega^{true}|}$
$t_0 - t_{\rm peak}$ in M