Running couplings in a higher derivative theory

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D.B., Roberto Percacci, *Renormalization group flows between gaussian fixed points,* arXiv:2207.10596v1

D.B., John Donoghue, Roberto Percacci, *Amplitudes and Renormalization Group Techniques: A Case Study*, arXiv:2307.00055v2

SISSA

Why higher derivative?

- Einstein general relativity as a QFT is not renormalizable
- Higher derivative operators R^2, $R_{\mu\nu}R^{\mu\nu}$ and $R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda}$ contain a fourth derivative kinetic term for the metric

⇓

The theory is now power counting renormalizable [Stelle, '77]

$$S = \int d^4x\sqrt{-g}\left[-2\Lambda + Z_N R - \frac{1}{2\lambda}\left(C^2 - \frac{2\omega}{3}R^2 + 2\theta E\right)\right]$$

$$Z_N = \frac{1}{16\pi G}$$

- Ostrogradsky instability, ghosts and breakdown of unitarity
- Quadratic gravity is Asymptotically Free [Avramidi & Barvinski, '85], but…

Regularization and running

- In perturbation theory, there are different definitions of running couplings:

μ-running

$$\beta_g^{\mu} = \mu \frac{\partial}{\partial \mu} g(\mu)$$

With μ unphysical parameter introduced by regularization procedure, to preserve dimensional analysis or as a cutoff

Physical running

$$\beta_g = \mu_R \frac{\partial}{\partial \mu_R} g(\mu_R)$$

Where $g(\mu_R)$ has absorbed non-polynomial dependence of the amplitude from external momentum $p = \mu_R$

- In a theory without mass scales in the propagator, they are equivalent, since logarithmic terms in μ appear in the form $\log(\mu^2/p^2)$
- In quadratic gravity we have introduced $[\lambda/G] = E^2$, hence the two runnings are inequivalent

Another definition for $\boldsymbol{\beta}$ functions: FRG

- One inserts an IR regulator $\Delta S_k[\phi] = \int D\phi\ \phi R(k)\phi$ in the Euclidean action, which suppresses modes below the cutoff scale k in functional integration
- k-dependent average effective action $\Gamma_k[\varphi]$

$k \to 0$ $\qquad\qquad$ $k \to \infty$

$\Gamma[\varphi]$ $\qquad\qquad$ $\mathrm{S}[\varphi]$

- Evolution of $\Gamma_k[\varphi]$ can be described with an exact closed differential equation: **k-running**

$$k\frac{d\Gamma_k}{dk} = \frac{1}{2} Tr\left[\Gamma_k^{(2)} + R(k)\right]^{-1} k\frac{dR(k)}{dk}$$

- Nonperturbative method, permits to quest for interacting fixed points

A toy model

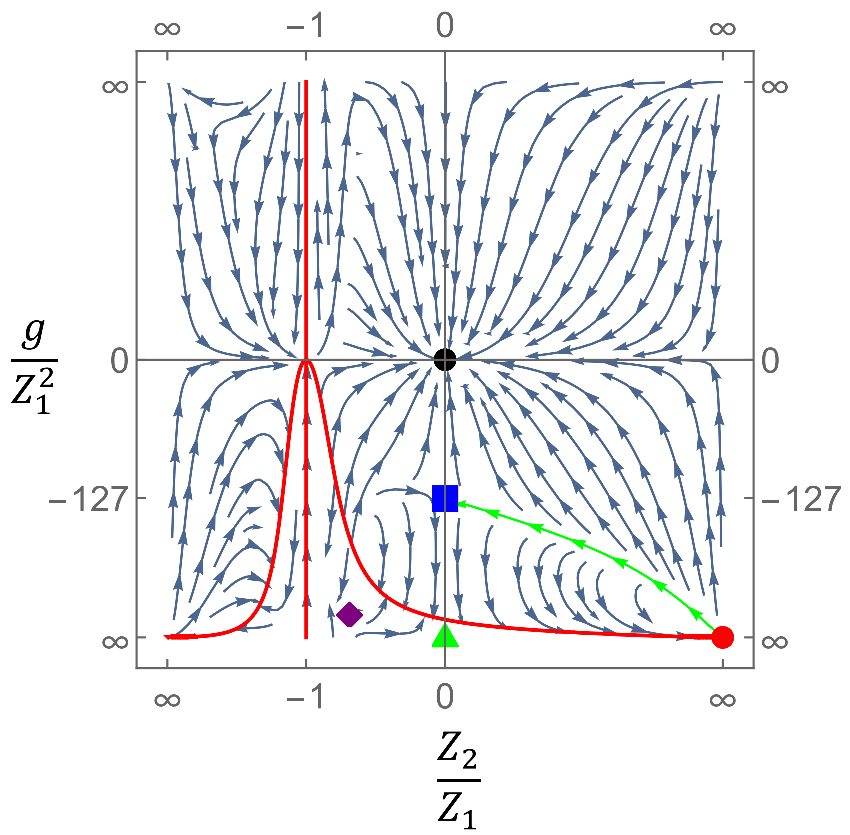
- To address the problem of definition of running couplings, we considered a simpler scalar model which could give some interesting insight
- Shift symmetry $\phi \to \phi + c$ and $\mathbb{Z}_2$ symmetry invariance

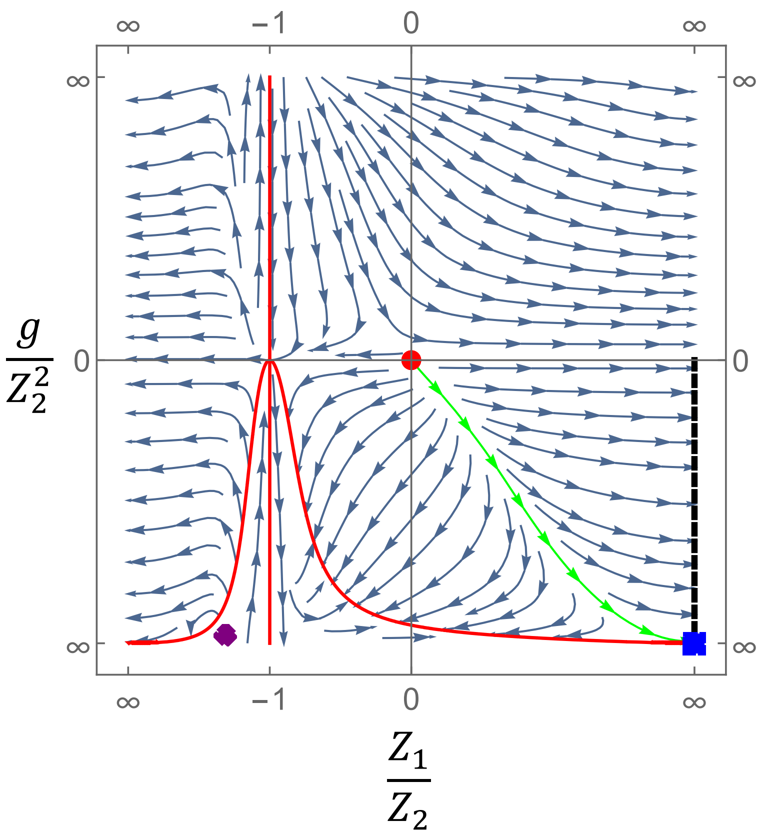
$$\mathrm{S}[\phi] = -\int d^4x \left[\frac{1}{2} Z_1 (\partial\phi)^2 + \frac{1}{2m^2} (\Box\phi)^2 + \frac{g}{4M^4} \left((\partial\phi)^2\right)^2 + \cdots \right]$$

Such derivative interaction can be generated by:

- spontaneous breaking of U(1) symmetry in a higher derivative linear sigma model
- interactions with gravity in the asymptotic safety scenario

FRG analysis





$$\beta_g = \frac{5(Z_1 + 2k^2/m^2)}{32\pi^2(Z_1 + k^2/m^2)^3}\frac{gk^4}{M^4} \, .$$

$$\beta_{Z_1} = -\frac{Z_1 + 2k^2/m^2}{16\pi^2(Z_1 + k^2/m^2)^2}\frac{gk^4}{M^4} \quad .$$

Regimes in perturbative computations

$$\overset{\vec{p}}{\rule{3cm}{1pt}} = -\frac{i}{p^2+\frac{1}{m^2}p^4} = -i\left[\frac{1}{p^2} - \frac{1}{p^2+m^2}\right]$$

Low Energy (LE):

the heavy ghost is not dynamically active and can be integrated out.

Intermediate Energy (IE):

the heavy ghost is dynamically active, but the interaction is small

High Energy (HE):

$gE^4/M^4 > 1$, one would expect a strongly interacting regime.

Effective Field Theory

- In the low energy regime, we consider only the quadratic kinetic term (Z_2=0), plus higher derivative operators

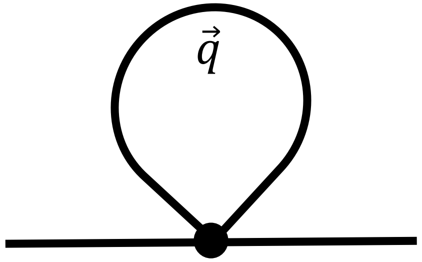
$$L_6 = \frac{g_6}{4M^6}\partial_\mu\phi\partial^\mu\phi\Box\partial_\nu\phi\partial^\nu\phi + \frac{g_6'}{4M^6}\partial_\mu\phi\partial_\nu\phi\Box\partial^\mu\phi\partial^\nu\phi$$

$$L_8 = -\frac{g_8}{4M^8}\partial_\mu\phi\partial^\mu\phi\Box^2\partial_\nu\phi\partial^\nu\phi - \frac{g_8'}{4M^8}\partial_\mu\phi\partial_\nu\phi\Box^2\partial^\mu\phi\partial^\nu\phi$$

- We have $\beta_{g_8} = 41g^2/480\pi^2, \beta_{g_8'} = g^2/240\pi^2$.
- At the same time, $\beta_g = 0$ and $\beta_{Z_1} = 0$.

The higher derivative two-point function

Tadpole diagram



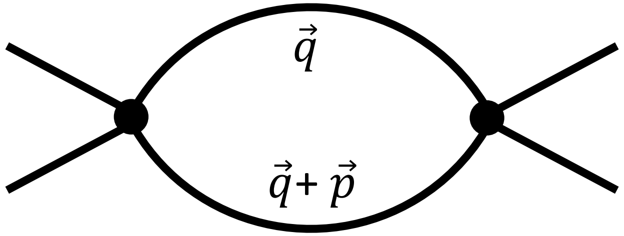
One-loop quantum correction to the propagator

$$i\,\frac{3}{2}\frac{1}{Z_1}\left(\frac{m}{M}\right)^4 p^2\,\frac{1}{(4\pi)^2}\left(\frac{1}{\epsilon}+\log 4\pi-\gamma-\log\frac{m^2}{\mu^2}+\frac{7}{6}+O(\epsilon)\right)$$

- Using $\boldsymbol{\mu}$**-running** : $\beta^{\mu}_{Z_1} = \frac{3}{16\pi^2}\frac{gm^4}{M^4}$, in agreement with $k \to \infty$ of FRG
- Logarithmic divergence independent of the momentum p: a field redefinition at a given energy is enough for all scales ⟹ no **physical running**

The higher derivative scattering amplitude

The full one loop 4-point function comes from a bubble diagram



Low energy limit:

$$\mathcal{M} = \frac{5g^2m^4(s^2+t^2+u^2)}{64\pi^2M^8\epsilon} - \frac{g^2}{11520\pi^2M^8}\left\{-900m^4(s^2+t^2+u^2)\log\left(\frac{4\pi\mu^2}{m^2}\right) + \dots\right\}$$

Divergencies and large logs can be reabsorbed in the renormalization of g

$$g(\mu) = g_B - \frac{5g^2m^4}{32\pi^2M^4}\left[\frac{1}{\epsilon} - \gamma_E - \log\left(\frac{4\pi\mu^2}{m^2}\right) + \frac{11}{30}\right]$$

$\beta_g^\mu = 5g^2m^4/16\pi^2$ in the ***μ*-running**

$\beta_g = 0$ in the **physical running**

High energy limit:
Using the low energy g,

$$\mathcal{M} = -\frac{g}{2M^4}\left[1 - \frac{17gm^4}{192\pi^2M^4}\right](s^2+t^2+u^2)$$
$$-\frac{g^2m^4}{192\pi^2M^8}\left[\log\left(\frac{-s}{m^2}\right)(13s^2+t^2+u^2) + \log\left(\frac{-t}{m^2}\right)(s^2+13t^2+u^2) + \log\left(\frac{-u}{m^2}\right)(s^2+t^2+13u^2)\right]$$

Large logs with growing u, s, t.

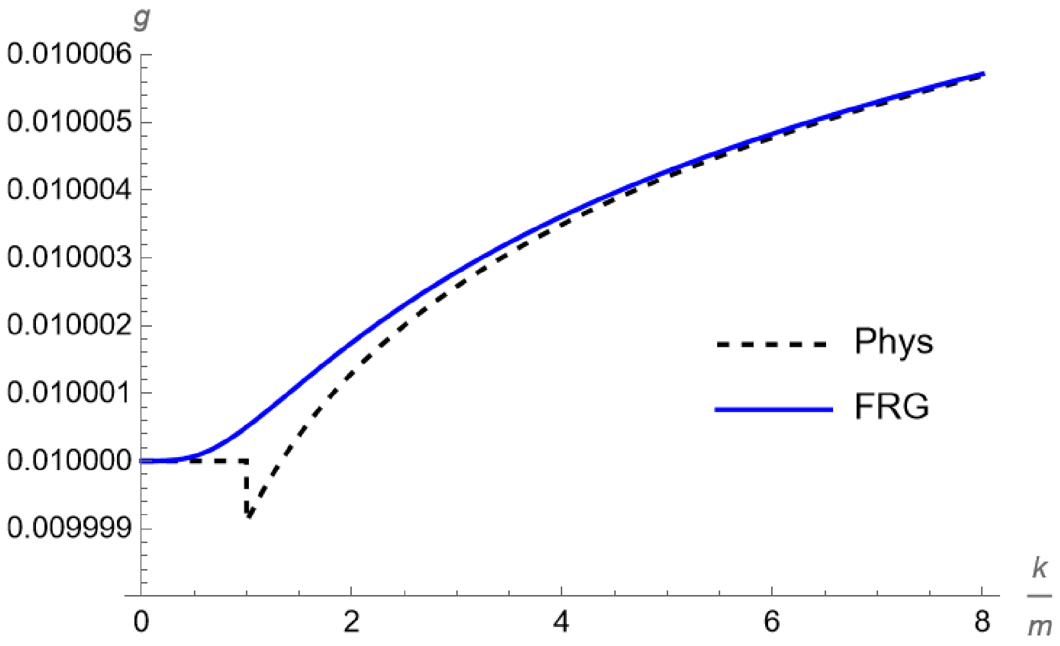
To avoid breakdown of perturbation theory, redefinition of g at $s = t = u = \mu_R$

$$\bar{g}(\mu_R) = g + \frac{5g^2m^4}{32\pi^2M^4}\left[\log\left(\frac{\mu_R^2}{m^2}\right) - \frac{17}{30}\right]$$

$\beta_g = 5g^2m^4/16\pi^2$ also in the **physical running**

The mass threshold

- When ghosts start to propagate, higher dimension operators do not run anymore.
- In the low energy limit, only the **physical running** of g matches the EFT, because **μ-running** can't manage the freezeout of ghosts
- The FRG β_g matches with the **physical running** in the asymptotic regions. When $E \sim m$, there is a strong dependence on the renormalization scheme.



Discussion

- In the low energy EFT at one loop there are higher order operators with 6 and 8 derivatives, which disappear above the mass threshold
- The physical running is only defined in asymptotic regions. The FRG running of g agrees there
- To match the FRG also with the two-point function, one should integrate the **k-flow** to zero and hence consider the dependence on external momenta: $\boldsymbol{p}$**-running** (see [A. Codello, R. Percacci, L. Rachwal and A. Tonero, ’16] for a ϕ^4 example)
- At high energy, with $g < 0$, the theory is AF and perturbative respect to the 4-derivative kinetic term, but also strongly interacting.

Physical running in R^2

- In Quadratic Gravity, heat kernel technique has been used to compute $\boldsymbol{\mu}$**-running** and $\boldsymbol{k}$**-running**:

Perturbative β functions

$$\beta_\lambda = -\frac{1}{(4\pi)^2}\frac{133}{10}\lambda^2$$
$$\beta_\omega = -\frac{1}{(4\pi)^2}\frac{25+1098\omega+200\omega^2}{60}\lambda$$
$$\beta_\theta = \frac{1}{(4\pi)^2}\frac{7(56-171\theta)}{90}\lambda$$

[I.G. Avramidi, A.O. Barvinski, Phys. Lett. **159 B**, 269 (1985).]

FRG β functions

$$\beta_\lambda = -\frac{133}{160\pi^2}\lambda^2 + \tilde{Z}_N\lambda^3\frac{251\xi-58\lambda}{120\pi^2\xi}$$
$$\beta_\xi = -\frac{5(72\lambda^2-36\lambda\xi+\xi^2)}{576\pi^2} + \tilde{Z}_N\frac{9720\lambda^3-1980\lambda^2\xi+489\lambda\xi^2-14\xi^3}{6480\pi^2}$$
$$\beta_\rho = -\frac{49}{180\pi^2}\rho^2 + \tilde{Z}_N\lambda\rho^2\frac{233\xi-58\lambda}{240\pi^2\xi}$$
$$\beta_{\tilde{Z}_N} = \left(-2+\frac{(30\lambda-\xi)(4\lambda+\xi)}{192\pi^2\xi}\right)\tilde{Z}_N + \frac{-3168\lambda^2+654\lambda\xi+35\xi^2}{1152\pi^2\xi(6\lambda+\xi)} - \frac{72\lambda^2-84\lambda\xi+65\xi^2}{192\pi^2(6\lambda+\xi)^2}\log\left(\frac{2}{3}-\frac{2\lambda}{\xi}\right).$$

[K. Falls , N. Ohta, R.P., arXiv:2004.04126 [hep-th]]

- Non-physical contributions in the UV region come from tadpole diagrams, as observed in our toy model. What remains if we neglect the contribution of tadpole diagrams?
- Physical running in HDNLSM has been studied in arXiv:2308.13704 by J. Donoghue and G. Menezes, we want to reproduce this study on Quadratic Gravity

Thank you