Testing the Second Postulate of Relativity using Gravitational Waves

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Outline of Talk

- Motivation
- Geometric Setup
- Gravitational wave Phasing
- Signal and Parameter estimation
- Conclusion and Outlook

Motivation and a bit history

- The two postulates of relativity are the bedrock of classical and quantum physics.

- Any violation of these principles would require a significant revision of the laws of physics. In our work, we want to test the 2nd postulate.

- How does one test such an important assertion: Does the speed of light depend on the speed of the emitter/observer?

- For EM radiation: Emission models [Ritz (1908, ’10), Tolman (1912), de Sitter (1913), Fox (1962, ’67)] .

Emission models,
$c' = c$+speed of source

→

Ritz (1908, ’10), Tolman (1910, ’12),
de Sitter (1913), Fox (1962, ’67), ...

Brecher's work: constraining k

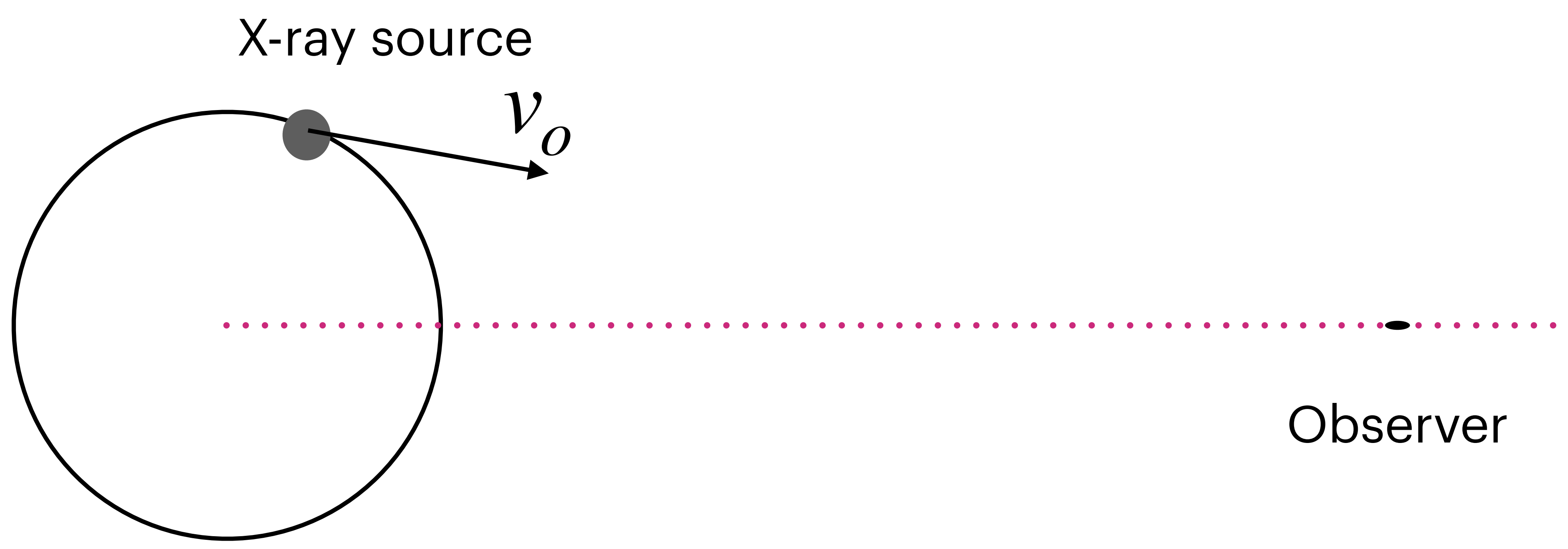
[Brecher, PRL 39, 1051 (1977)]

- Fox (1962): Observations at optical wavelengths is not suitable because of interstellar extinction.

- Therefore, Brecher used hard X-ray sources (larger extinction length) like Her X-I.

- Using the arrival time of X-ray pulses, he obtained the strongest bound for EM radiation so far: $k < 2 \times 10^{-9}$.

- Now, we build an emission model for GWs.

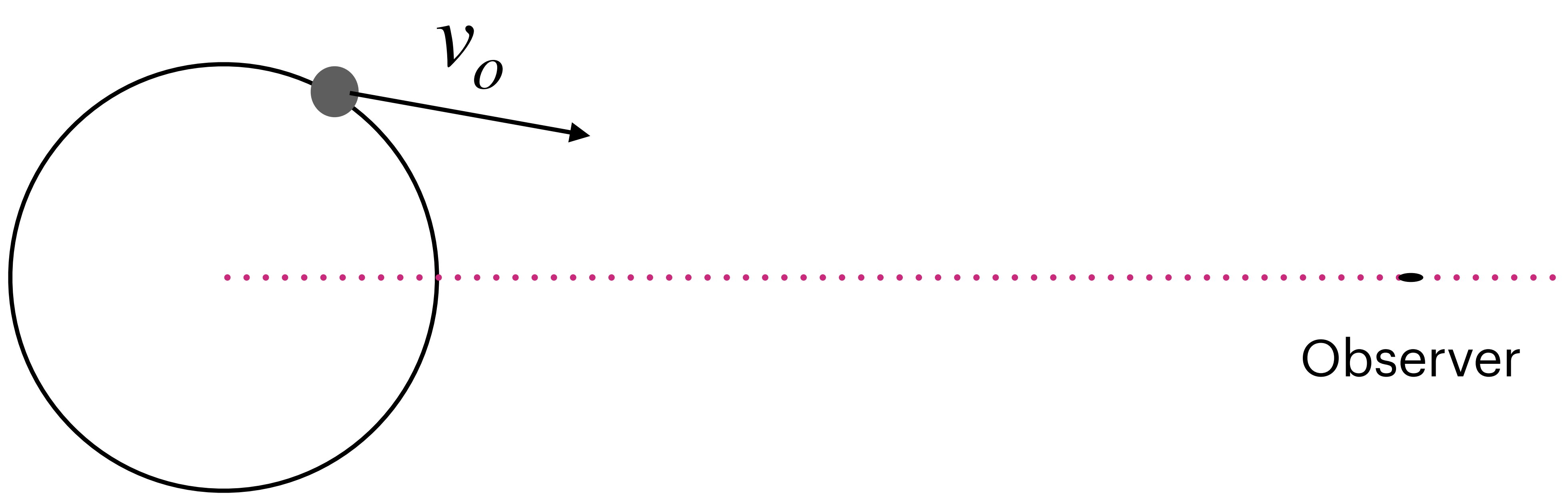


Observed speed of light, $c' = c + k\, v_o$

Emission models for GWs

Since GWs propagate in flat background with the speed of light, one can use emission-theory motivated phenomenological models to test the 2nd postulate.

For light: source emitting photons



Observed speed of light, $c' = c + k\, v_o$

For GWs: inspiralling objects in binary

- Natural extension: reduced one-body motion of the binary

$$v_o \propto v(f) = (\pi G m f)^{1/3} \quad .$$

- This sets the scale of velocity for a particular system.
- In fact, the CoM-frame orbital velocities are proportional to $v(f)$. In case of comparable component masses (as in GW170817), the proportionality constant is $\mathcal{O}(1)$.

Thus, our result will not change much even if we use orbital velocities. Even for extreme mass ratio case (important for LISA), the velocity of the smaller mass is well approximated by $v(f)$.

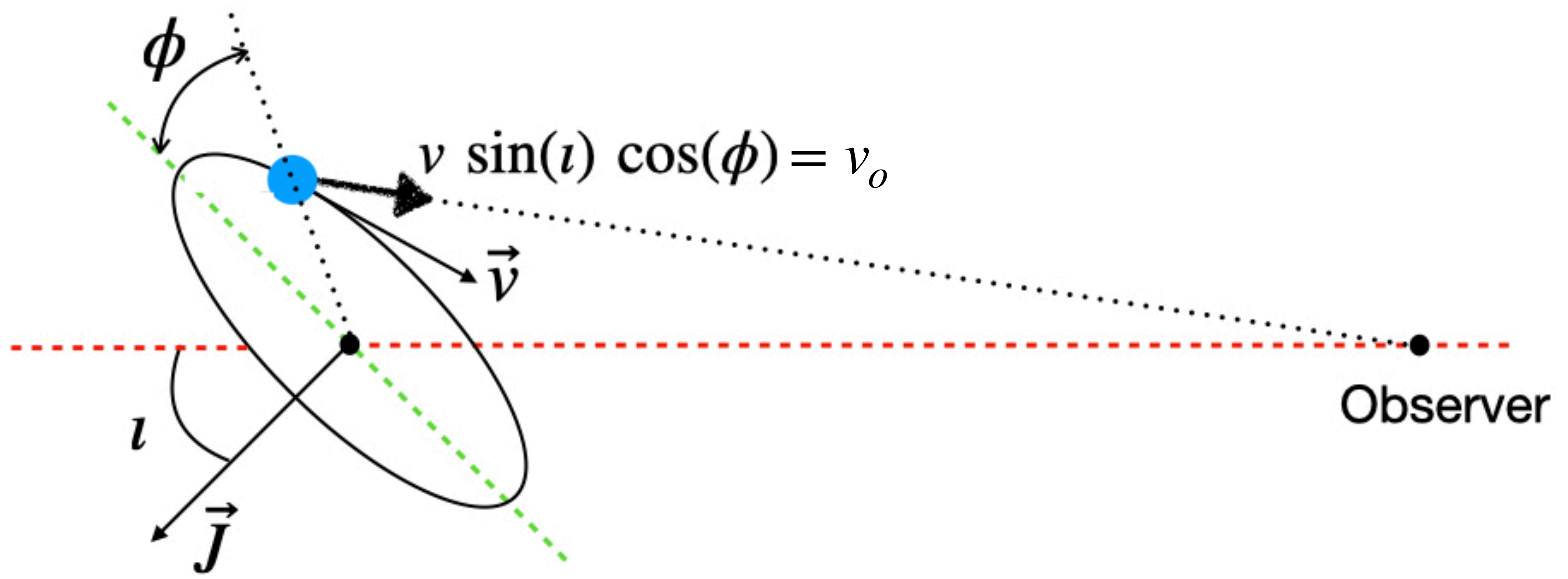
Geometric setup
of a compact binary GW source

- Let's have a closer look on the reduced binary system.

- We shall treat the radius of the circular orbit to vary adiabatically due to an induced GW radiation reaction: post-Newtonian (PN) formalism.

- Emission model:
$$c'(f) = c + k\, v(f)\ \sin(\iota)\ \cos(\phi(f)).$$
v_o

- Thus, effect of k is realisable only when there is component of velocity along the observer.



Geometric orientation of the reduced binary system relative to a distant observer. For `GW170817`, $\iota = 151^\circ$ and redshift, $z = 0.0099 \ll 1$.

Imprint of k on GW phasing

- The effects due to k are captured prominently in the GW propagation, which is $\propto k\,d_L$.

- Time (t_e) of GW emission and time (t_o) of reception of the signal by the observer is given by

$$t_o \simeq t_e + \frac{d_L}{c} - \frac{R_e\ \sin(\iota)}{c} \sin\left(\omega_e\, t_e\right) - \frac{k\, d_L\, v_e\ \sin(\iota)}{c^2} \cos\left(\omega_e\, t_e\right) .$$

- Let us consider the emission of two successive GW signals at source times t_e' and $t_e = t_e' + \Delta t_e$.

- Due to the frequency-dependent speed of GW propagation, these signals will reach the distant observer within a duration given by $\Delta t_o \neq \Delta t_e$.

Distinctions from previous works

Our analysis is distinct from previous works that

either

(a) constrain GW speed c (**We rather constrain** k**,** not to be interpreted as a bound on c),

[R. Abbott et al. (LIGO scientific, Virgo, Fermi-GBM, INTEGRAL), ApJL 848, L13 (2017);
R. Abbott et al. (LIGO scientific, Virgo), PRD 103, 122002 (2021)]

or,

(b) consider Lorentz-violating dispersion relation (We independently test the violation of the Second postulate)

[Mirshekari et al., PRD 85, 024041 (2012); Kostelecky et al., PLB 757, 510 (2016);
R. Abbott et al. PRD 103, 122002 (2021); R. Abbott et al. arXiv: 2112.06861[gr-qc] (2021)]

or,

(c) consider the effects of massive graviton: GW speed is controlled by its mass.

[Will, PRD 57, 2061 (1998)]

Modified GW phasing formula

- GW phasing equation: $\psi\left(f_e\right) = 2\pi \int_{f_e'}^{f_e} \Delta t_o \, df + 2\,\pi f_e \, t_c - \phi_c - \dfrac{\pi}{4}$ $(z \ll 1)$.

- Leads to modifications in the GW phasing expression (say, between $[f_e', f_e]$) beyond the standard PN-expression:

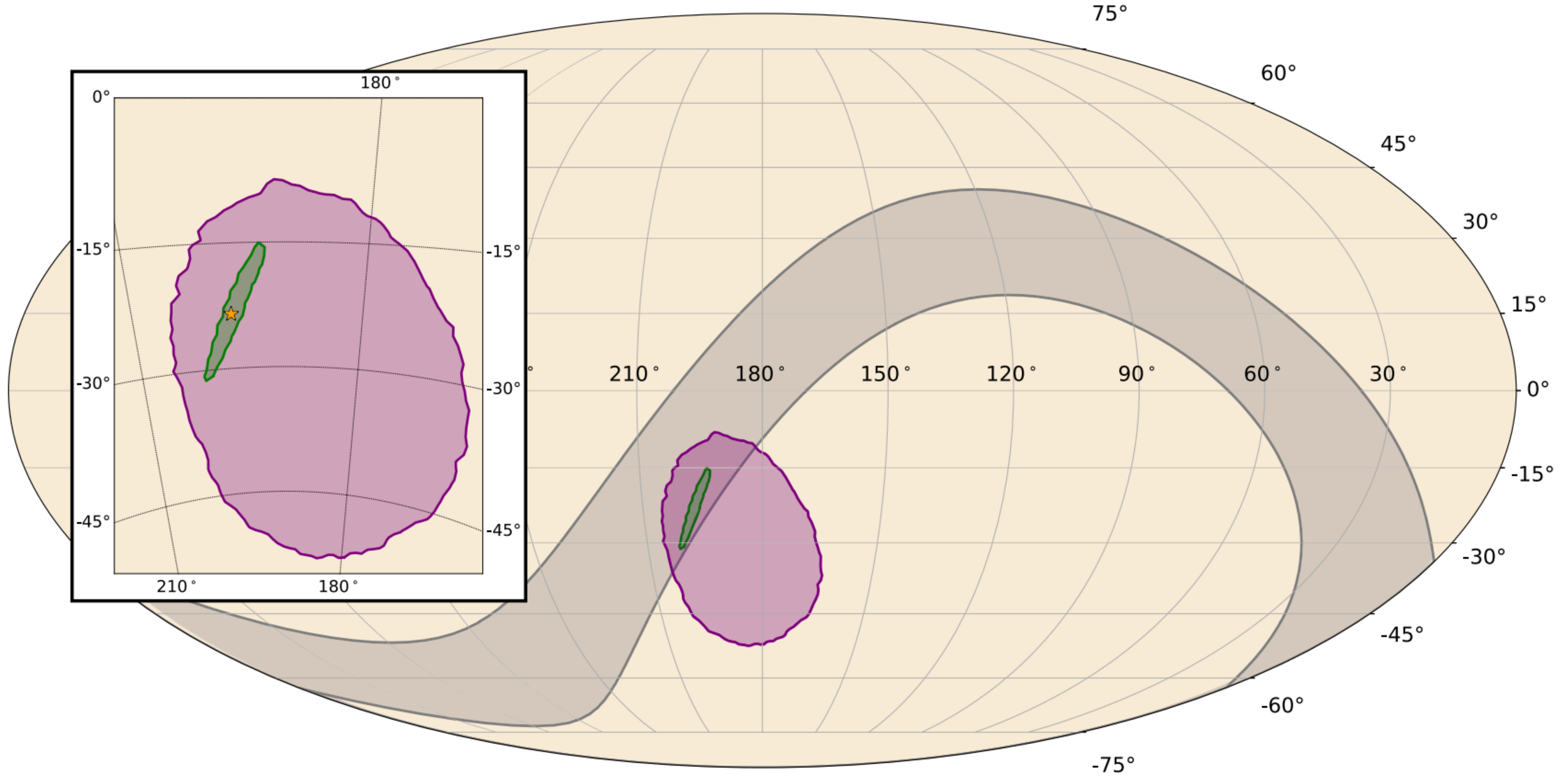
$$\psi(f_e) = \psi_{PN} + 3\left(\frac{k\, d_L\, v_e^4\, \sin(\iota)}{\sqrt{2}\,\pi\, c^2\, G\, m}\right)\left[1 + x^2 + \frac{x^4}{2}\log(x) - \frac{x^6}{8} + \ldots\right],$$ where

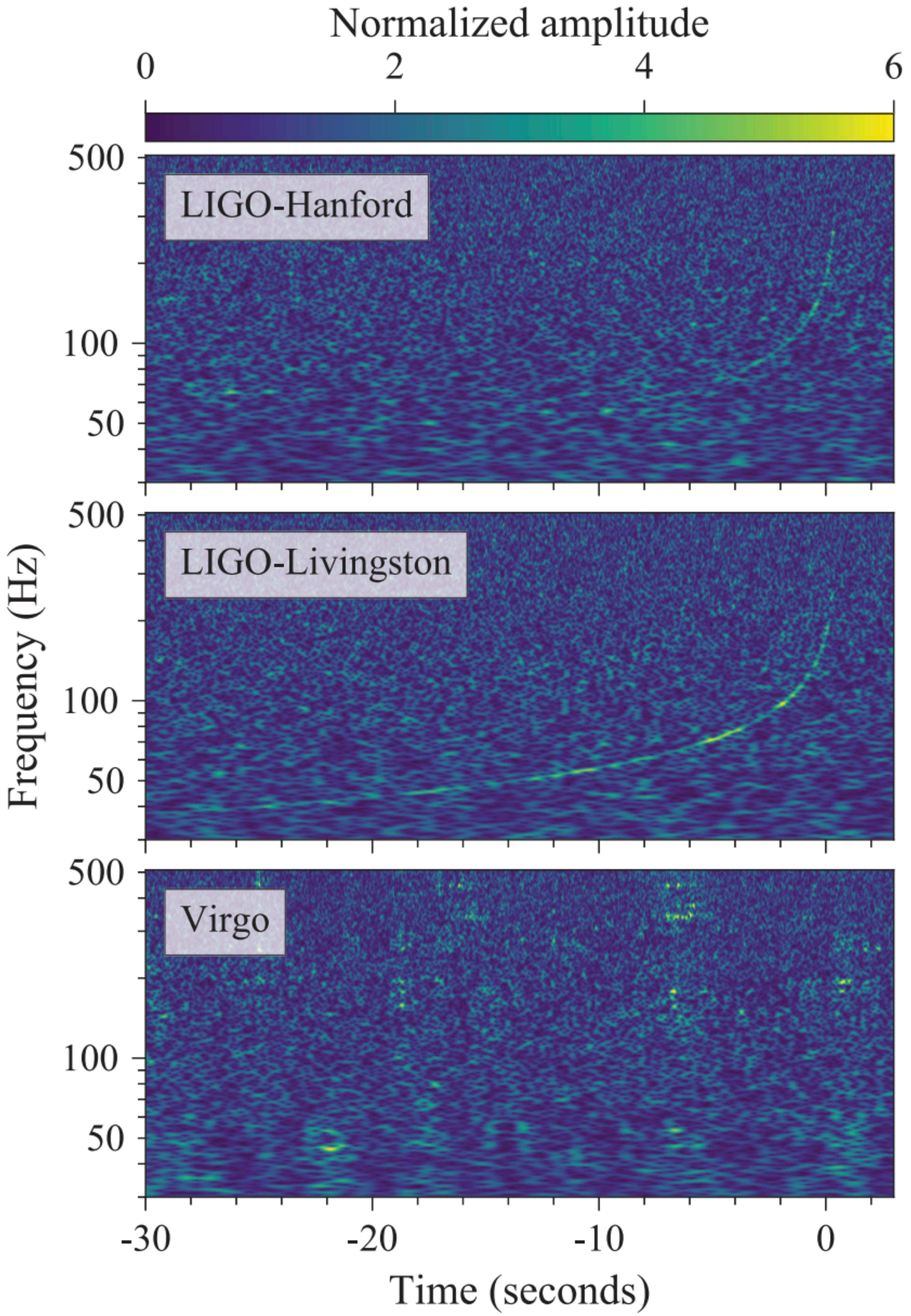
$x = v_e'/v_e$.

GW170817

B.P. Abbott *et al.* (LIGO Scientific and Virgo Collaboration)
ApJL **848** L13 (2017)

- Binary neutron star (BNS) event (large number of in-band cycles)
- Precisely estimated sky (α, δ) location and luminosity distance d_L from EM counterpart GRB 170817A





Estimation of *k* from GW data

Aim

measure k and,

calculate the Bayes factor $\mathscr{B}_{01}$ to check if data favours the null hypothesis ($k = 0$) over $k \neq 0$.

Priors considered

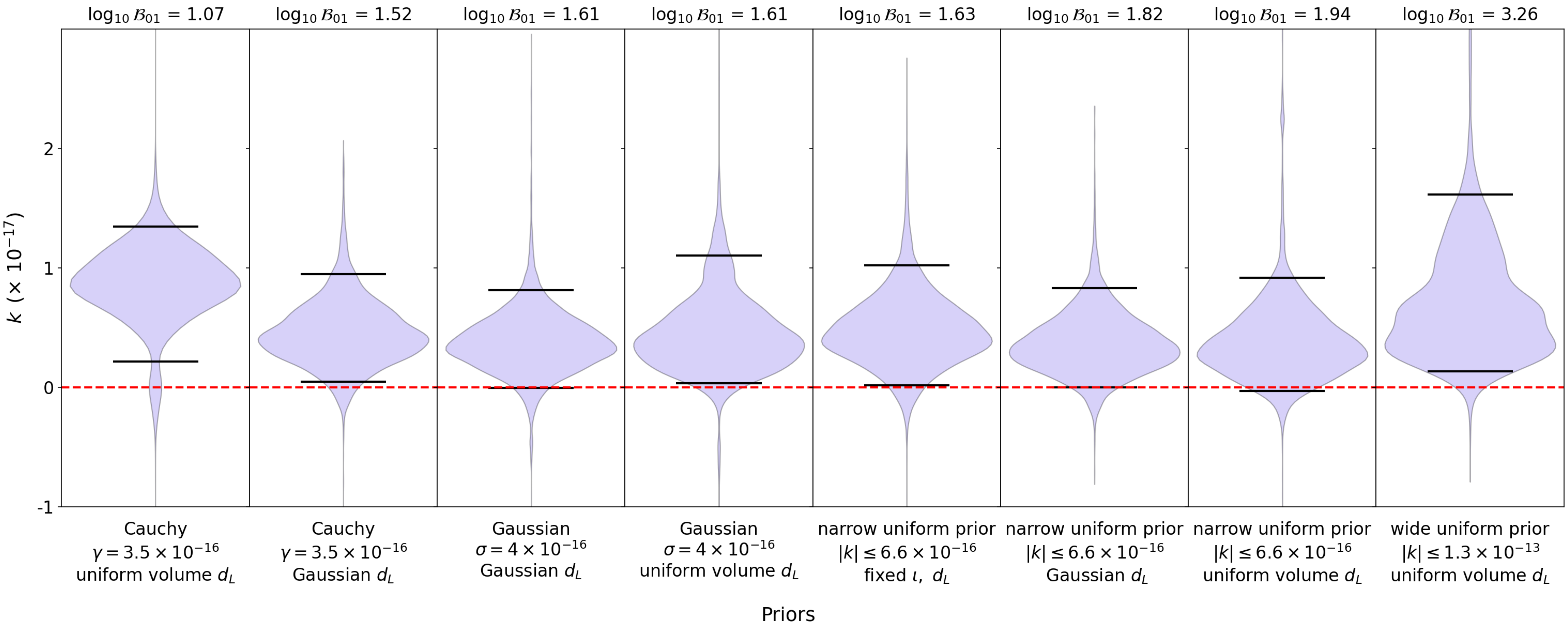
- Uniform over $m_1, m_2, \chi_{1z}, \chi_{2z}, \cos\iota, t_c$ (around the trigger's geocentric coalescence time)
- Prior over d_L (two cases): Uniform in volume, Gaussian. [Cantiello et al, ApJL 854, L31 (2018)]
- Prior over k (four cases): Narrow/Wide non-informative priors, Weak/Strongly informative

Data and Sampling

- `TaylorF2` `3.5PN` waveform model with modified phase.
- 360 s of cleaned LIGO/Virgo data containing the `GW170817` event.
- Used phase-marginalized likelihood, interfaced with the `Dynesty` sampler

Results

Range of upper limits of 90% CI: $k \leq (0.8 - 1.6) \times 10^{-17}$



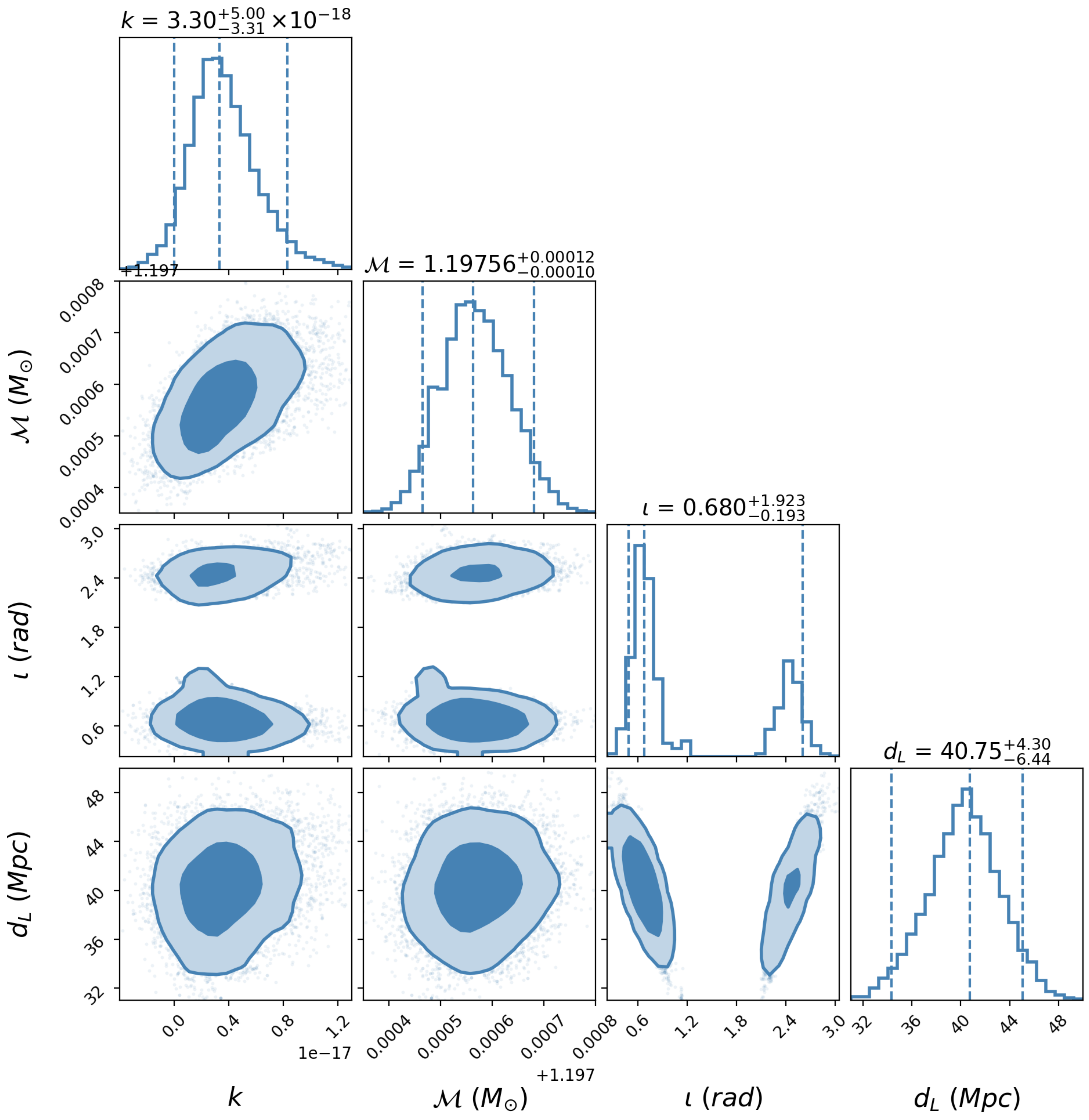
Results continued...

- Narrow uniform prior over k
- Gaussian prior over d_L

$$d_L \sim \mathcal{N}(40.7\,\text{Mpc}, 3.3\,\text{Mpc})$$

[Cantiello et al, ApJL 854, L31 (2018)]

- Degeneracy in ι
- Recovered posterior distributions over model parameters are consistent with LVK analysis when $k = 0$



Conclusion and Outlook

- Tested the second postulate of relativity
- Estimated k using Bayesian analysis of `GW170817` event
- Found $k \leq (0.8 - 1.6) \times 10^{-17}$ (These are the most stringent upper bound on k so far) for a wide range of non-informative, weakly informative and strongly informative priors over k.
- $\log_{10} \mathscr{B}_{01} \sim 1.82$, upholding the second postulate $(k = 0)$.

Implies that the data is $\times 66$ more likely under the null hypothesis $(k = 0)$

Thank You

Any questions?

Backup slides

Comments on GW phasing formula

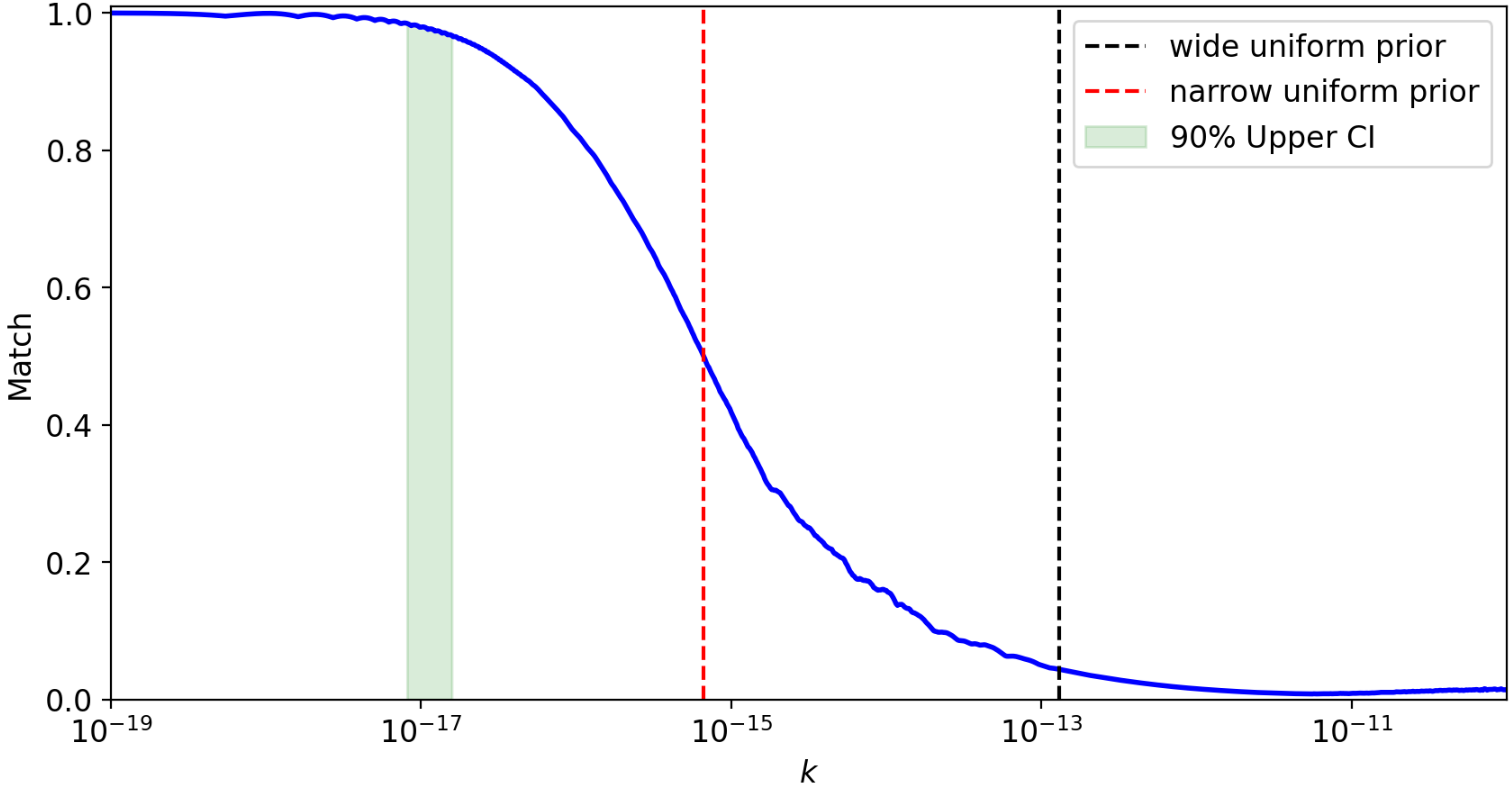
- The leading order correction term denotes a non-shrinking circular orbit. And, as $x \ll 1$, this leading order term alone gives a good estimate for k .

- Since the correction contains a series of terms (capture radiative effect) , we need to check whether the estimate of k is stable, i.e., does not vary much as we include more terms in the series.

- Since $d_L \sim \mathcal{O}(10)$ Mpc, it is obvious that value of k should be very small to match with observation.

- Use this modified phasing expression to look for any deviation from the observed GW signal.

Match vs k



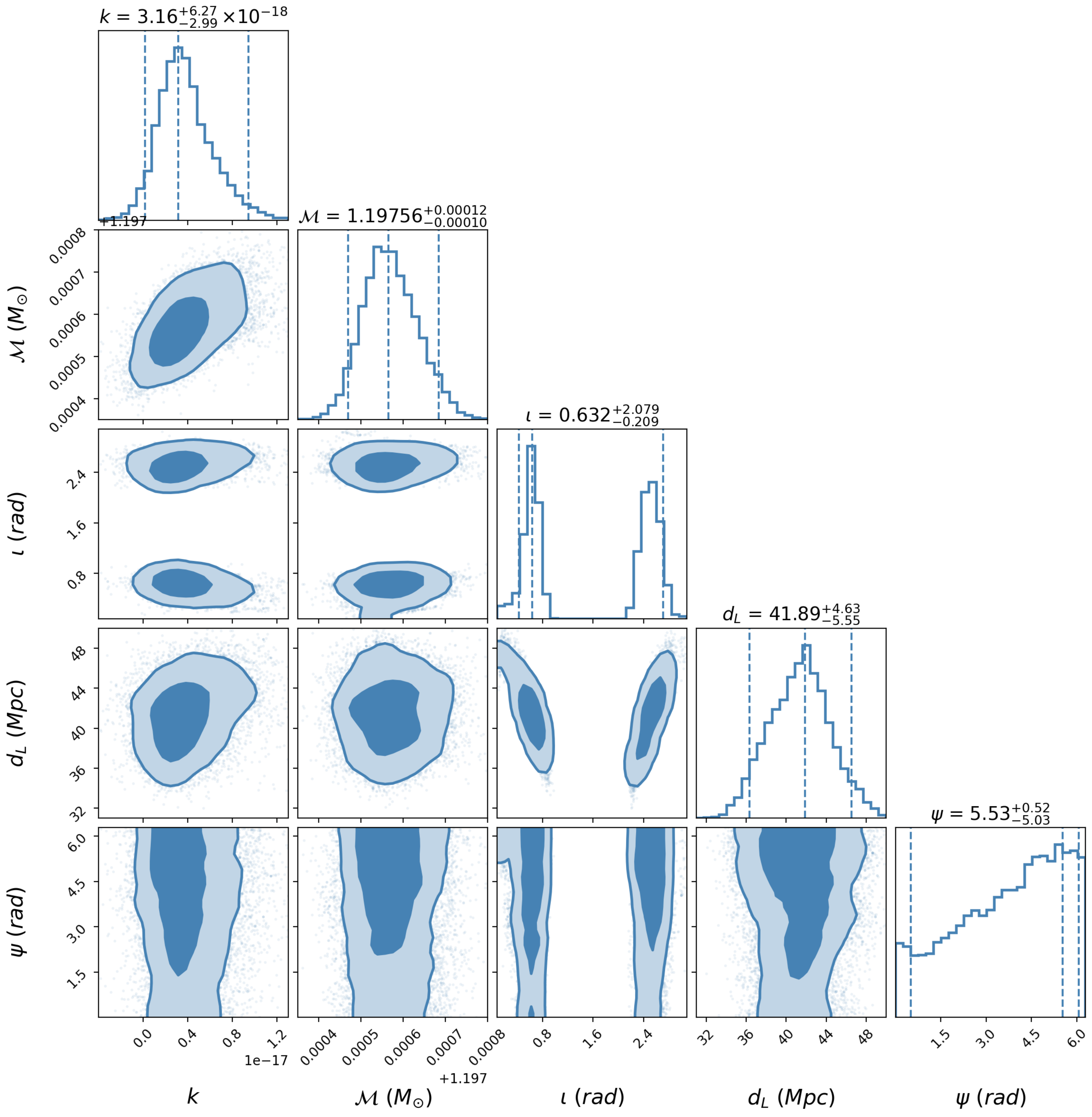
wide uniform prior
narrow uniform prior
90% Upper CI
Match
1.0
0.8
0.6
0.4
0.2
0.0
10^{-19}
10^{-17}
10^{-15}
10^{-13}
10^{-11}
k

Varying ψ

- Narrow uniform prior over k
- Uniform over $m_1, m_2, \chi_{1z}, \chi_{2z}, \cos\iota, \psi, t_c$ (around trigger)
- Gaussian over d_L

[Cantiello et al, APJL 854, L31 (2018)]

- $\log_{10} \mathscr{B}_{01} = 1.75$



The Role of priors

A toy study

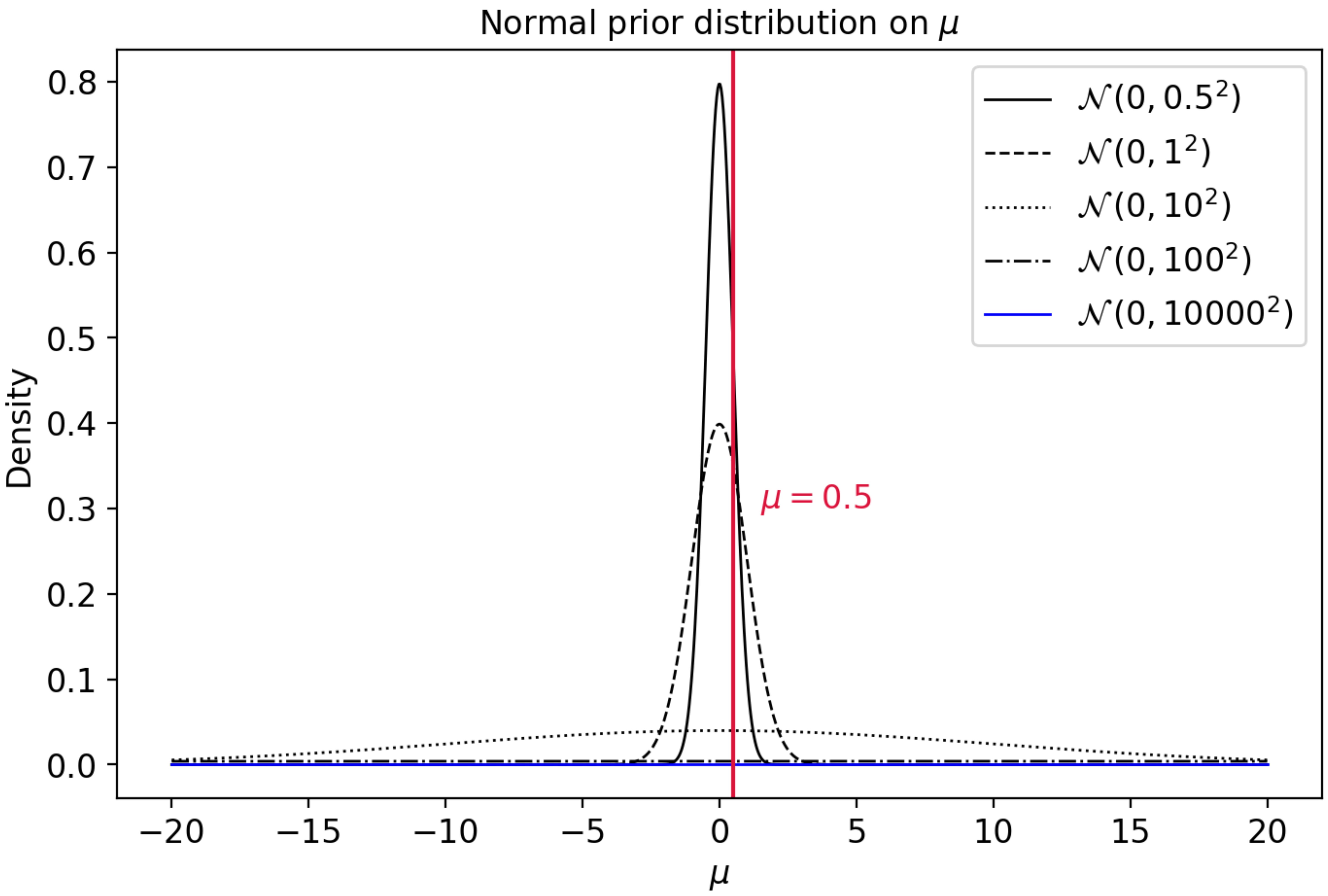
Observations

$d \sim \mathcal{N}(\mu = 0.5, \sigma^2 = 1)$

$\mathcal{H}_0 : \mu = 0$ Null hypothesis

$\mathcal{H}_1 : \mu \neq 0$ Alternative hypothesis

Observing the effects of prior's variance on Bayes factor

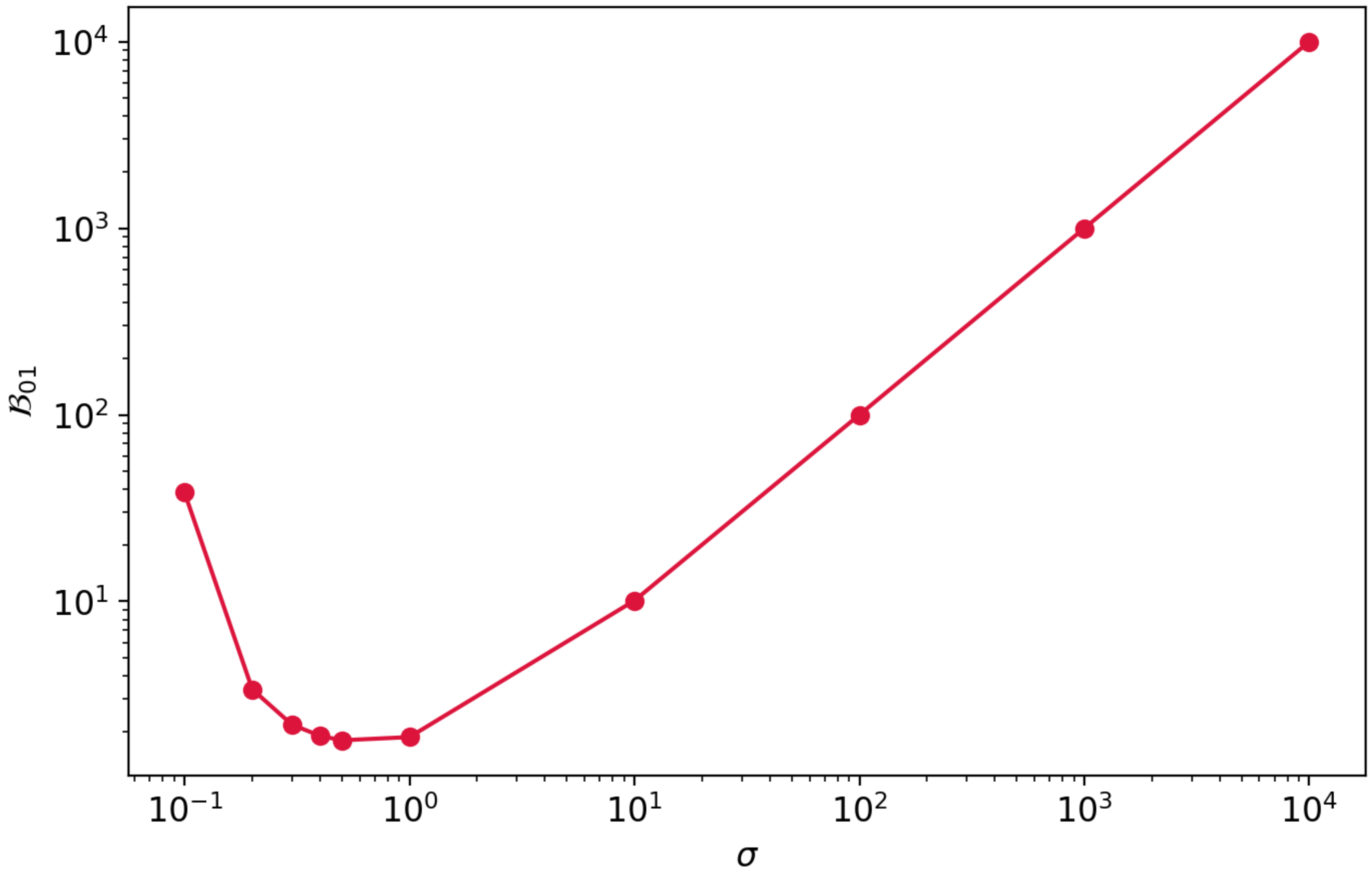


The Role of priors

Results

Bayes factor $\mathscr{B}_{01} = \frac{p(d \mid \mathscr{H}_0)}{p(d \mid \mathscr{H}_1)}$

- Priors with large variance tends to favour the null hypothesis.
- Priors should be carefully chosen, especially their width (variance)



[Rouder, J.N. *et al. Psychonomic Bulletin & Review* **16**, 225–237 (2009)]

[Du et al., *Behav Res* **51**, 1998–2021 (2019)]