# Mapping the landscape of gravity theories

**Serena Giardino**
Max Planck Institute for Gravitational Physics
Heidelberg University

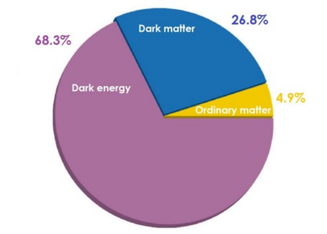
In collaboration with Valerio Faraoni (Bishop's U.), Andrea Giusti (ETH Zurich), Lavinia Heisenberg (Heidelberg U.)

XXV SIGRAV Conference

07/09/2023

# Picturing the landscape

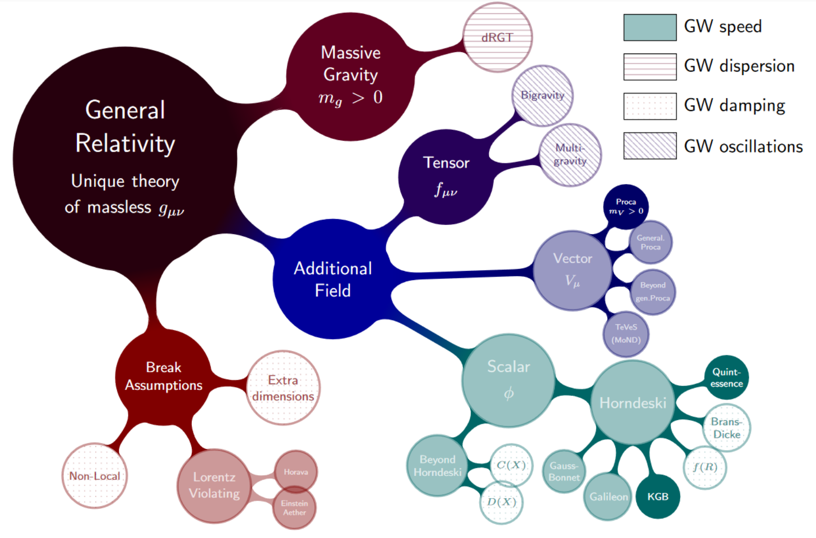
- Einstein's GR: not the only theory of gravity we can construct
- UV problems: quantum gravity
- IR problems: dark energy, dark matter



Source: NASA (after Planck data)

- Generalisations/extensions: higher-order curvature terms, additional degrees of freedom...
- Basic principles (equivalence, covariance, causality... unless explicitly broken) and observational constraints
- Landscape: GR + generalisations/extensions

# (A portion of) the landscape




Source: [1]

[1] J. M. Ezquiaga, M. Zumalacarregui, Front. Astron. Space Sci. 5:44 (2018) - arXiv:1807.09241

# Big-picture goal

Explore generalisations of GR

$\Downarrow$

Understand GR as a special case in a broader framework

$\Downarrow$

Attempt to overcome limitations of GR (and learn about universe in the process)

# Inspiration: thermodynamics and gravity

- Thermodynamics $\Leftrightarrow$ gravity?
- GR $\Leftrightarrow$ thermodynamical equilibrium [2]
  Modified gravity $\Leftrightarrow$ non-equilibrium [3]
- ”Thermodynamics of gravitational theories”
- Interpretation: internal entropy production due to viscous dissipative effects from purely gravitational degrees of freedom [4]
- Question: what exactly is the dissipative process leading to equilibrium from non-equilibrium?
- Question: what is the order parameter measuring closeness to equilibrium?

[2] T. Jacobson, PRL 75 (1995) - arXiv:gr-qc/9504004
[3] C. Eling, R. Guedens, T. Jacobson, PRL 96 (2006) - arXiv:gr-qc/0602001
[4] G. Chirco & S. Liberati, PRD 81 (2010) - arXiv:0909.4194

# The plan

- Take scalar-tensor gravity, containing $f(\mathcal{R})$ as subclass
- Model scalar contribution as effective dissipative fluid [5]
- Apply irreversible non-equilibrium thermodynamical description, such as Eckart's [6]
- Extract thermodynamical quantities such as temperature and viscosity
- Understand dissipative process

[5] V. Faraoni & J. Coté, PRD 98 (2018) - arXiv:1808.02427
[6] C. Eckart, Phys. Rev. 58 (1940)

# Imperfect fluid description of modified gravity

- Brans-Dicke scalar-tensor action (Jordan frame) ($G_{\text{eff}} = 1/\phi$)

$$S_{\text{ST}} = \frac{1}{16\pi} \int d^4x\sqrt{-g}\left[\phi\mathcal{R} - \frac{\omega(\phi)}{\phi}\nabla^c\phi\nabla_c\phi - V(\phi)\right] + S^{(\text{m})}$$

- Effective Einstein equations

$$G_{ab} = 8\pi G_{\text{eff}} T^{(m)}_{ab} + 8\pi T^{(\phi)}_{ab}$$

- Study thermodynamics of imperfect fluid
$T^{(\phi)}_{ab} = \rho^{(\phi)} u_a u_b + q^{(\phi)}_a u_b + q^{(\phi)}_b u_a + \Pi^{(\phi)}_{ab}$

- $\nabla_a\phi$ timelike + future-oriented $\Rightarrow$ natural fluid interpretation

- 4-velocity $u^a = \dfrac{\nabla^a\phi}{\sqrt{-\nabla^e\phi\nabla_e\phi}}$ with $u^a u_a = -1$

# Eckart's thermodynamics

$$P^{(\phi)}_{\text{vis}} = -\zeta\,\theta$$

$$q^{(\phi)}_a = -\mathcal{K}\left(h_{ab}\nabla^b\mathcal{T} + \mathcal{T}\dot{u}_a\right)$$

$$\pi^{(\phi)}_{ab} = -2\eta\,\sigma_{ab}$$

- Non-equilibrium (irreversible) thermodynamics
- "1st order": simplest (linear) assumptions to satisfy $S^{\alpha}_{;\alpha} \geq 0$
- Relativistic generalisations of Stokes' law, Fourier's law and Newton's law of viscosity

# Temperature of $\phi$-fluid

- Comparing $\dot{u}_a$ and $q_a^{(\phi)}$

$$q_a^{(\phi)} = -\frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{8\pi\phi}\,\dot{u}_a \quad \Rightarrow \quad \mathcal{K}\mathcal{T} = \frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{8\pi\phi} \geq 0$$

- Temperature ”of scalar-tensor gravity”, relative to GR [7]

- GR limit: $\mathcal{K}\mathcal{T} \to 0$ when $\phi = \text{const.} \Rightarrow G_{\text{eff}} = G_{\text{N}}$, no $\phi$-fluid

[7] V. Faraoni, A. Giusti, PRD 103 (2021) - arXiv:2103.05389

# Fixed points of dissipation equation

- Approach to equilibrium governed by effective heat equation

$$\frac{d(\mathcal{K}\mathcal{T})}{d\tau} = 8\pi(\mathcal{K}\mathcal{T})^2 - \theta\mathcal{K}\mathcal{T} + \frac{\Box\phi}{8\pi\phi}$$

↙ ↘

$$\mathcal{K}\mathcal{T} = 0 \qquad\qquad \mathcal{K}\mathcal{T} = \text{const.}$$

GR + theories with non-dynamical $\phi$ [8]

Scalar-tensor stealth solutions[9]

- Rewriting heat equation $\Rightarrow$ stability criterion: $\Box\phi + m_{\text{eff}}^2 \geq 0$
- GR seems to be the only stable equilibrium!

[8] V. Faraoni, A. Giusti, S. Jose, S. Giardino, PRD 106 (2022) - arXiv:2206.02046
[9] S. Giardino, A. Giusti, V. Faraoni, EPJC 83 (2023) - arXiv:2302.08550

# Mapping the landscape

- Thermodynamical analogy $\Rightarrow$ map of gravity theories landscape
- GR: special role as the only stable equilibrium at $\mathcal{KT} = 0$
- Modified theories with additional dynamical degrees of freedom always have $\mathcal{KT} > 0$
- Nordström gravity with less degrees of freedom than GR has $\mathcal{KT} < 0$ (pathological)
- Dissipative relaxation process to GR in most cases, but not guaranteed

## Intuitive picture

# Application to cosmology

- Goal: test thermodynamical formalism and gain physical intuition in natural arena for scalar-tensor theories [10]

- FLRW spacetime: homogeneity & isotropy $\Rightarrow$ only contributions to $T_{ab}^{(\phi)}$ are $\rho^{(\phi)}$ and $P^{(\phi)}$

- Result: most cosmological solutions of scalar-tensor approach GR equilibrium as universe expands...

- Physical understanding of formalism confirmed (relaxation to equilibrium, singularity behaviour)

- ...but not always (e.g. Big Rip, $\mathcal{KT} = \mathrm{const}$) $\rightarrow$ different equilibrium states possible

[10] S. Giardino, V. Faraoni, A. Giusti, JCAP 04 (2022) - arXiv:2202.07393

# Viable Horndeski gravity

- Horndeski: most general scalar-tensor theory with 2nd-order equations of motion

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 \,,$$

$$\begin{aligned}
\mathcal{L}_2 &= G_2 \,, \\
\mathcal{L}_3 &= -G_3 \, \Box\phi \,, \\
\mathcal{L}_4 &= G_4 \, R + G_{4X} \left[ (\Box\phi)^2 - (\nabla_a \nabla_b \phi)^2 \right] , \\
\mathcal{L}_5 &= G_5 \, G_{ab} \, \nabla^a \nabla^b \phi - \frac{G_{5X}}{6} \left[ (\Box\phi)^3 - 3 \, \Box\phi \, (\nabla_a \nabla_b \phi)^2 + 2 \, (\nabla_a \nabla_b \phi)^3 \right] ,
\end{aligned}$$

- Horndeski constrained by multi-messenger event GW170817 $\Rightarrow$ ”viable” Horndeski

# Thermodynamics of Horndeski gravity

- Surprising finding: formalism only works for viable Horndeski! [11]
- $\mathcal{K}\mathcal{T} = \frac{\sqrt{2X(G_{4X} - XG_{3X})}}{G_4} \qquad \eta = -\frac{\sqrt{XG_{4X}}}{\sqrt{2}G_4}$
- $T_{ab} \subset \zeta(\phi, X) R_{abcd} \nabla^c \phi \nabla^d \phi \Rightarrow$ constitutive eqs. no longer hold
- Derivative non-minimal couplings $\Rightarrow$ failure of local separation between "gravity" and "matter" d.o.f.

[11] A. Giusti, S. Zentarra, L. Heisenberg, V. Faraoni, PRD 105 (2022) - arXiv:2108.10706

# Summary

- Connection between thermodynamics and gravity
- GR $\Rightarrow$ equilibrium state, modified gravity $\Rightarrow$ non-equilibrium
- New perspective: effective fluid approach to S-T theories + Eckart's non-equilibrium thermodynamics $\rightarrow \mathcal{T}$ of modified gravity, relative to GR
- Mapping the landscape of gravity theories with goal of understanding GR in more general framework
- $\mathcal{K}\mathcal{T} > 0$ whenever additional dynamical scalar d.o.f.
- GR special as the only stable equilibrium state!

# Outlook

- Extension to vector-tensor (Einstein-aether, generalized Proca). Limits of applicability, breakdown similar to Horndeski
- Complementary picture based on chemical potential instead of temperature (allowing for Einstein frame description) [12]
- Extension to space-like gradients $\Rightarrow$ BH stealth solutions + scalarization, no-hair theorem
- Long-term goal: ”2nd order” causal and stable thermodynamics

[12] V. Faraoni, S. Giardino, A. Giusti, R. Vanderwee, EPJC 83 (2023) - arXiv:2208.04051

Thank you for your attention!