Amplitudes in AdS from Conformal Field Theory

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September 4, 2023, SIGRAV conference

Conformal invariance

An interesting class of transformations are conformal transformations, which preserve angles.

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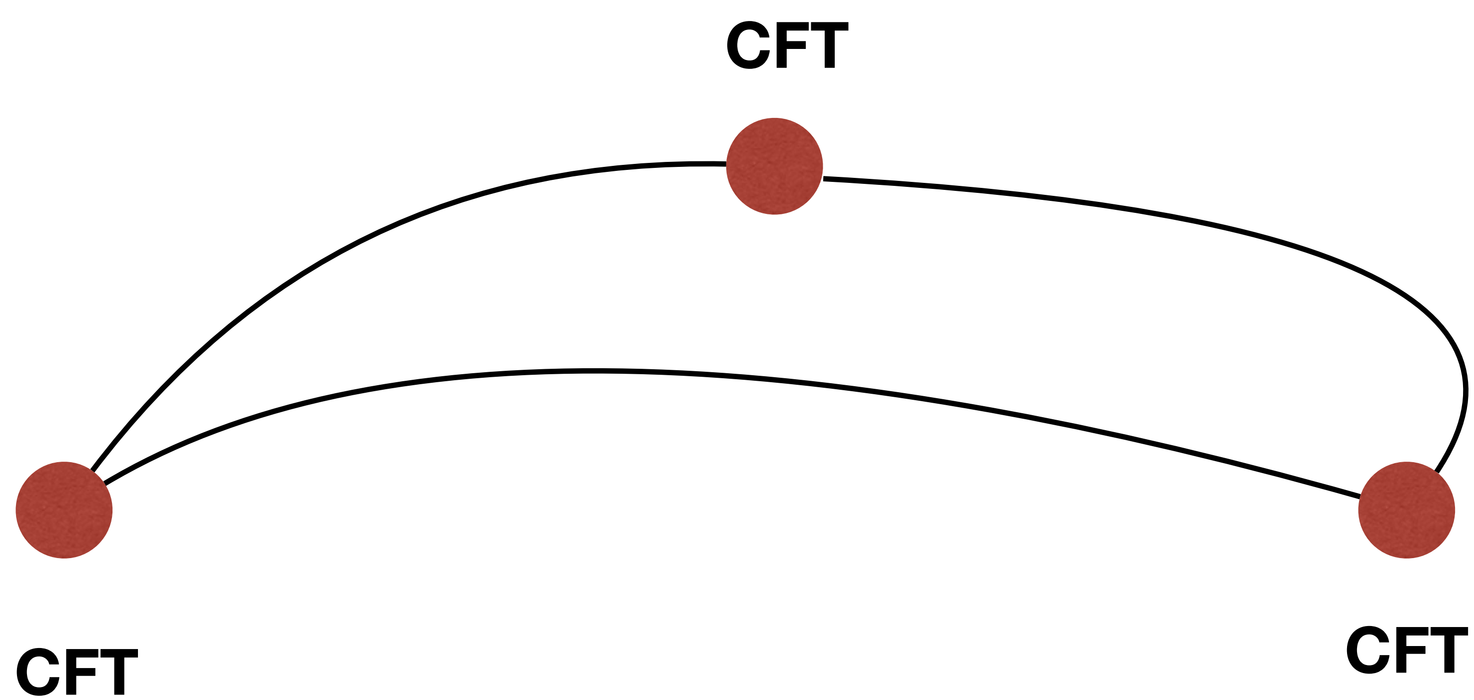
The description of fixed points boils down to classifying conformal field theories.

Centrality of CFTs

Conformal Field Theories (CFTs) are central also in the characterisation of QFTs.

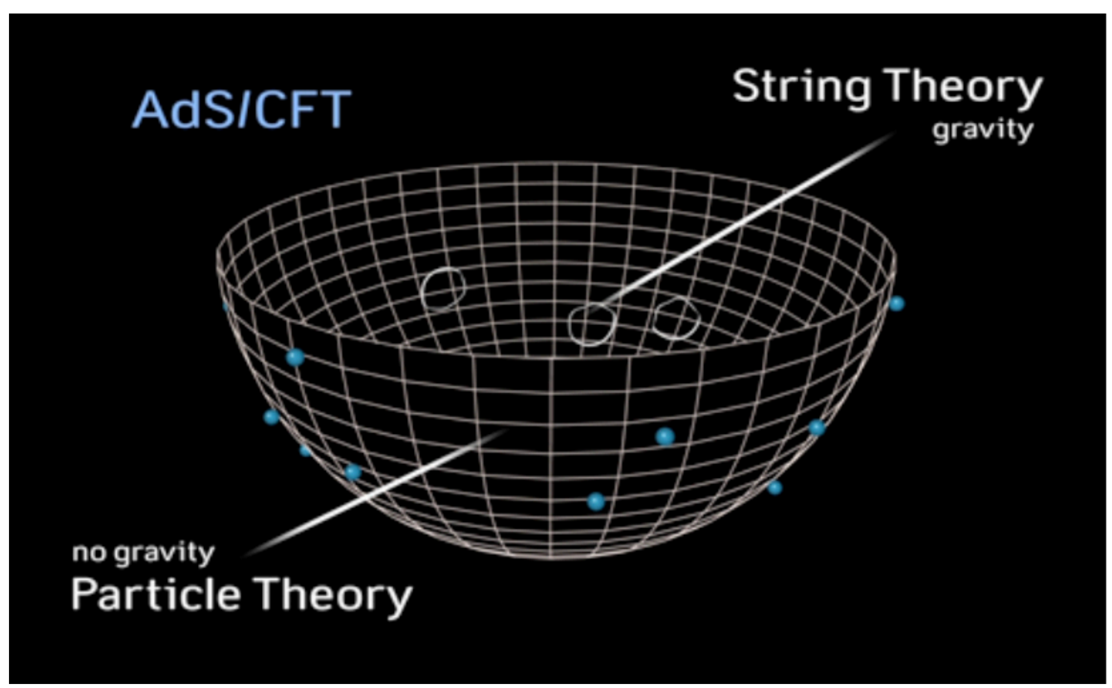


Large classes of QFTs can be seen as RG flows which emerge from a CFT (UV fixed point) and another non trivial CFT (IR fixed point)

Centrality of CFTs

They are related to theories of quantum gravity via the AdS/CFT correspondence

Operative mapping: observables (correlation functions and scattering amplitudes) in both theories are related in a very specific way.



Maldacena 1998

Main aim

Study conformal field theories using an approach which is based on symmetries and with very little/no input from the microscopical description of the theory

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classify the possible CFTs

find the critical exponents

conformal invariance

+

existence of the operator product expansion (OPE)

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Here it is shown for scalars, but it is similar for spinning operators.

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cross ratios $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$ $v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$

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Using the OPE inside correlators of *n*-points, with $n \geq 4$, it is possible to reduce them to two point functions.

Conformal blocks

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

Dolan Osborn 2002

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$$\frac{\delta_{mn}}{y_{12}^{2\Delta_m}}$$

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conformal blocks

What is m? It denotes the quantum numbers of the exchanged operators, which in this particular case are the conformal dimension and the Lorenz spin (Δ, ℓ).

Dolan Osborn 2002

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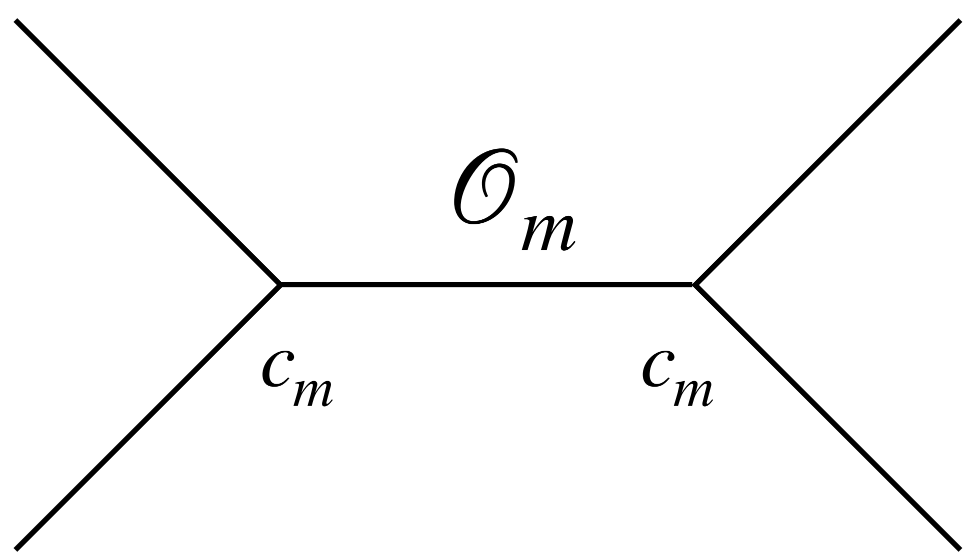
$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4) \rangle$$

$$\sum_m$$



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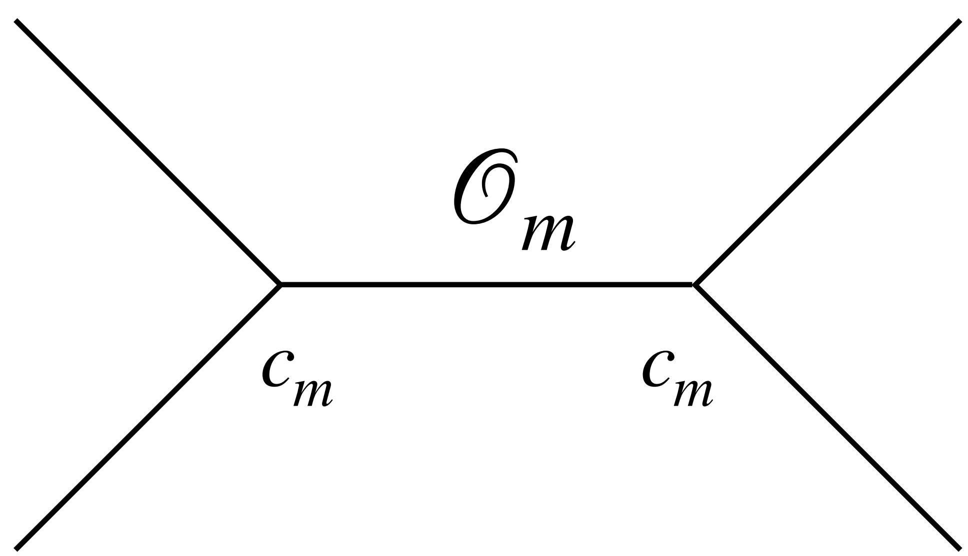

$= \sum_m c_m^2 \, g_m(u, v)$

Consistency

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Polyakov 1974, Ferrara Grillo Gatto 1973

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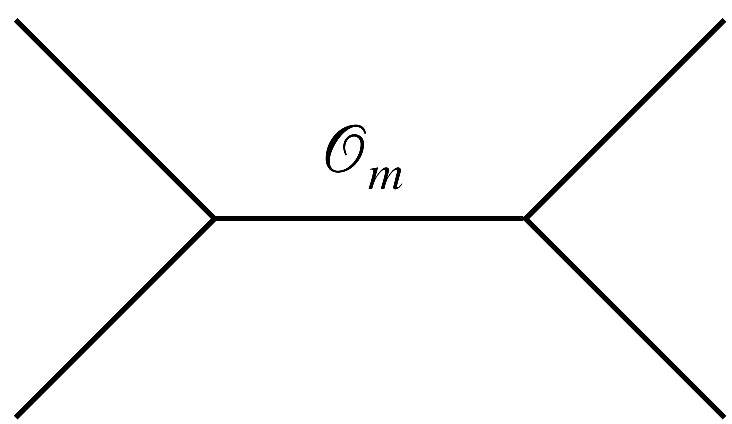
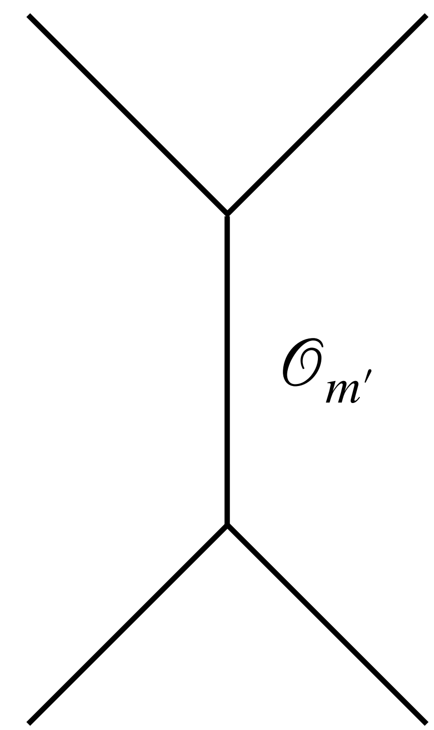
$\sum_{m}$



$=$ $\sum_{m'}$



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We can write it in terms of conformal blocks

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We won't be able to solve them completely, but we will discuss some approaches to find solutions consistent with the crossing relations.

Conformal Bootstrap

$$\sum_m c_m^2 \, g_m(u,v) = \left(\frac{u}{v}\right)^{\Delta_O} \sum_{m'} c_{m'}^2 \, g_{m'}(v,u)$$

Rattazzi Rychkov Tonni Vichi 2008
Kos Poland Simmons Duffin 2014

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It is related to the fact that the norm of a state is positive. In this context it means that $\Delta \geq \frac{d-2}{2}$ for scalars and $\Delta \geq d + \ell - 2$ for operators of spin ℓ, and that the $c_m \in \mathbb{R}$

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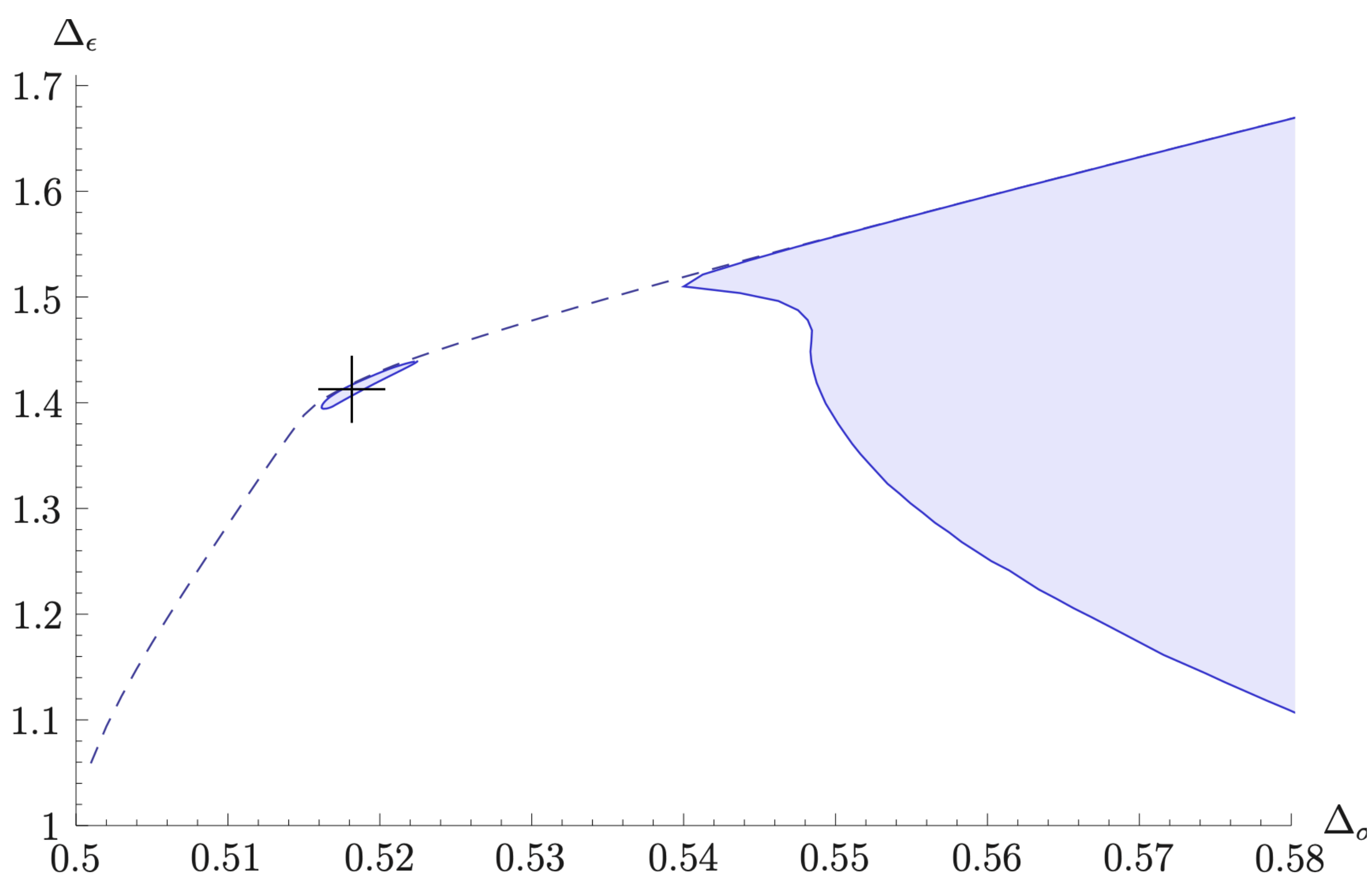

3d Ising Model

Rattazzi Rychkov Tonni Vichi 2008
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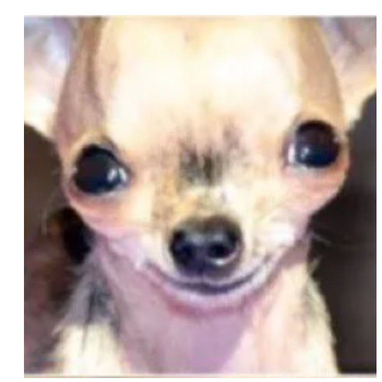
Numerical Bootstrap

The idea is to use the crossing relations as necessary conditions for conformal dimensions and OPE coefficients to belong to a CFT.

Tentative CFT data

crossing relations

MAYBE



NO

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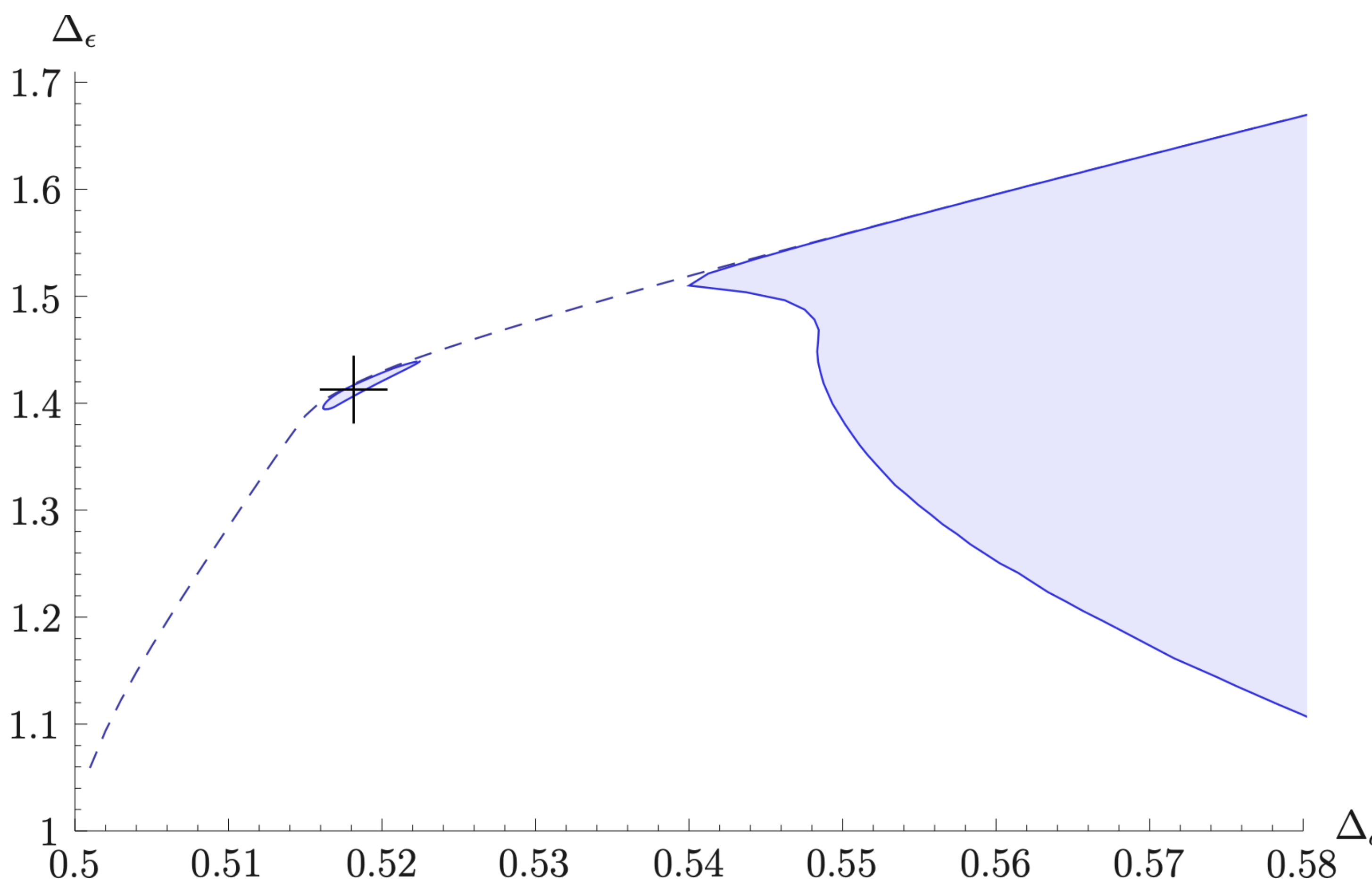
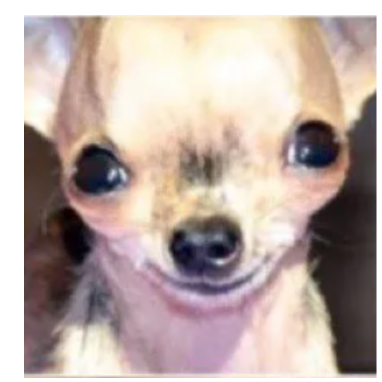


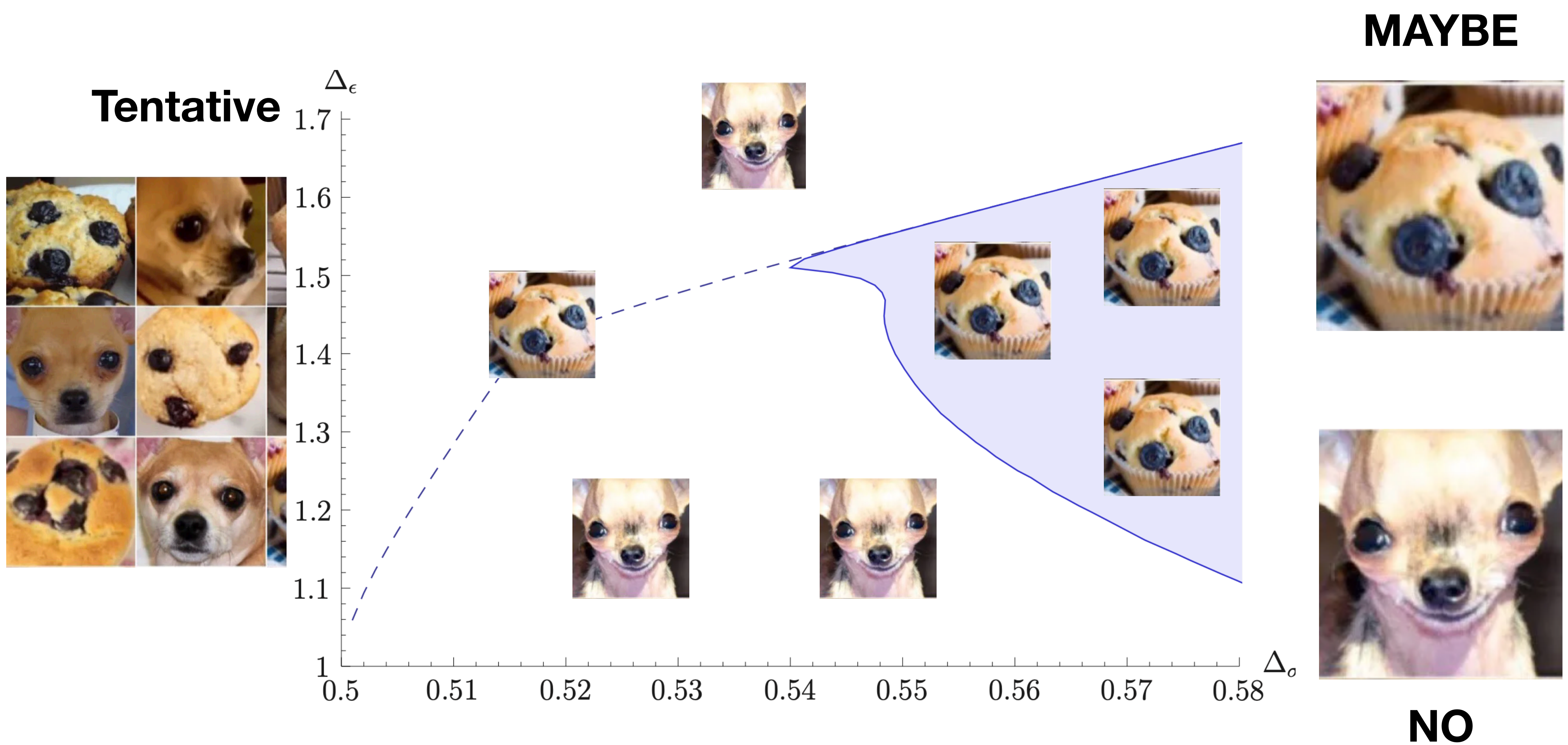
MAYBE



NO

Numerical Bootstrap

The idea is to use the crossing relations as necessary conditions for conformal dimensions and OPE coefficients to belong to a CFT.



Analytical approach

Komargodski Zhiboedov 2013
Fitzpatrick Kaplan Poland Simmons Duffin 2012
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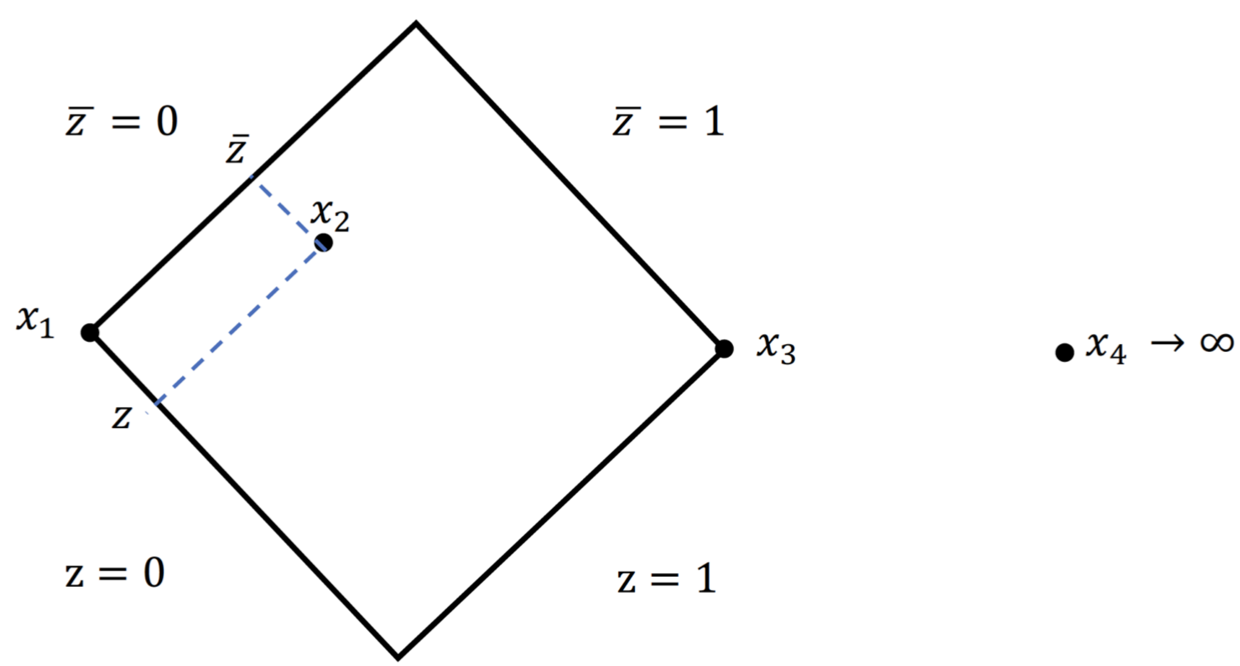

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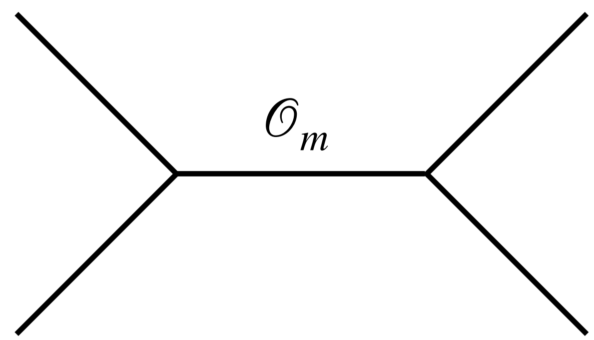
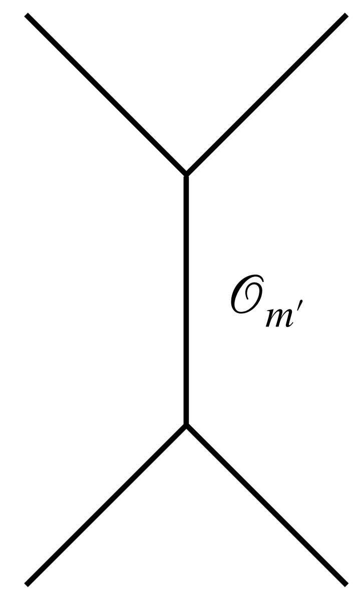
Large spin sector

$\sum_m$ 

$=$ $\sum_{m'}$ 

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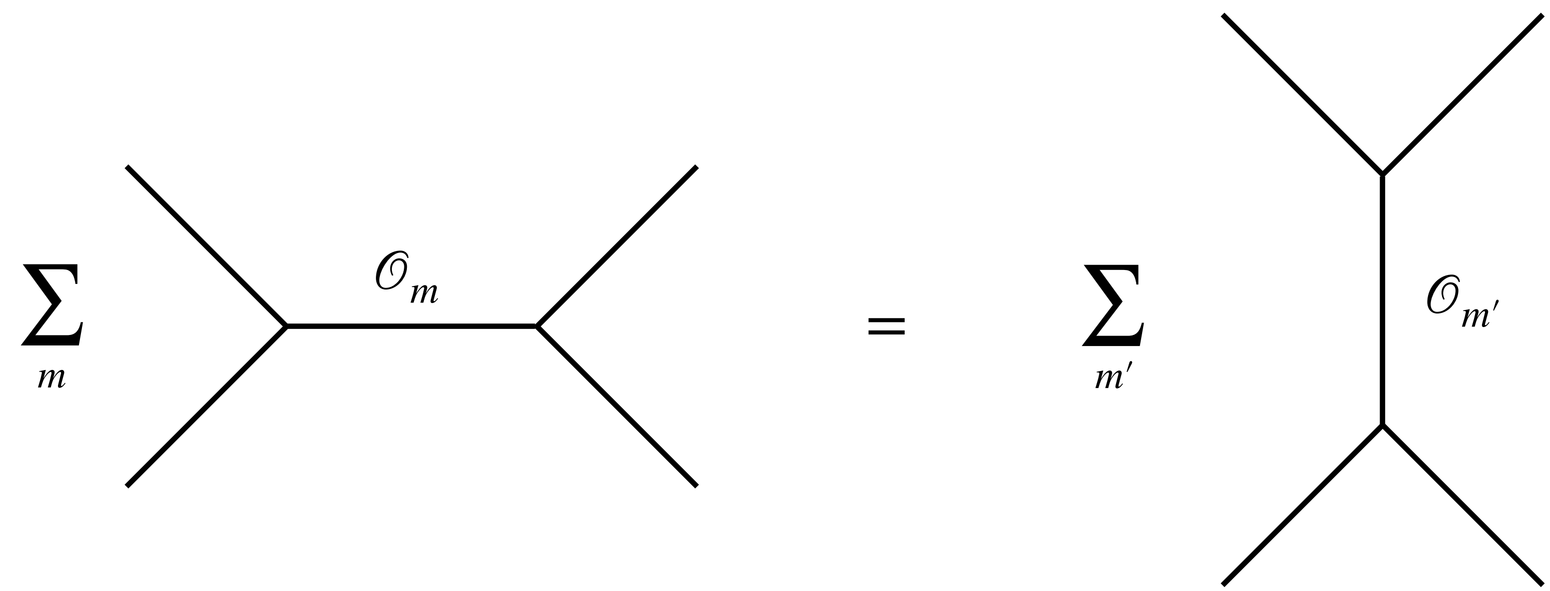

Identity Operator

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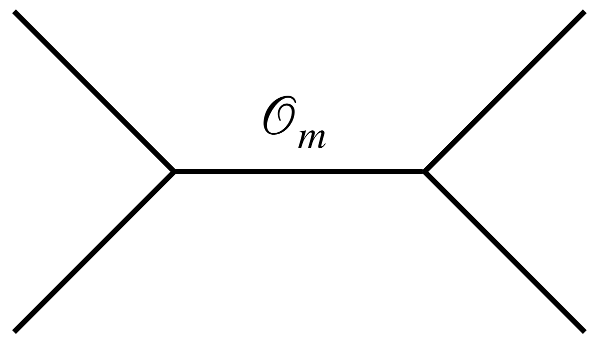

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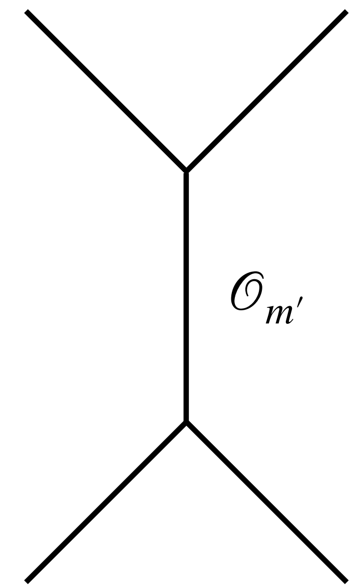

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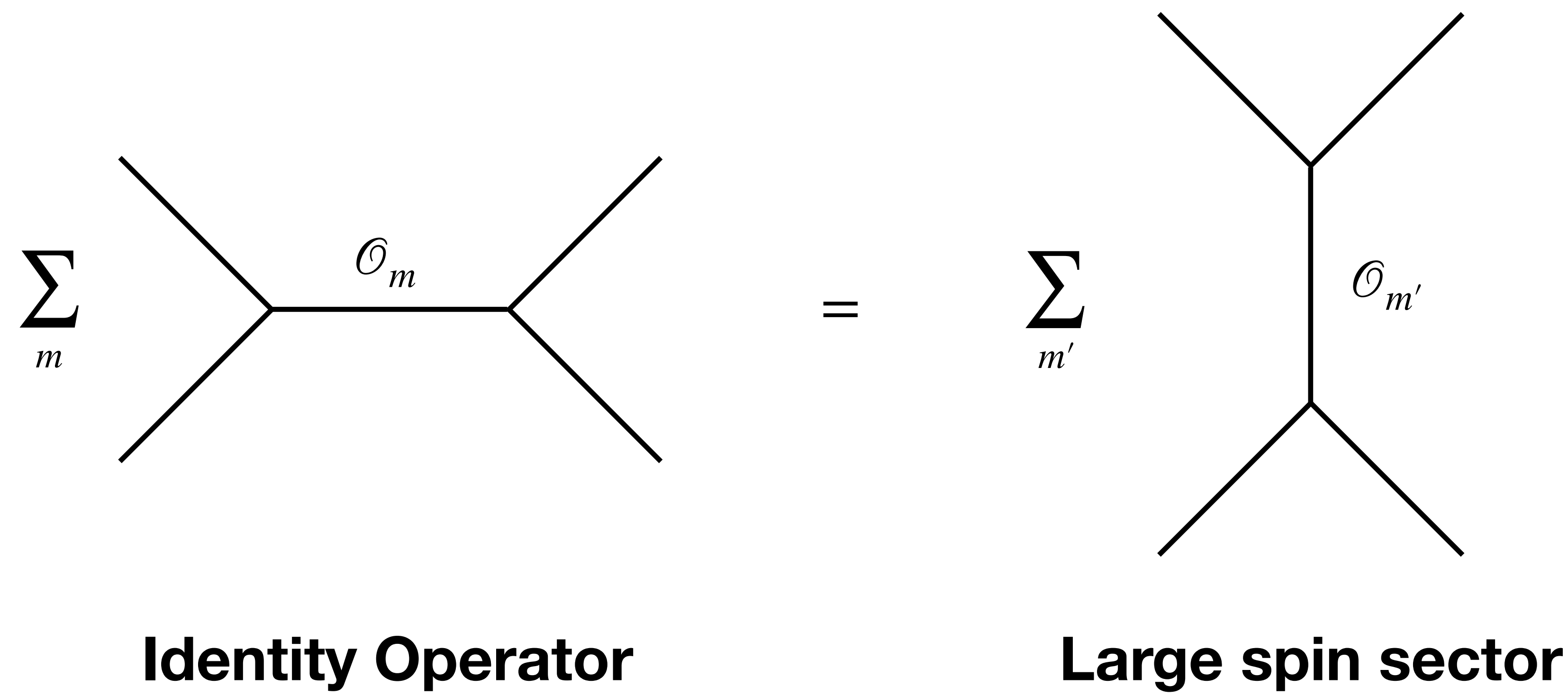

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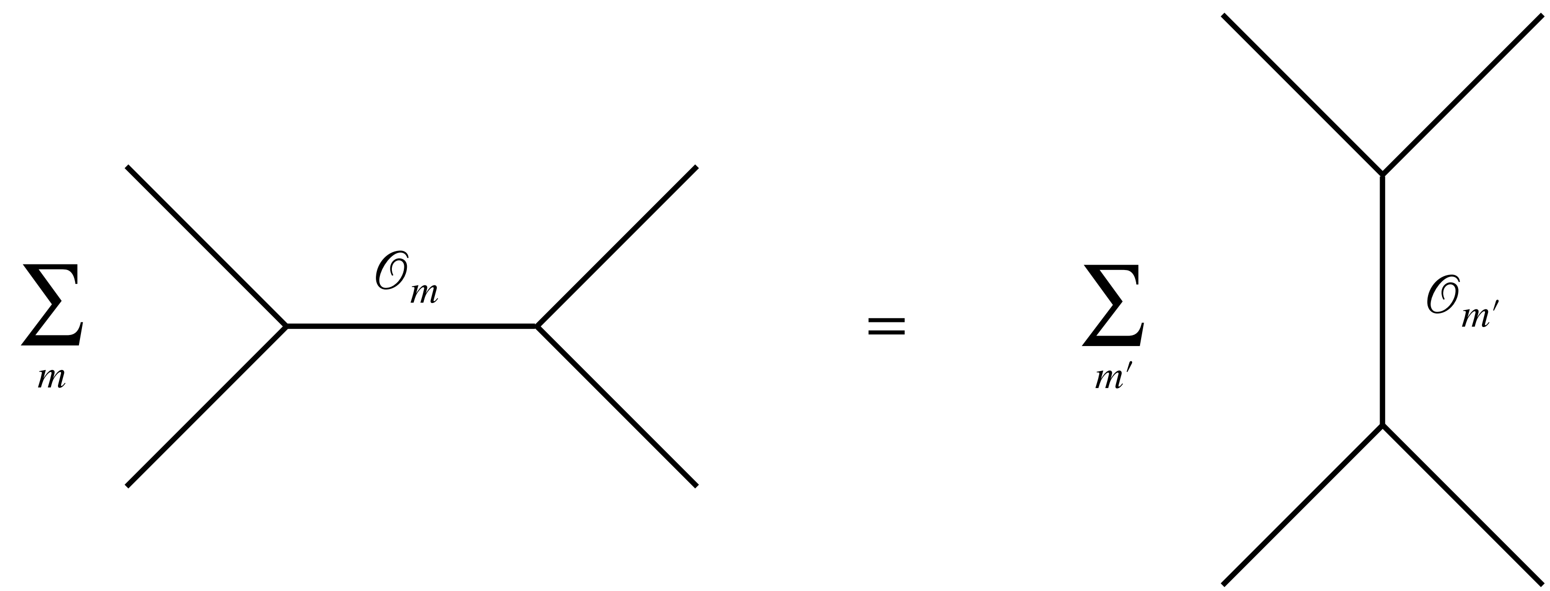

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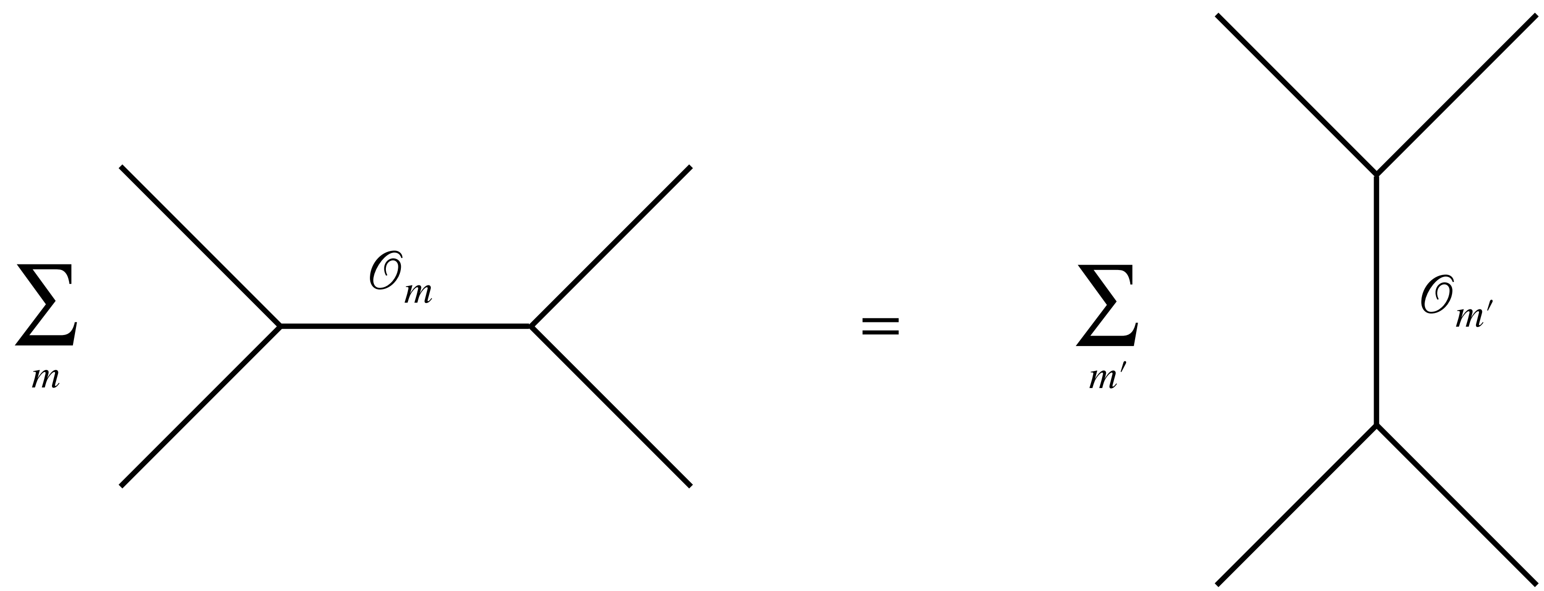

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It is possible to use the Casimir equation to iteratively find all the $1/\ell$ corrections and resum them to extrapolate for finite values of the spin.

Inversion formula

There is another approach to compute these quantities, less intuitive but computationally more powerful.

This approach allows inverting the OPE and prove why it is possible to resum the large spin series up to finite values of the spin.

Caron Huot 2017

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$$\textbf{dDisc}[\mathcal{G}(z,\bar{z})] = \mathcal{G}_{Eucl}(z,\bar{z}) - \frac{1}{2}\mathcal{G}^{\circlearrowright}(z,\bar{z}) - \frac{1}{2}\mathcal{G}^{\circlearrowleft}(z,\bar{z})$$

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It has poles at the dimensions of the exchange operators with residues the square of the three point functions. The function is analytic in the spin for $\ell \geq 2$.

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Applicability

The applicability of these methods is pretty vast, and it mostly efficiently used when the theory has a small parameter (perturbation theory)

number of dimensions	$d = 4 - \epsilon$		
rank of the gauge group	N		
coupling constant	g	or	λ
...			

AdS/CFT correspondence

CFT

AdS

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4 dimensional $\mathcal{N} = 4$
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$$N \sim g_s^{-1}$$

$$\lambda = g_{YM}^2 N = (\alpha')^{-2}$$

AdS/CFT correspondence

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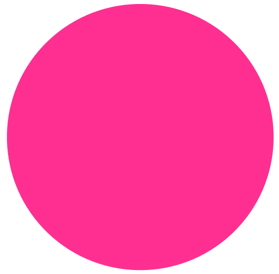
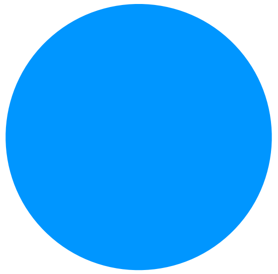
Correlators in CFTs



Amplitudes in AdS



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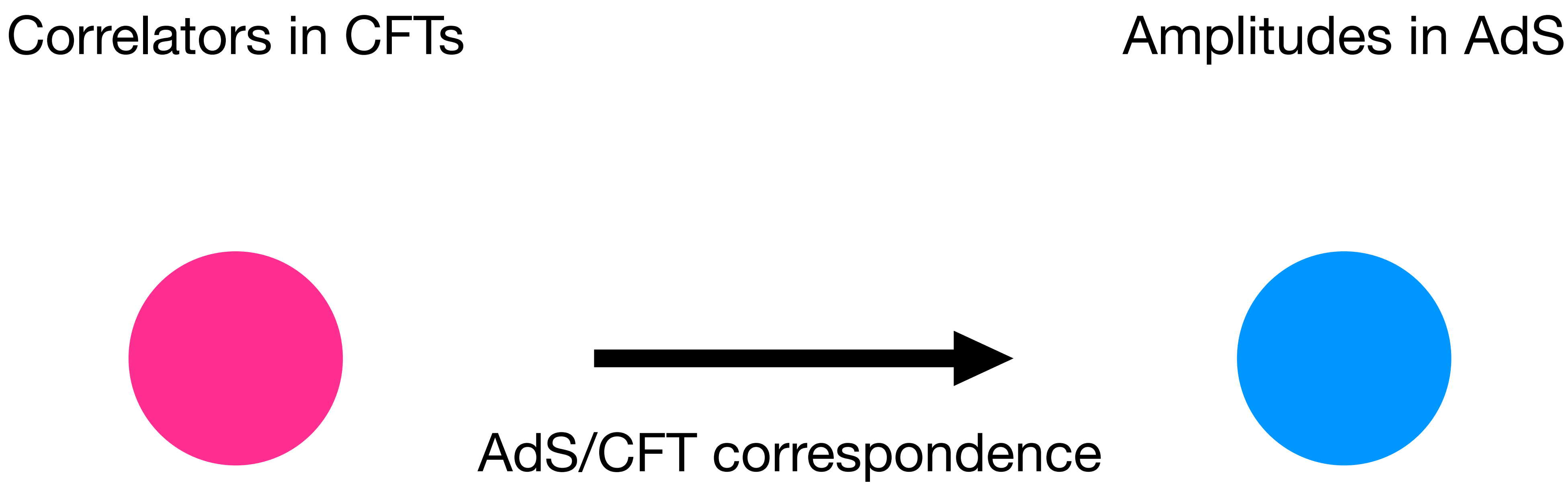

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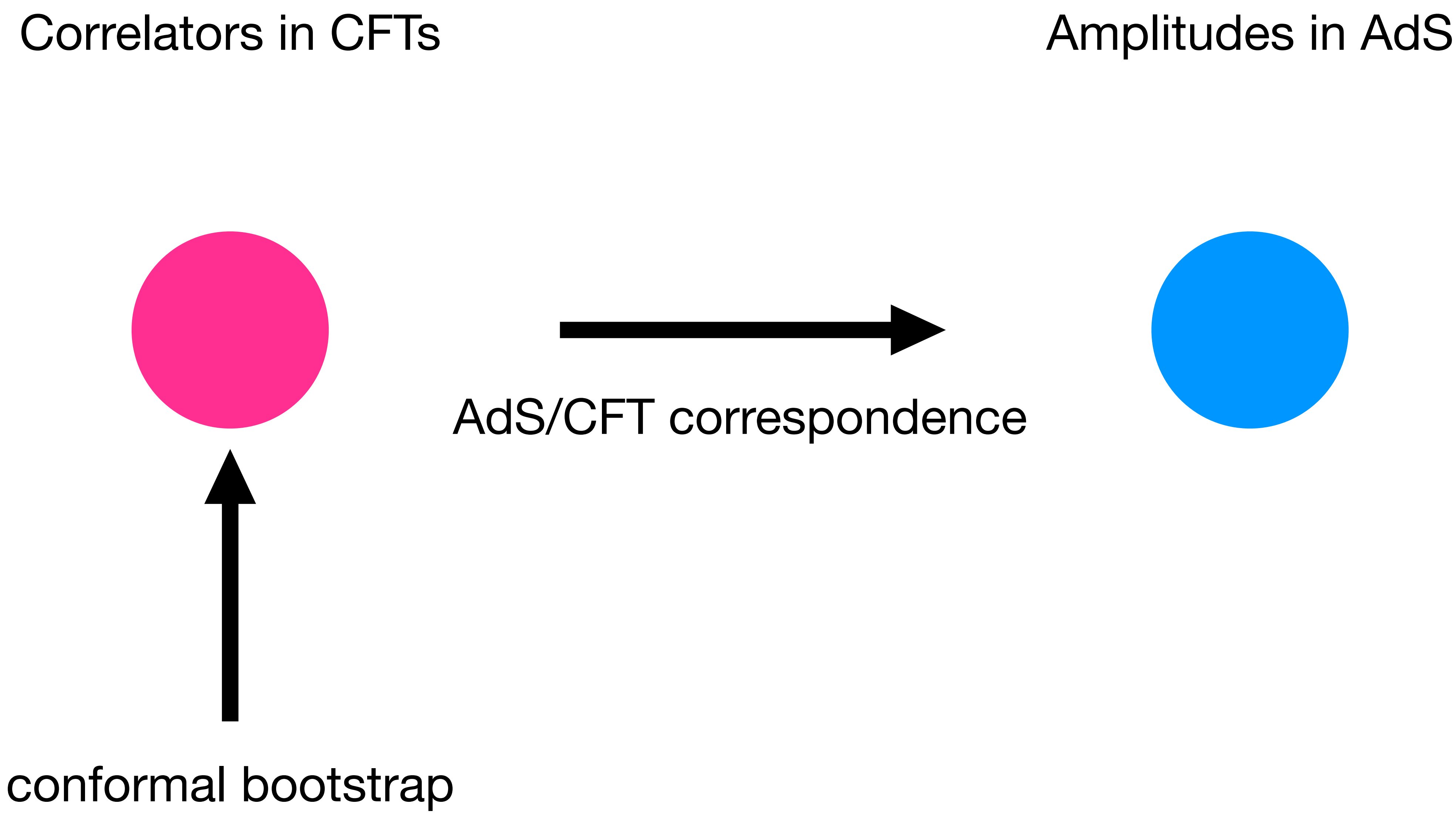

AdS/CFT correspondence

Correlators in CFTs

Amplitudes in AdS

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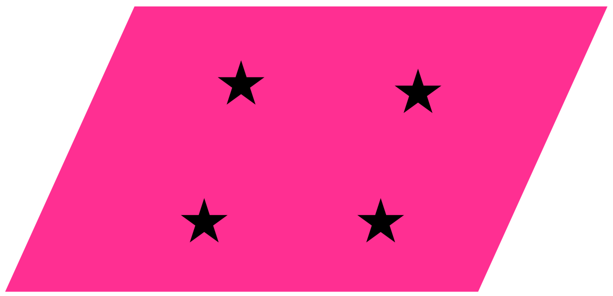

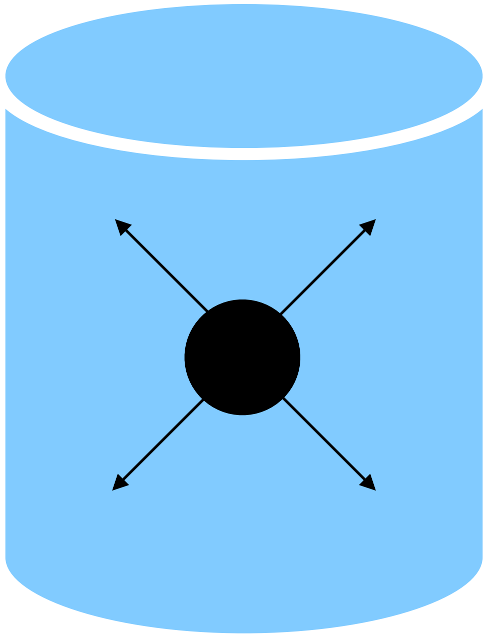

$\mathbb{R}^{d-1,1} = \partial \mathrm{AdS}_{d+1}$

$\mathrm{AdS}_{d+1} \times \mathrm{S}^q$

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ignore the stress tensor

Large N

We expand all the quantities up to order N^{-4}:

$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \dots$$

$$\Delta = \Delta^{(0)} + \frac{1}{N^2}\gamma^{(1)} + \frac{1}{N^4}\gamma^{(2)} + \dots$$

$$c^2_{\Delta,\ell} = k^{(0)}_{\Delta,\ell} + \frac{1}{N^2}k^{(1)}_{\Delta,\ell} + \frac{1}{N^4}k^{(2)}_{\Delta,\ell} + \dots$$

The idea is to compute order by order in N, they CFT data. The main aim is to understand if we can predict the order N^{2k} using the $N^{2(k-1)}$ one.

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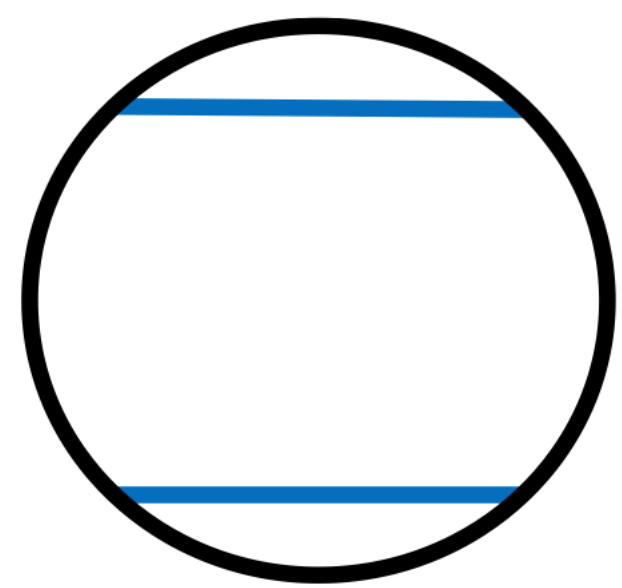

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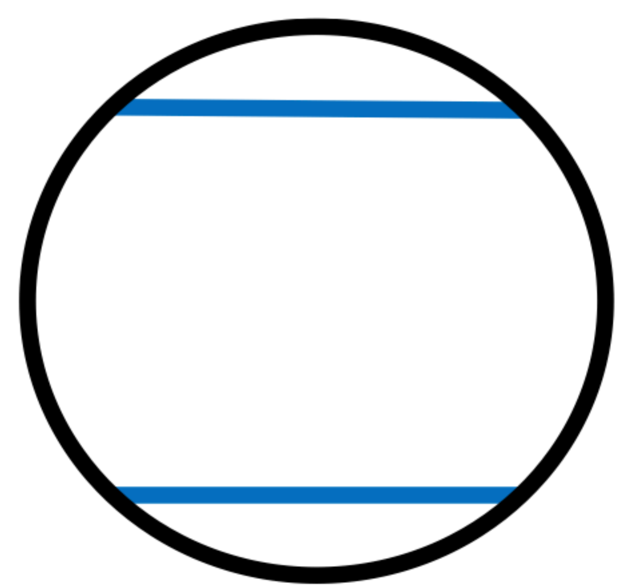

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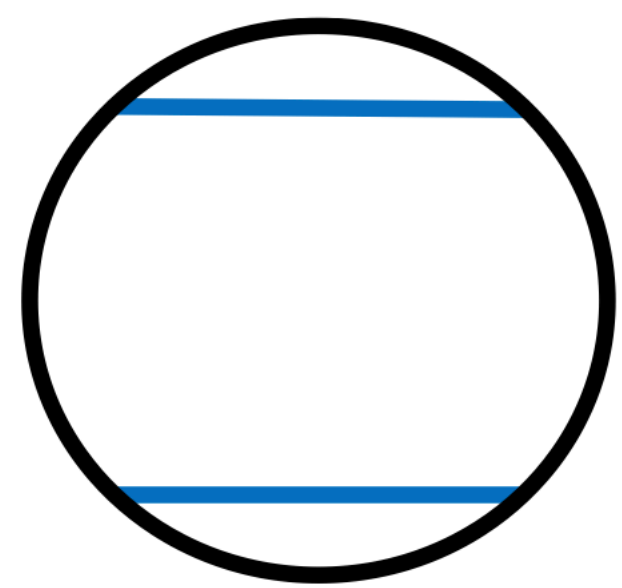

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$\Delta^{(0)} \rightarrow 2\Delta_O + 2n + \ell$

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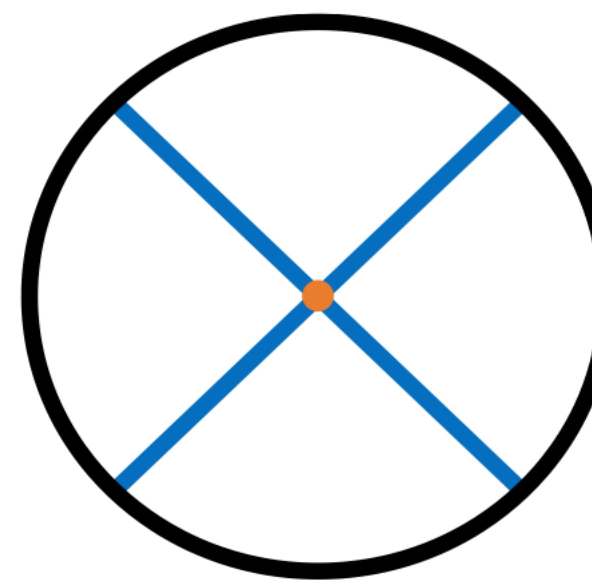

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Aharony Alday AB Perlmutter 2016

Fitzpatrick Poland 2012

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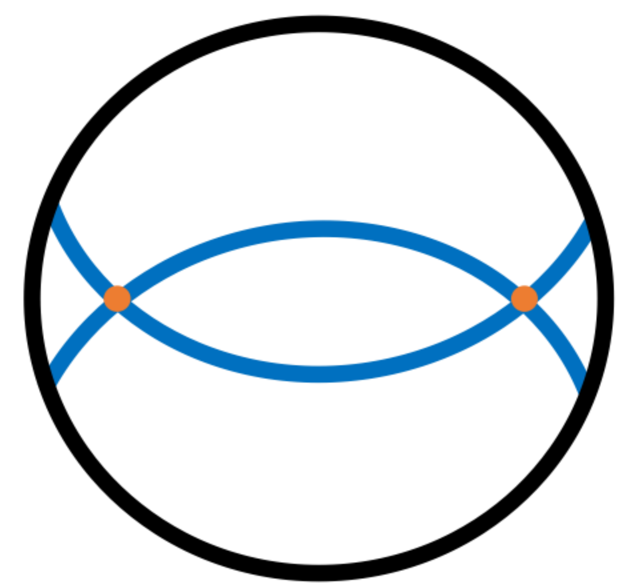

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In pictures

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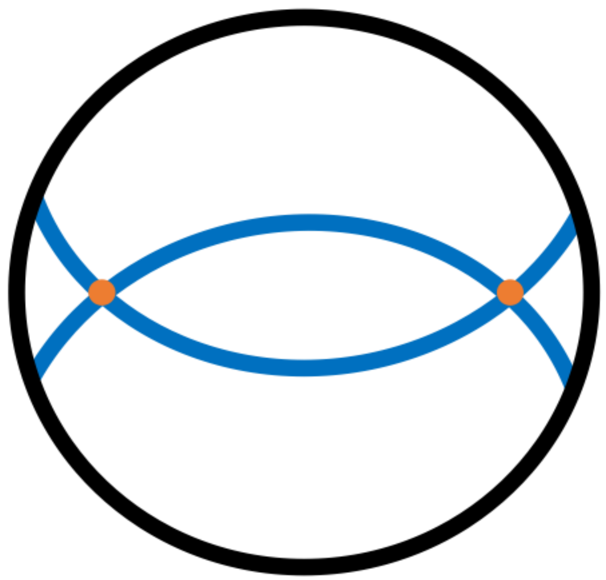

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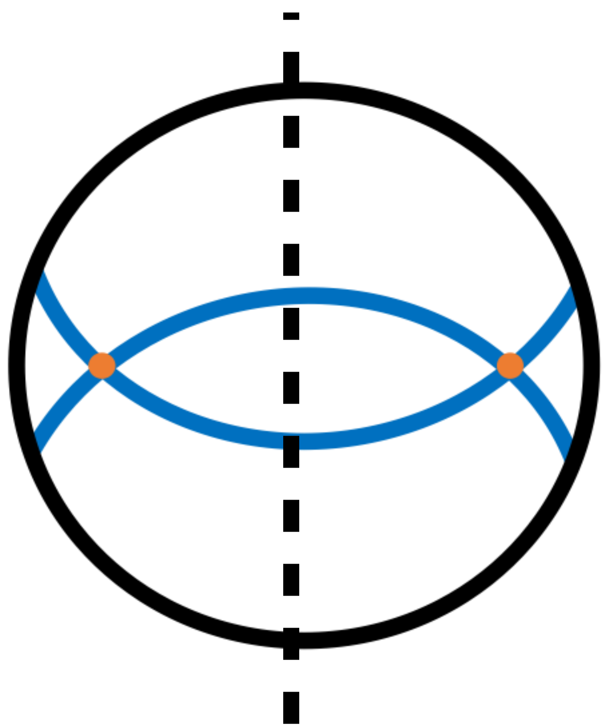

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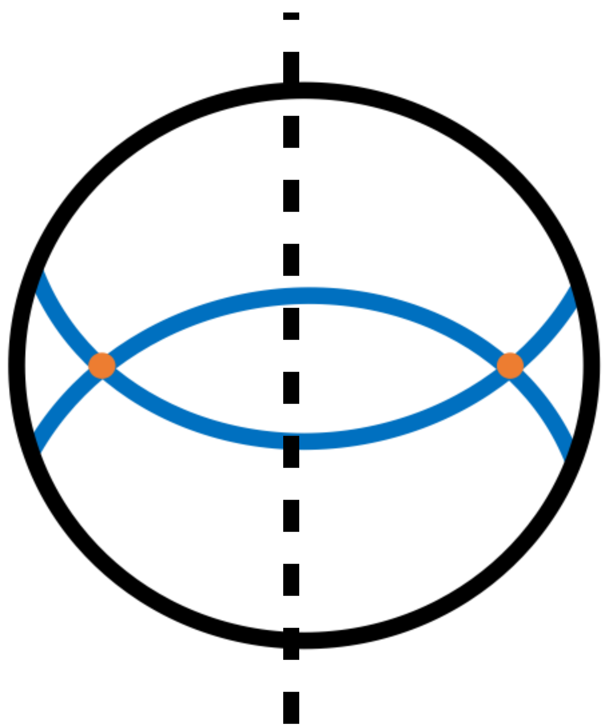

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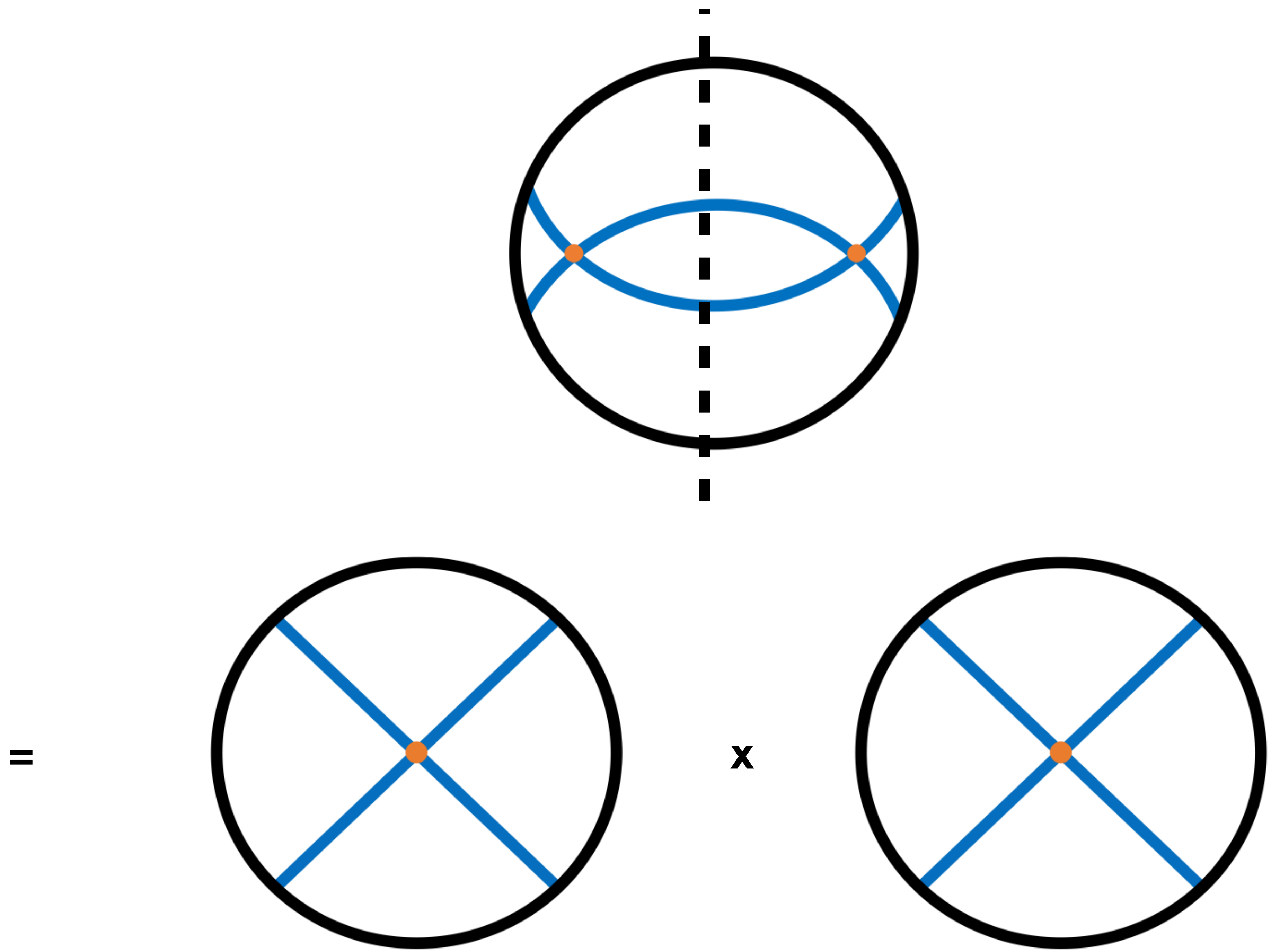
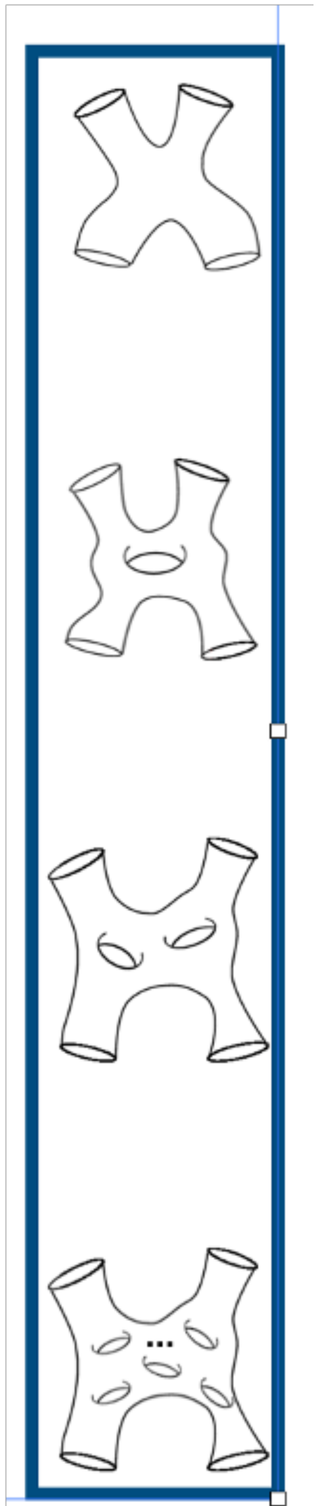


Other applications



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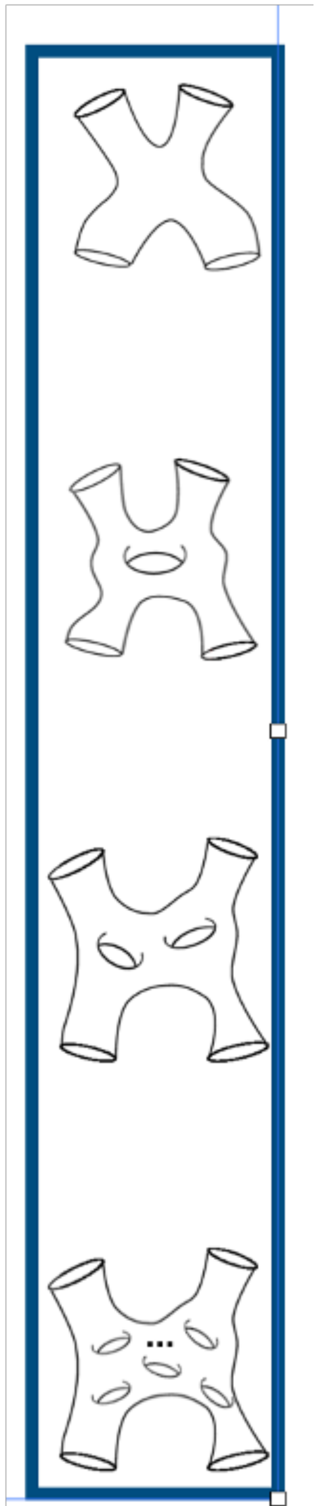

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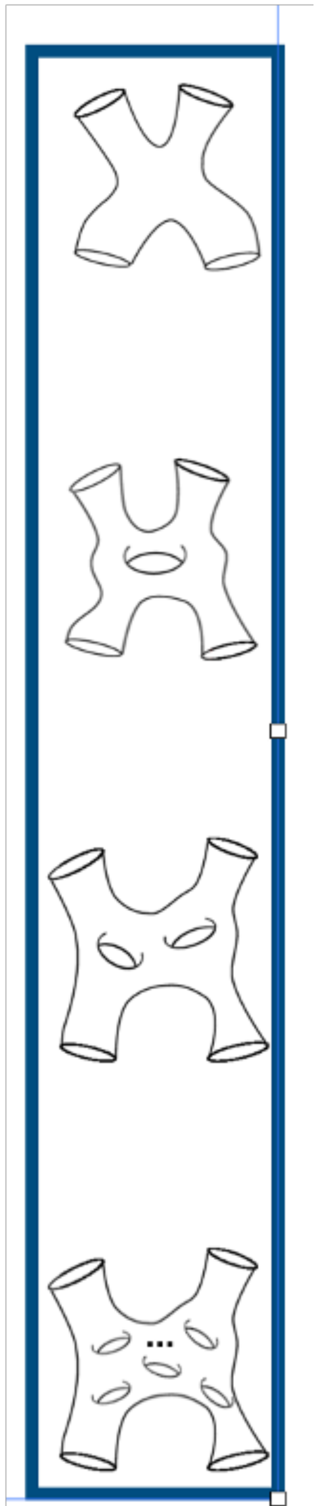

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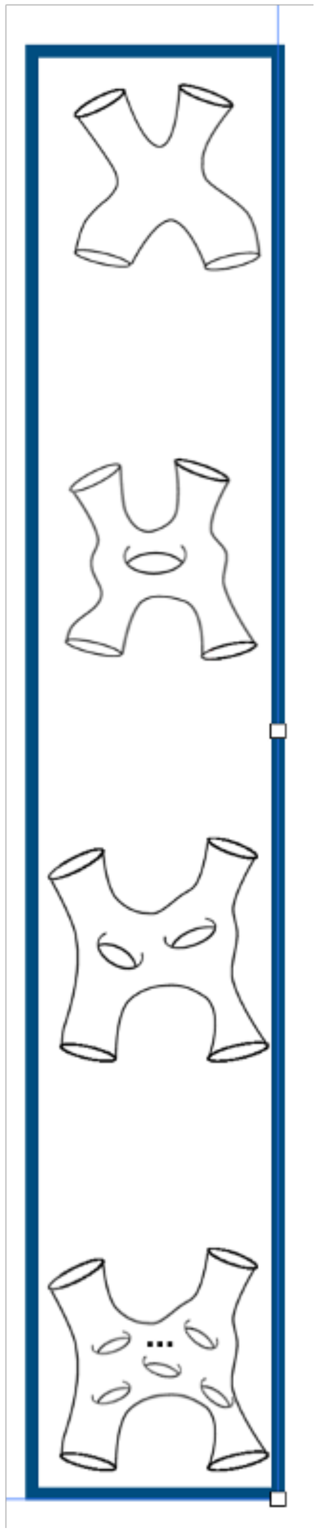

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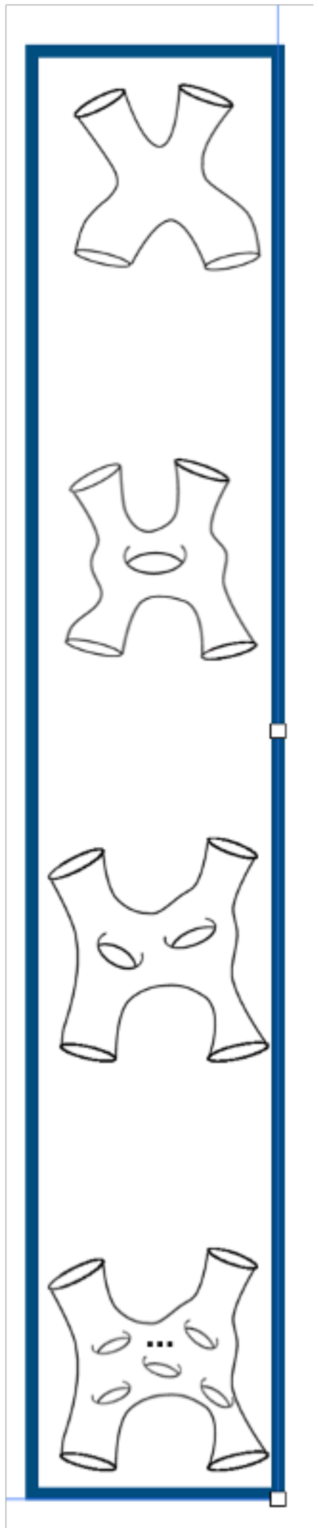

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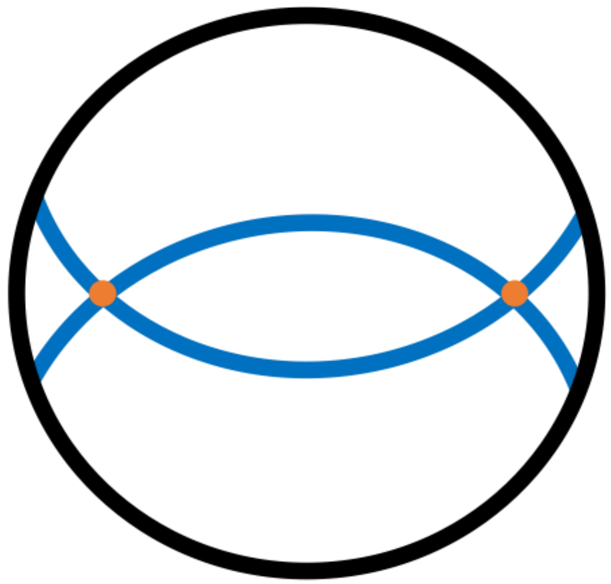
Provides a unique framework to access scattering amplitudes in curved space-times, which are generically very hard/impossible to compute with other methods.

Conclusions

I presented a framework to analytically study CFT, using only the symmetries and the presence of an OPE expansion.

Mapping between singularities and OPE data.

Access amplitudes in AdS using the AdS/CFT correspondence



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We are interested in the behaviour of the correlator at large distances $|x_i - x_j| \gg a$

Scale invariance

Extend the long distance behaviour to any distance: **continuous limit**