PRIMORDIAL FLUCTUATIONS FROM QUANTUM GRAVITY

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ENGAGING SCIENCE.

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UNIVERSITY · CANADA

QUANTUM GRAVITY IN COSMOLOGY

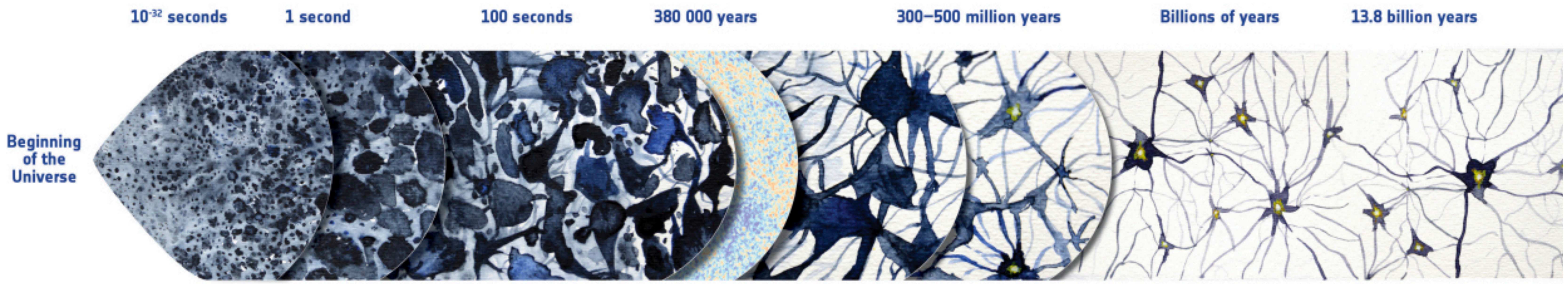


Image credit: ESA

- **Quantum Regime**
- Graph State *as truncation of d.o.f.*
- Lorentzian Transition Amplitude *numerically computable*
- Hartle-Hawking State *as initial vacuum state*
- *Comparison with QFT computations*

QUANTUM COSMOLOGY

[Bianchi, Rovelli,Vidotto'10]

canonical / covariant
quantization

gravity

$$R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

quantum gravity

$$W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma\ \psi_v)(\mathbb{1})$$

symmetry
reduction

cosmology

$$ds^2 = dt^2 - a^2(t)\, d^3\vec{x}$$

+ perturbations

quantum
cosmology

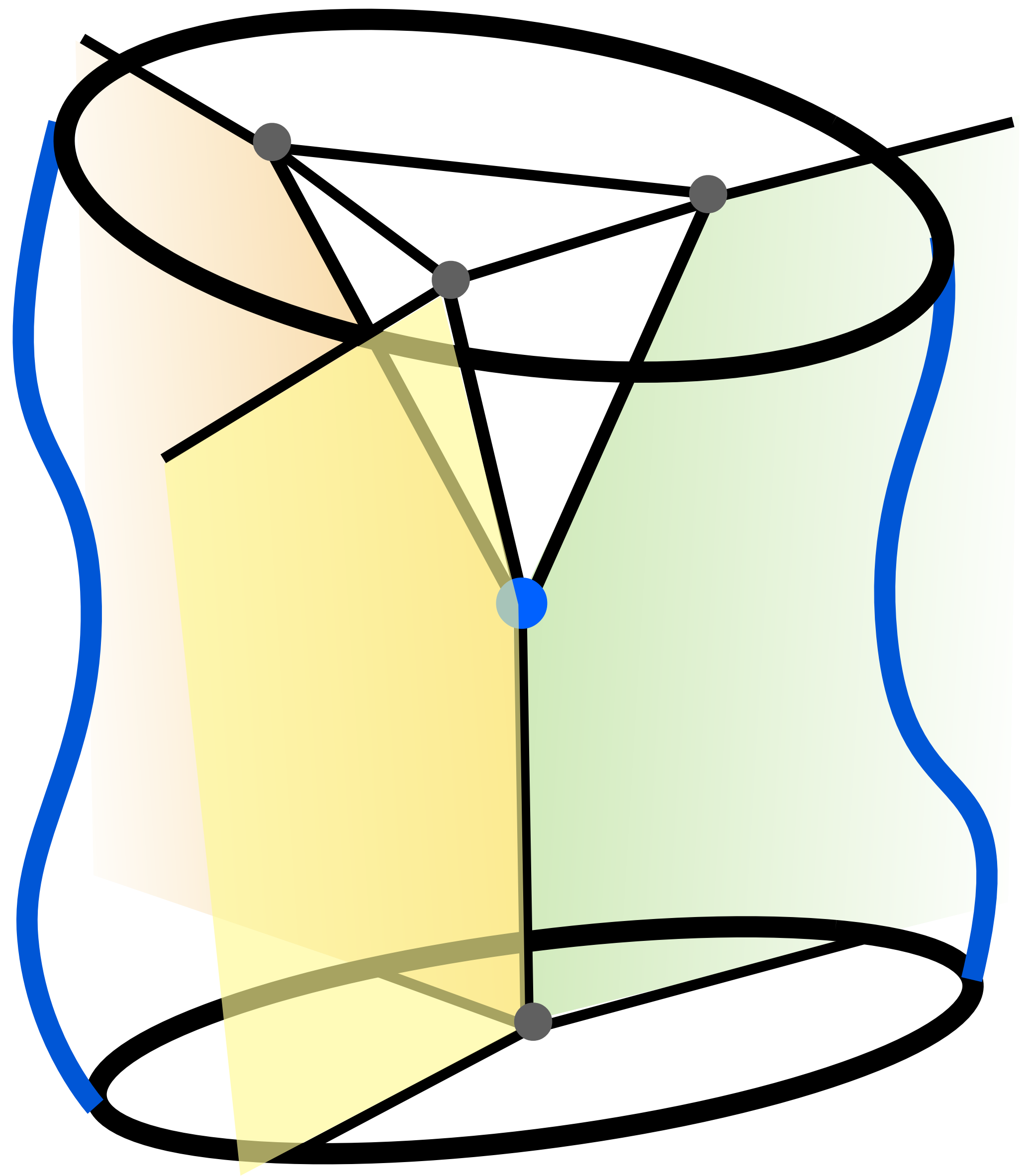
SPINFOAM AMPLITUDES

[Engle-Pereira-Livine-Rovelli, Freidel-Krasnov '08]

Probability amplitude $P(\psi) = |\langle W|\psi\rangle|^2$

for a state ψ associated to the boundary of a 4d region

$$W(q'_{ij}, q_{ij}) \sim \int_{\partial g = q', q} Dq\; e^{i\,S}$$



- Superposition $\langle W|\psi\rangle = \sum_\sigma W(\sigma)$
- Local vertex expansion $W(\sigma) \sim \prod_v W_v.$
- Lorentz covariance $W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma\; \psi_v)(\mathbb{1})$
- UV and IR finite (with Λ)
- Classical limit: discretized GR (with Λ) Barrett et al. '09

www.cpt.univ-mrs.fr/~rovelli/IntroductionLQG.pdf

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$$W(q) \approx \int_{\partial g=q} Dq\ e^{iS[q]}$$

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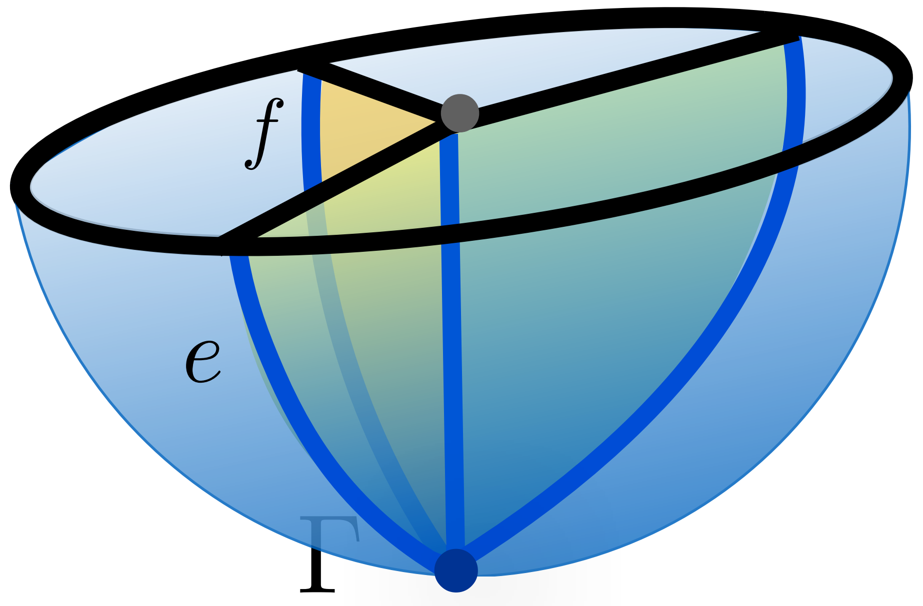
Spinfoam Hartle-Hawking state



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CARLO ROVELLI AND FRANCESCA VIDOTTO

COVARIANT LOOP QUANTUM GRAVITY

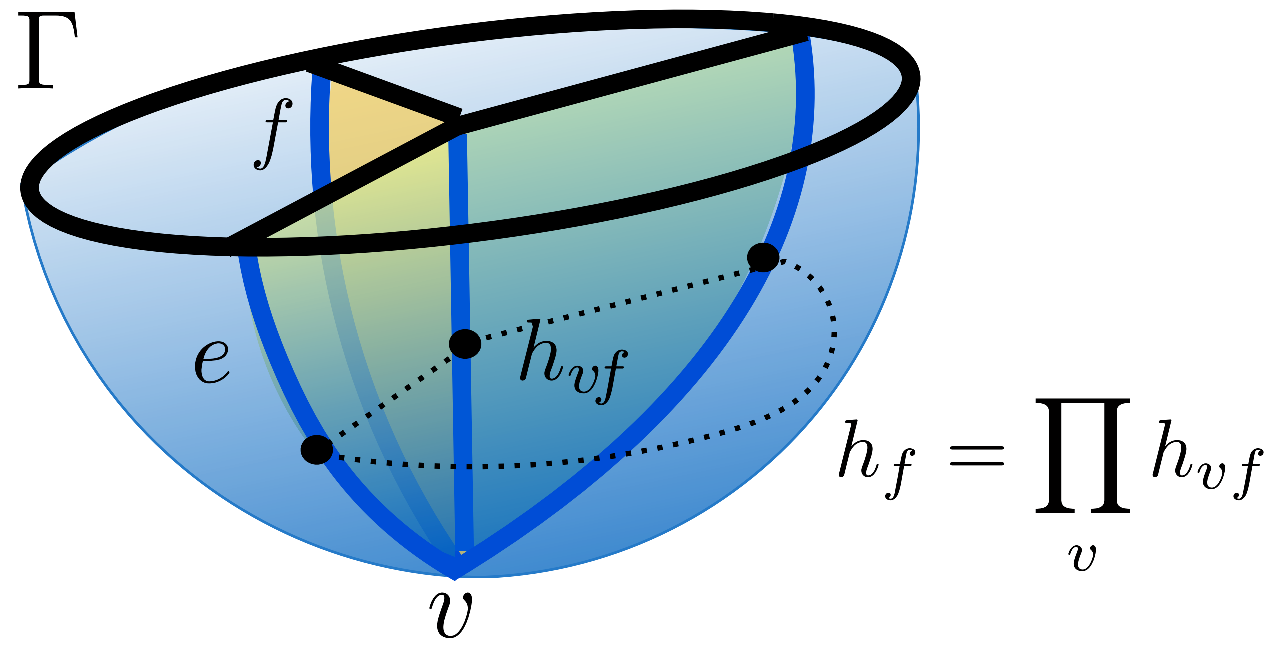
AN ELEMENTARY INTRODUCTION TO QUANTUM GRAVITY AND SPINFOAM THEORY

SPINFOAM HARTLE-HAWKING STATES

[Bianchi, Rovelli,Vidotto'10]

- Hartle-Hawking states:

$$\psi_H(q) = \int_{\partial g = q} Dg\, e^{iS[g]}$$



- Spinfoam HH states:

$$W_{\mathcal{C}}(h_l) = \int_{SU(2)} dh_{vf}\, \prod_f \delta(h_f)\, \prod_v A(h_{vf})$$

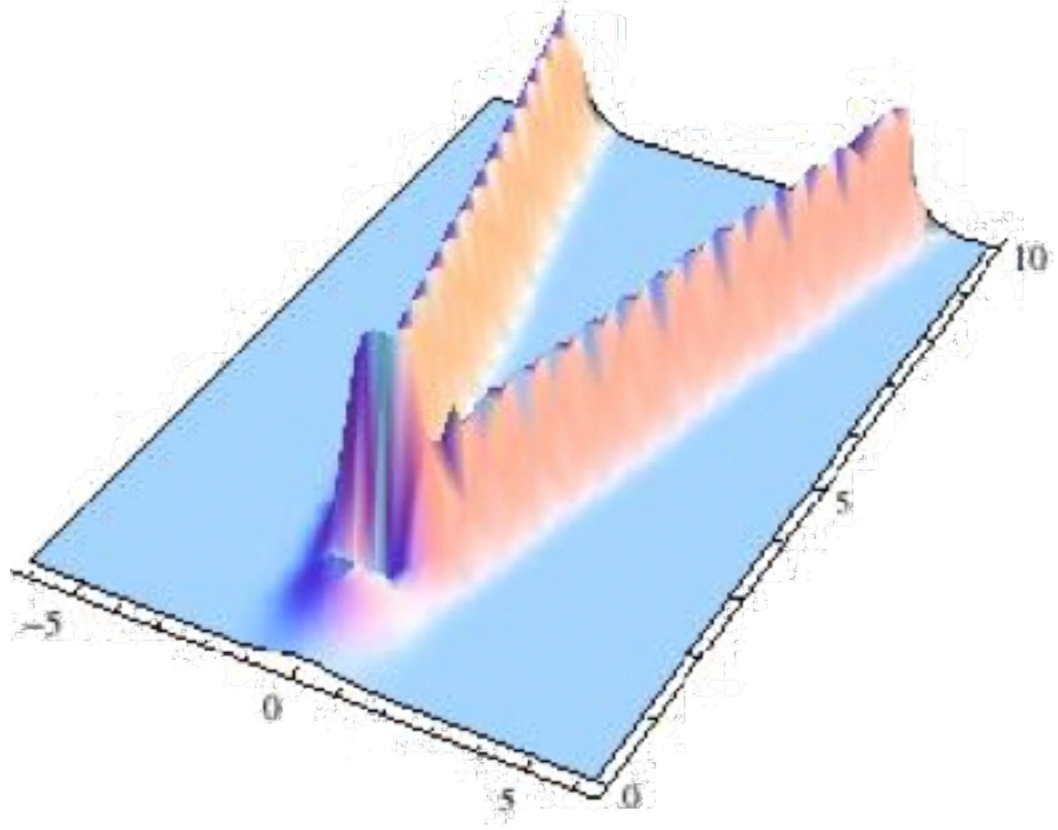
SEMICLASSICAL REGIME

[Bianchi, Krajewski, Rovelli,Vidotto'11]

- LQG coherent states
  peaked on a homogenous and isotropic geometry

- Spinfoam amplitude with an effective Λ:

$$Z_{\mathcal{C}} = \sum_{j_f, \mathtt{v}_e} \prod_f (2j+1) \prod_e e^{i\lambda \mathtt{v}_e} \prod_v A_v(j_f, \mathtt{v}_e)$$



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FEW-NODE THEORY: REGGE CALCULUS

[Collins & Williams '72]

- IDEA — Evolve one or few tetrahedra, triangulating a 3-sphere.
- PROBLEM — Compare the evolution for 5, 16 and 600 tetrahedra.
- RESULT — The qualitative behaviour is the same!

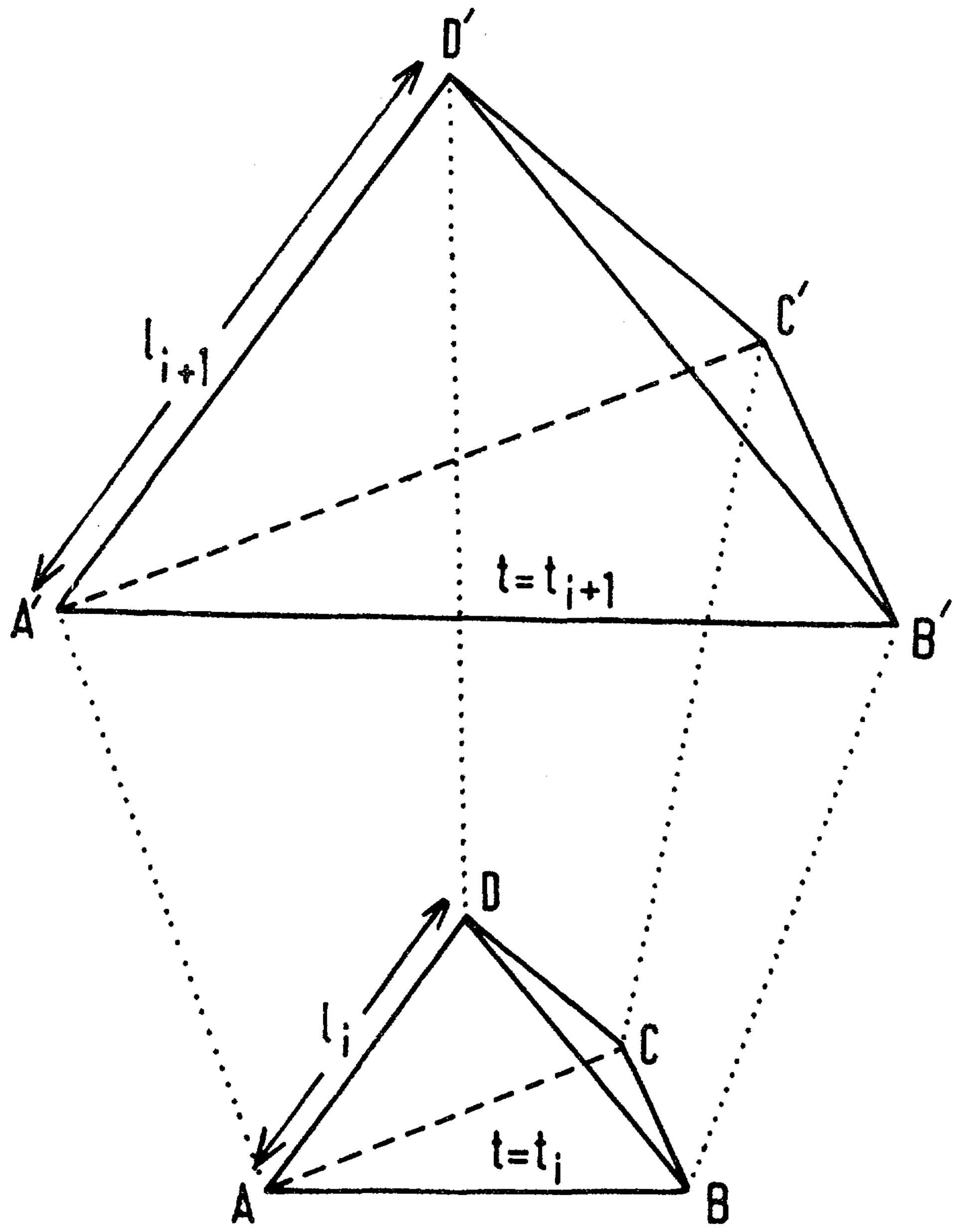


FIG. 1. Diagram illustrating a 4-dimensional block.

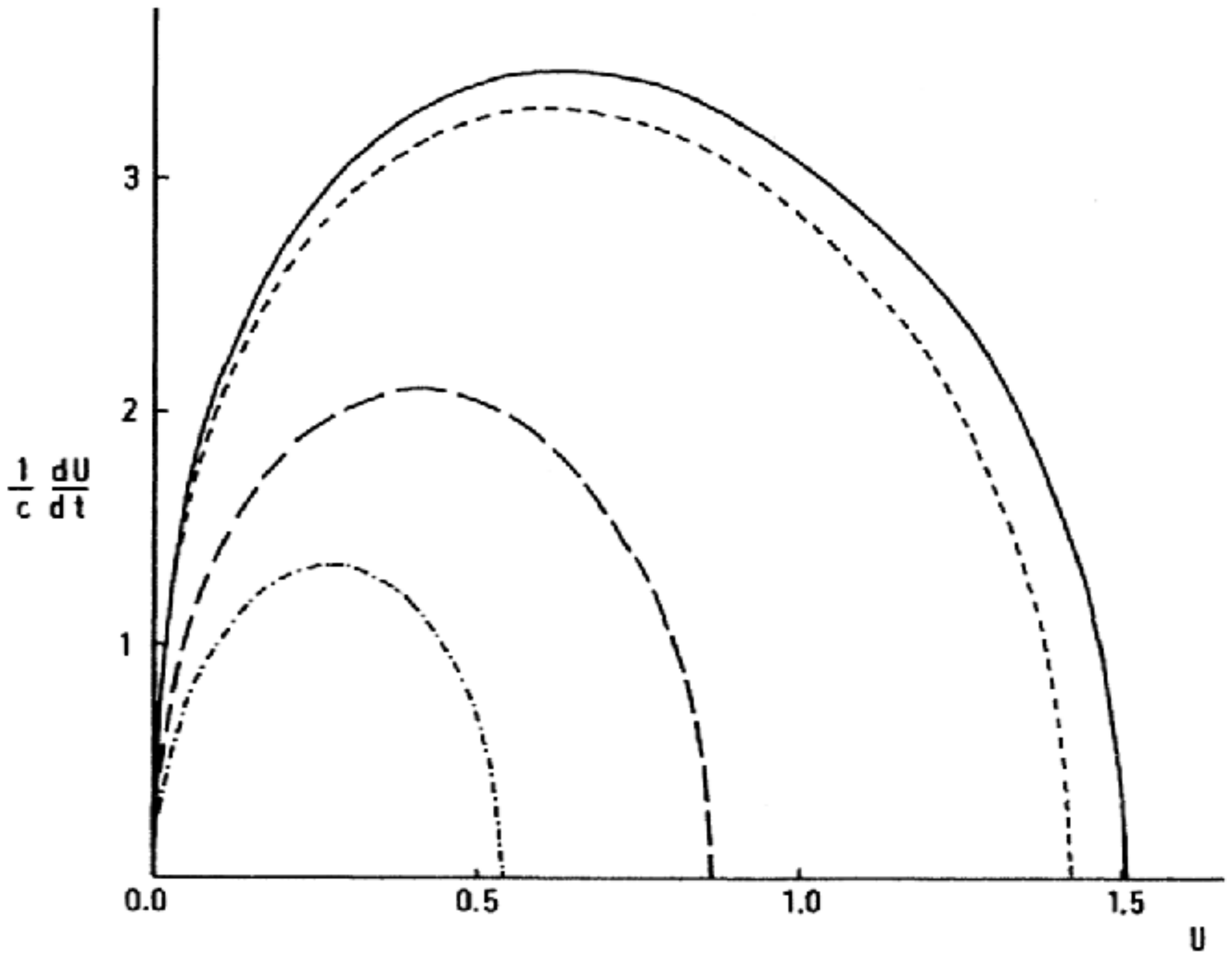
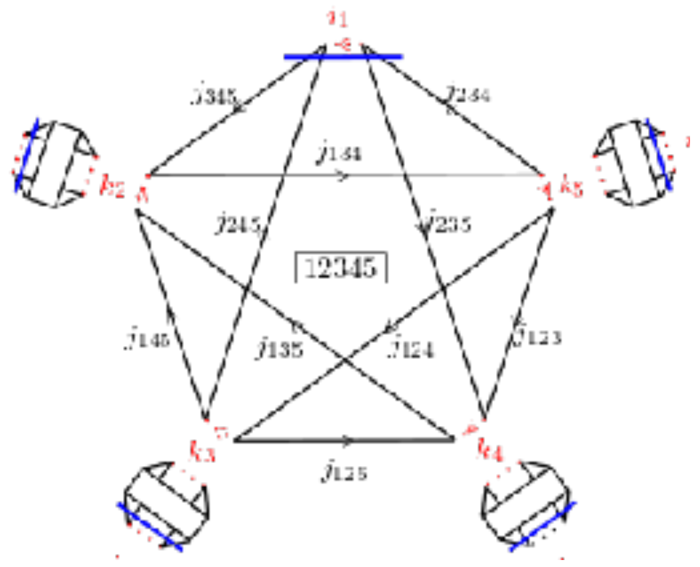


FIG. 2. Rate of change of the volume of the universe plotted against the volume for $MG/c^2=1$; analytic solution ———, 600-tetrahedron model - - - - - - - -, 16-tetrahedron model — — — —, 5-tetrahedron model —·—·—· .

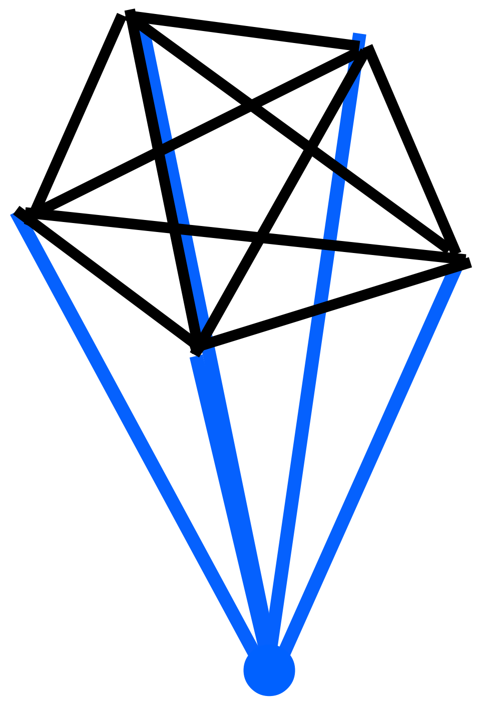
5-CELL PENTACHORDS

Frisoni, Gozzini, Vidotto 2207.02881

- Simplest regular 4-polytope



- Regular triangulation of S_3



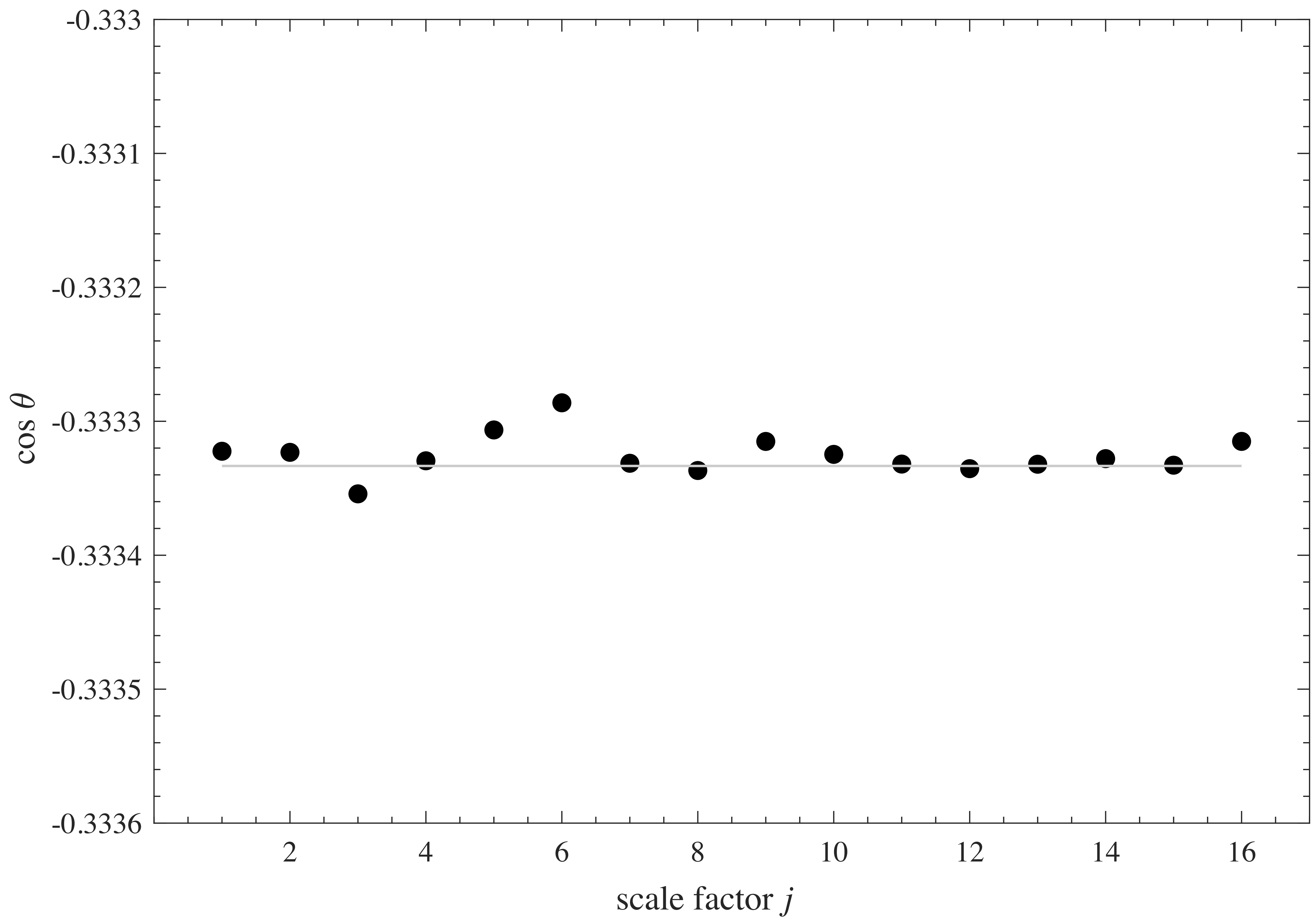
OBSERVABLES

- Area
- Volume $\qquad \langle O \rangle = \langle \psi_o | O | \psi_o \rangle$
- Dihedral Angles $\Rightarrow$ Curvature $\qquad$ *spread*
- **Correlations** $\qquad C(O_1, O_2) = \dfrac{\langle \psi_o | O_1 O_2 | \psi_o \rangle - \langle O_1 \rangle \langle O_2 \rangle}{(\Delta O_1)\ (\Delta O_2)} \qquad \Delta O = \sqrt{\langle \psi_o | O^2 | \psi_o \rangle - \langle O \rangle^2}$
- **Entanglement**

RESULTS

Gozzini, Vidotto 1906.02211

1. 3-sphere as emerging geometry
2. large fluctuations
3. large correlations



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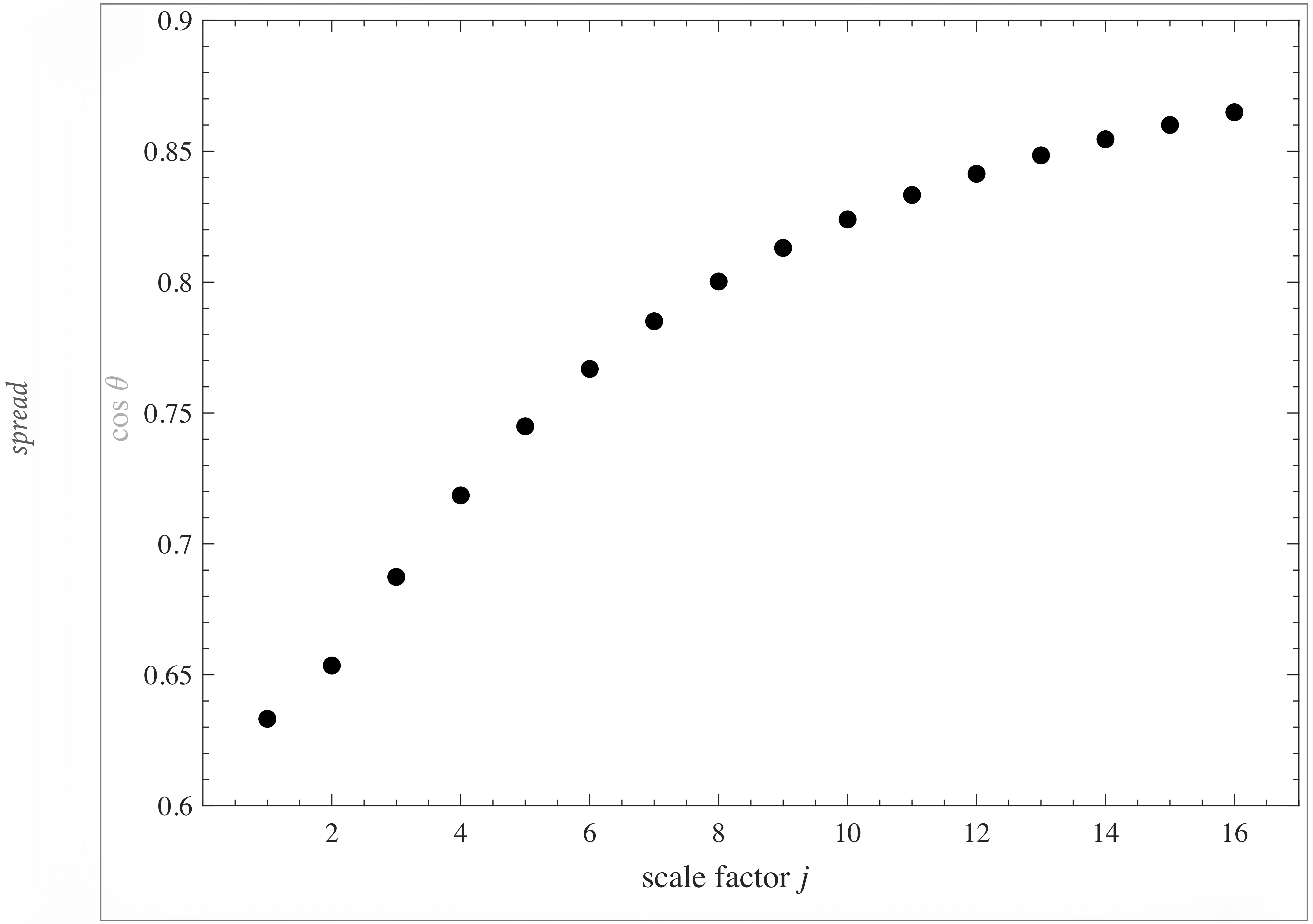

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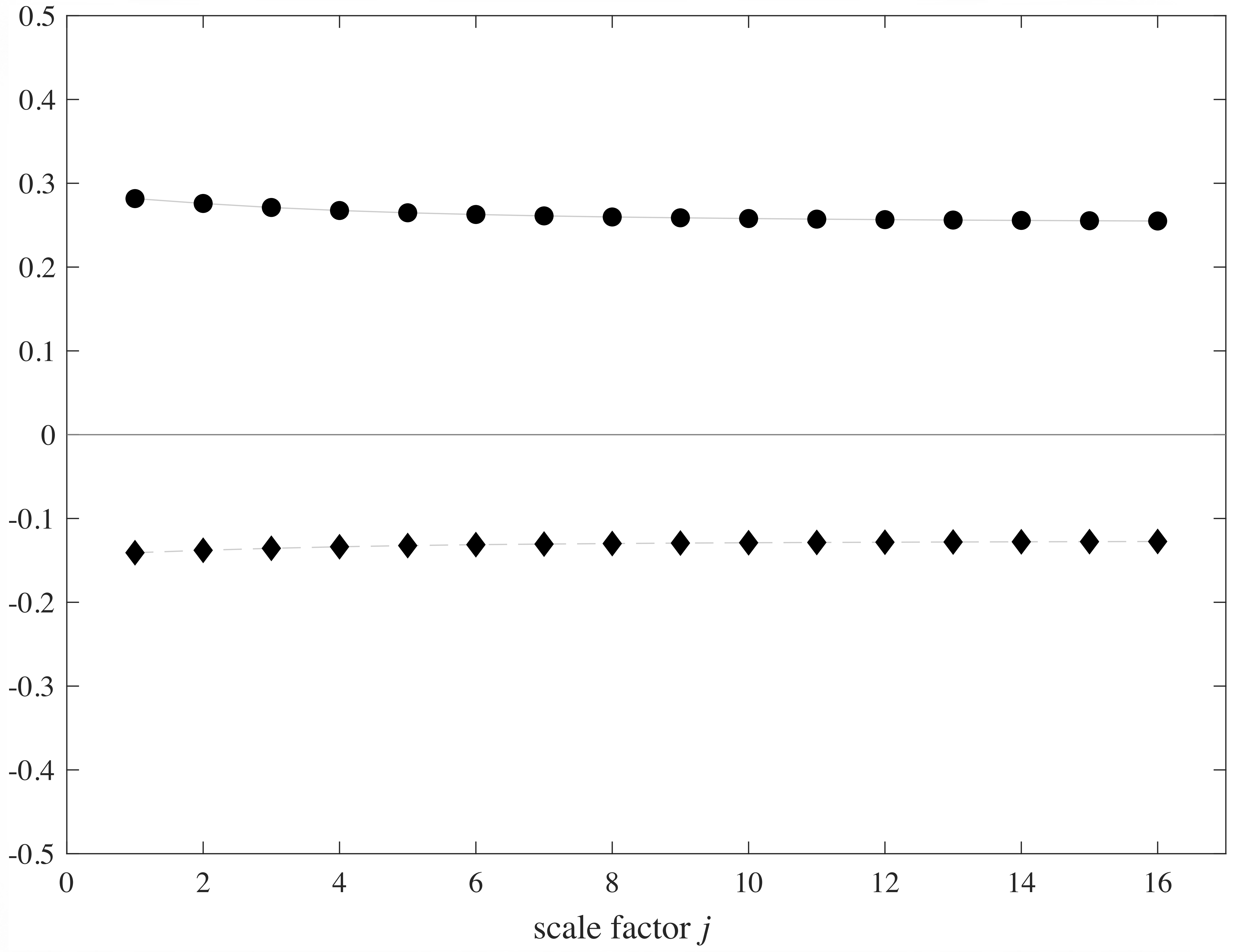

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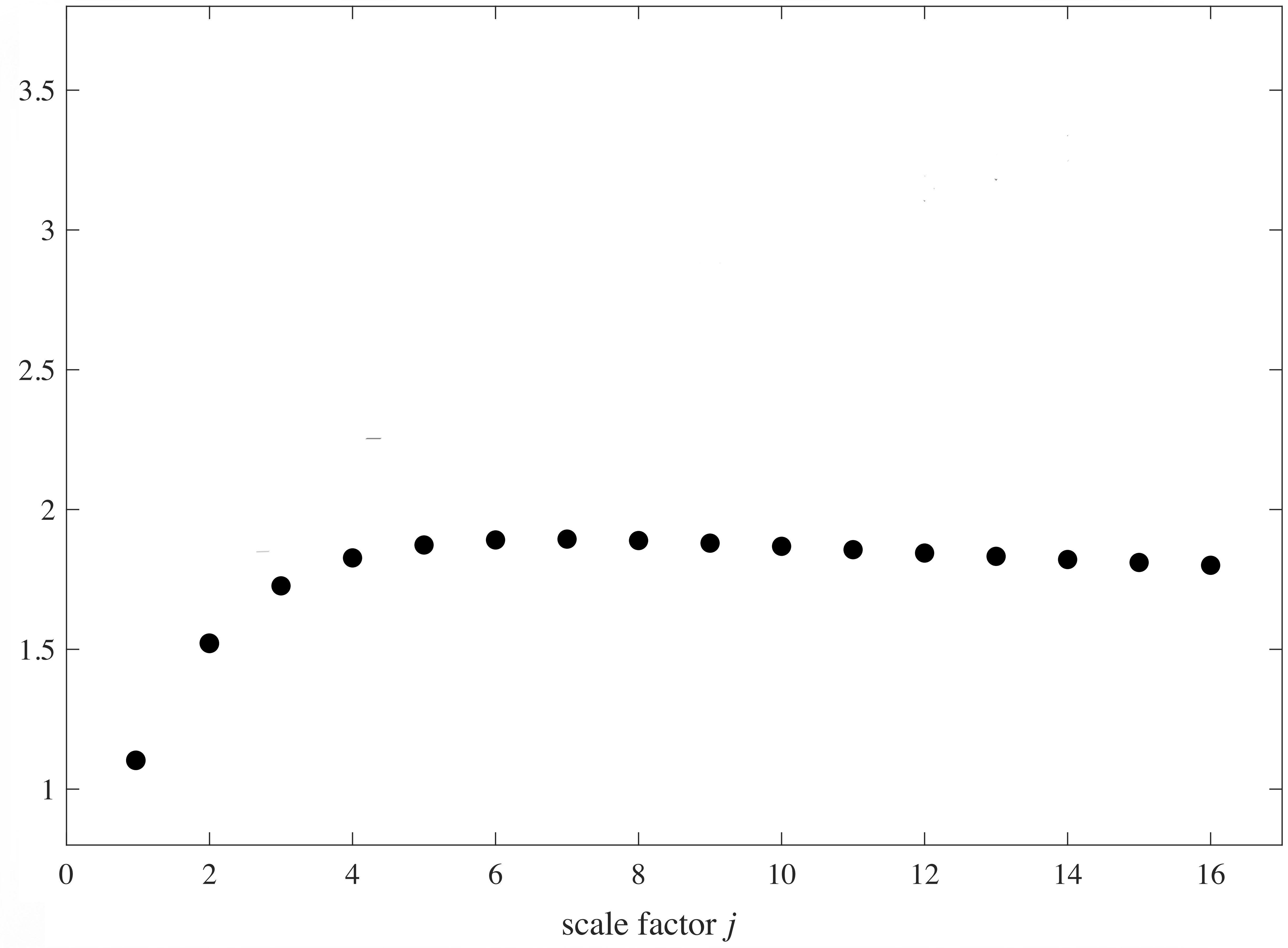
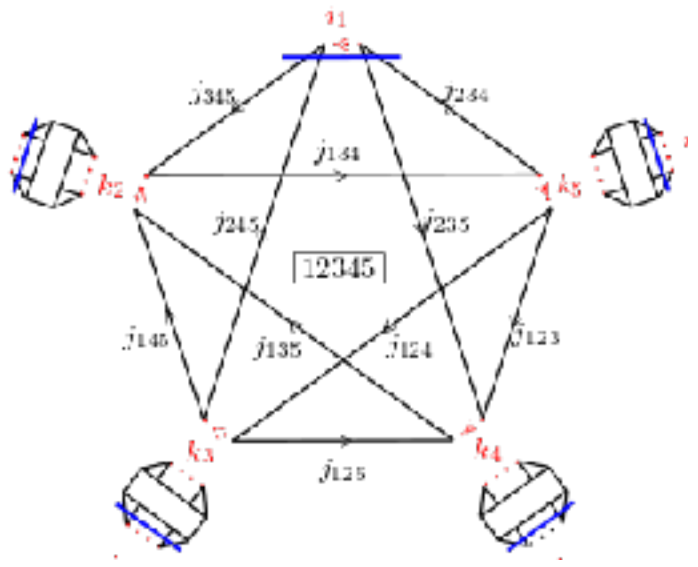
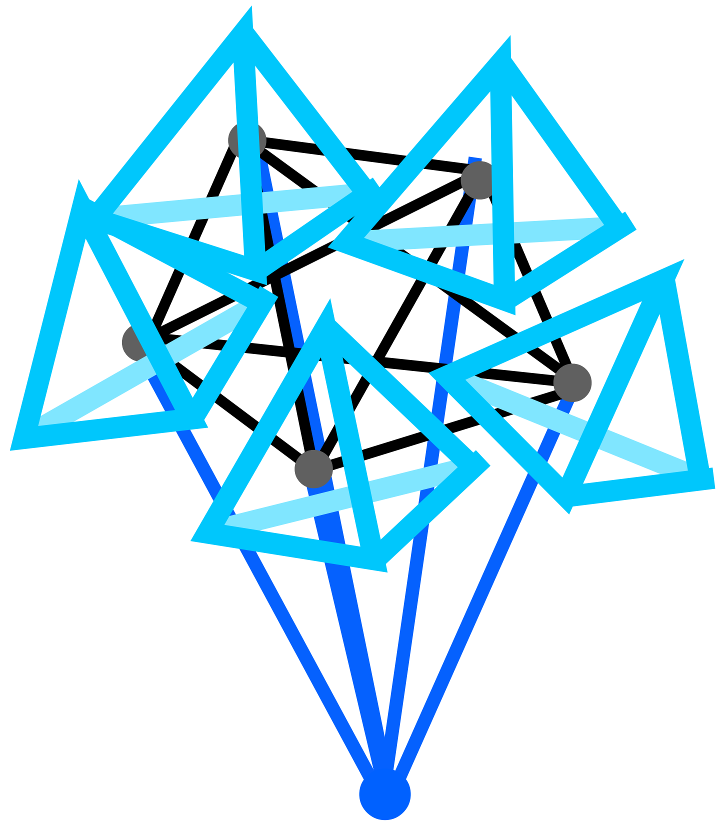


GRAPH REFINEMENT

Frisoni, Gozzini, Vidotto 2207.02881





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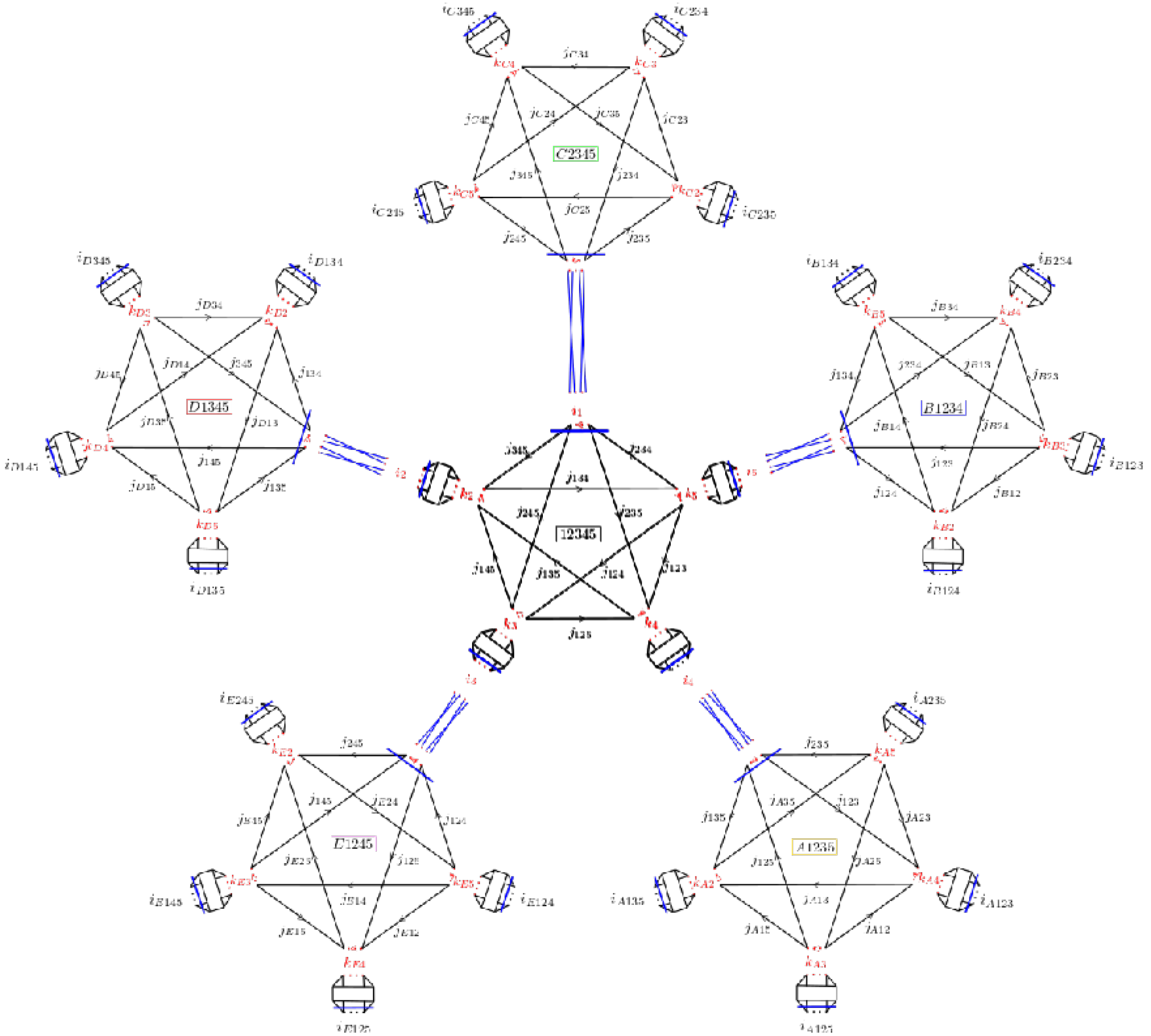
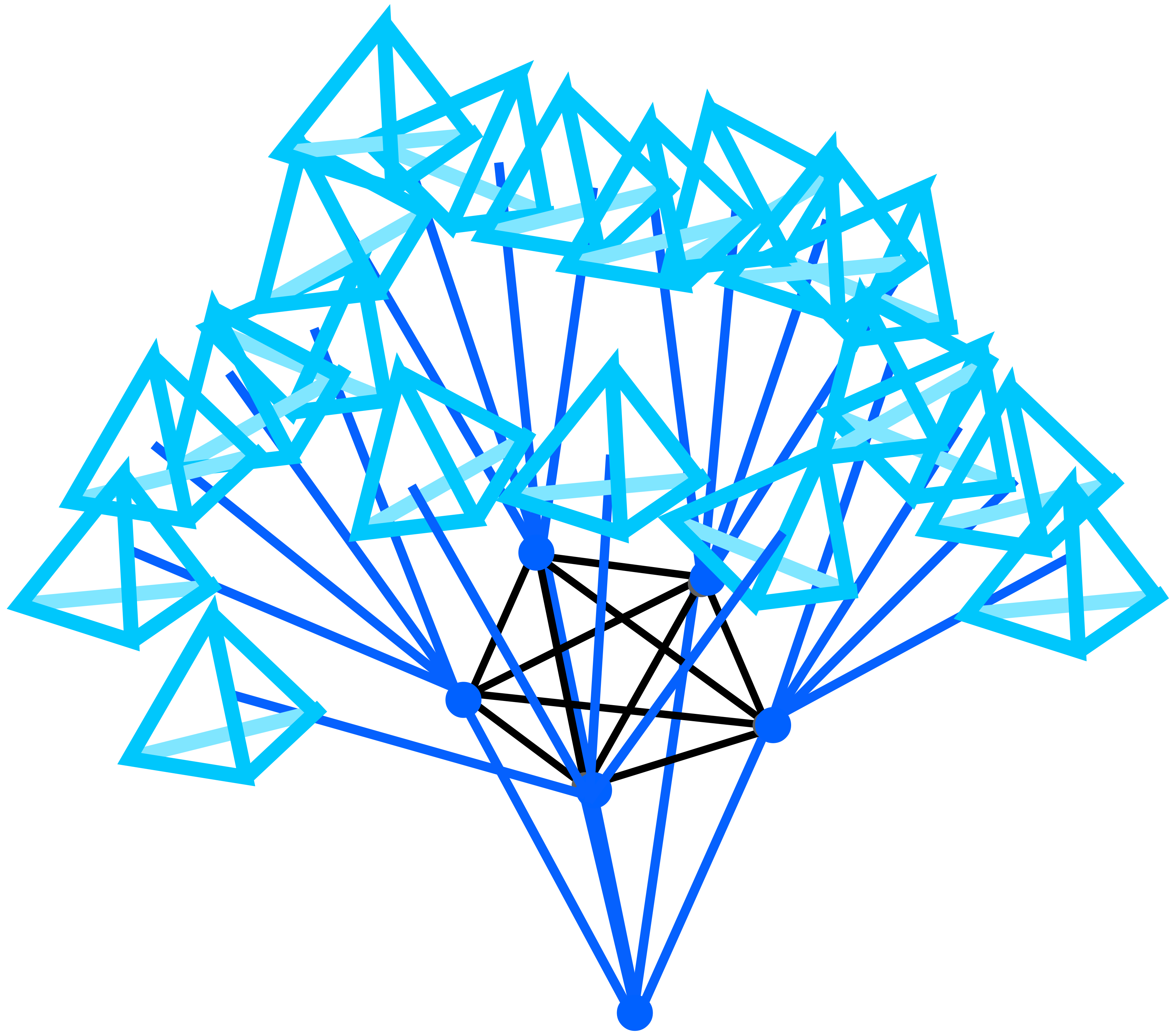
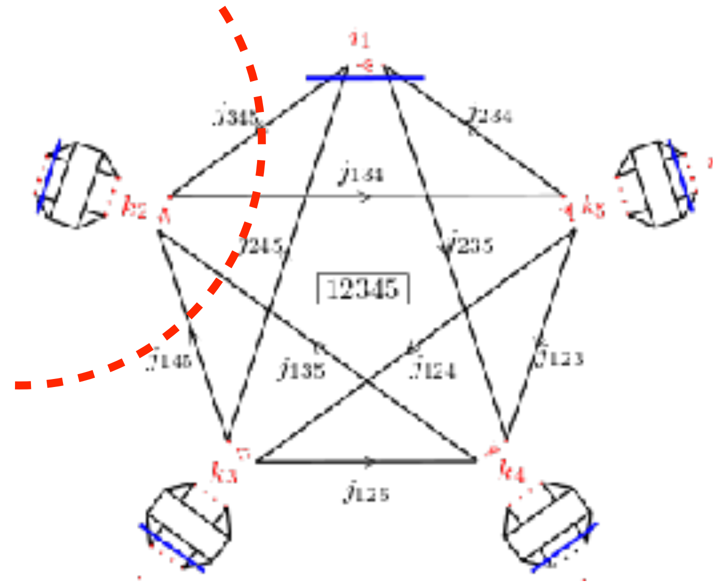




ENTANGLEMENT ENTROPY

Frisoni, Gozzini, Vidotto 2207.02881

- Partition: $\mathscr{H} = \mathscr{H}_A \otimes \mathscr{H}_{\bar{A}}$
- Reduced density matrix: $\rho_A = \dfrac{1}{Z} Tr_{\bar{A}} \, |\psi_0\rangle\langle\psi_0|$
- Entanglement entropy: $S_A = - Tr \left(\rho_A \log \rho_A\right)$



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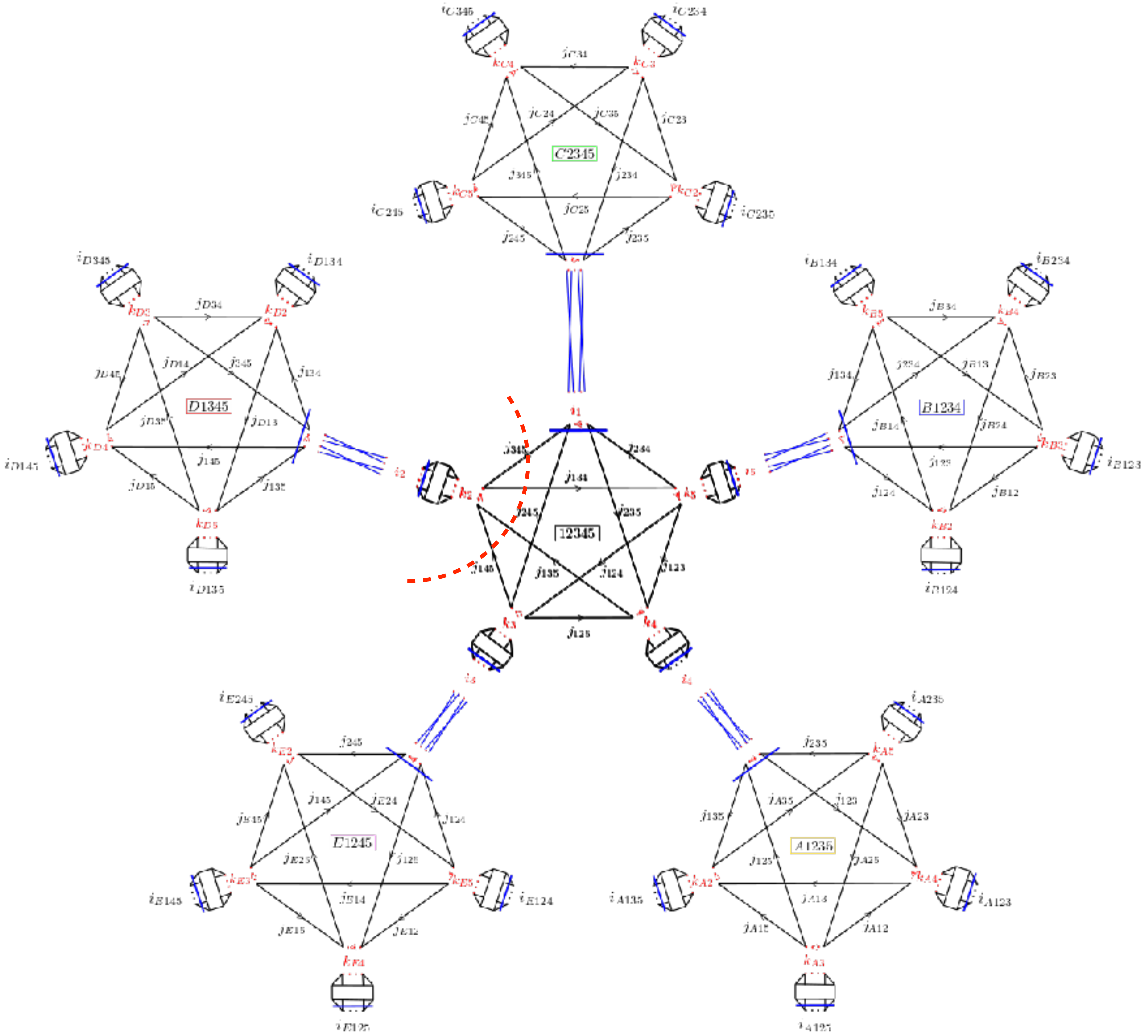

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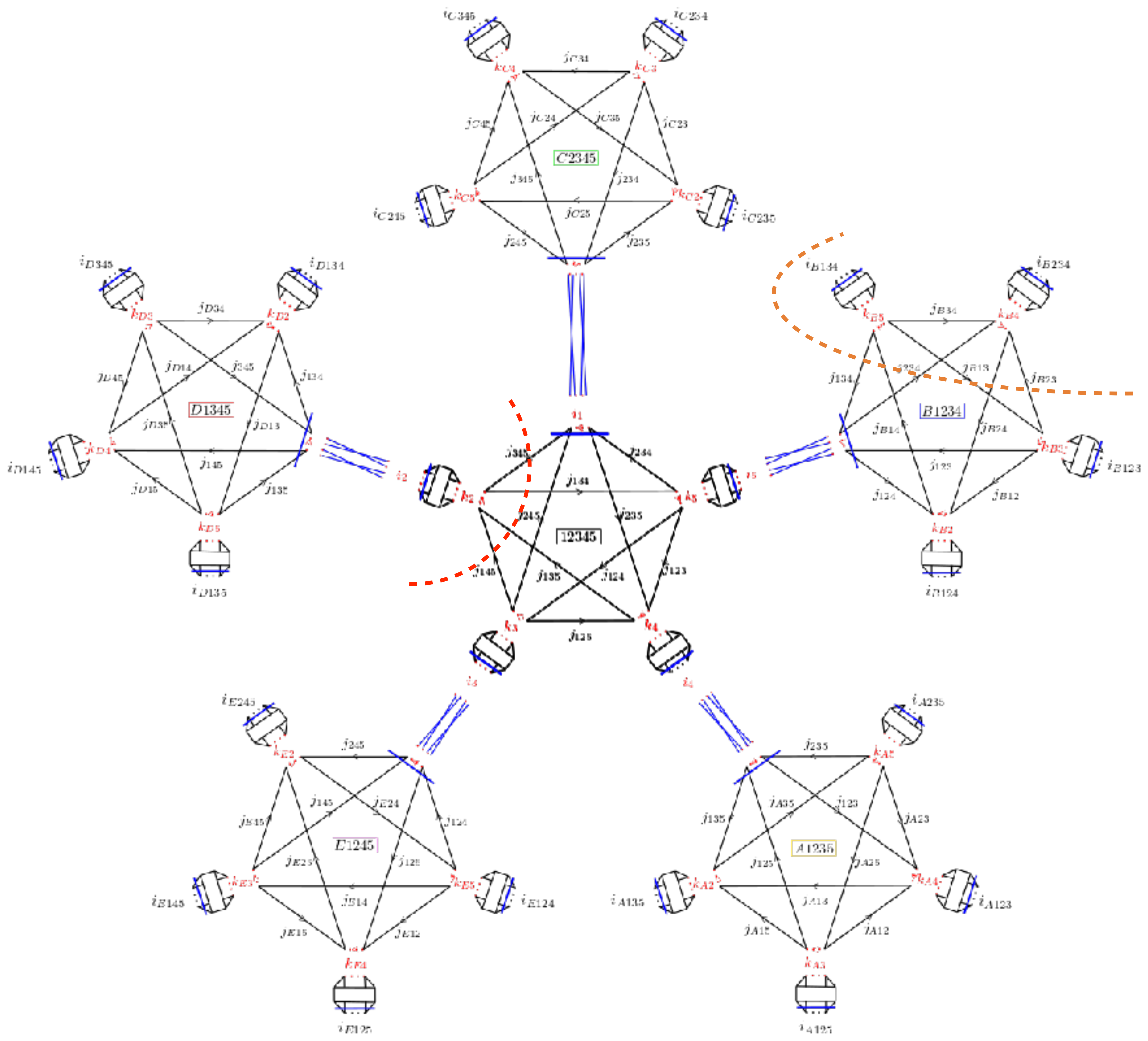

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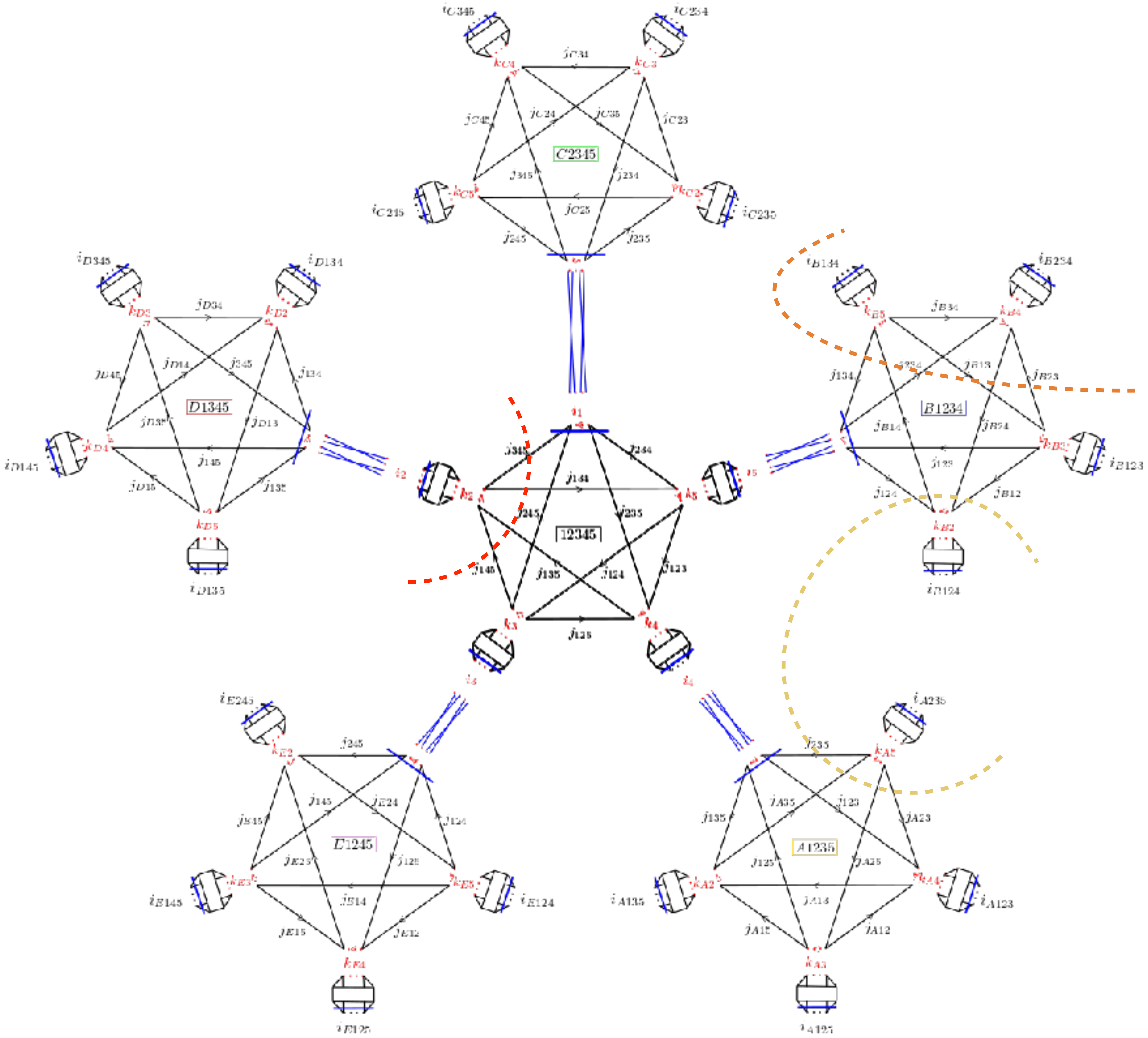
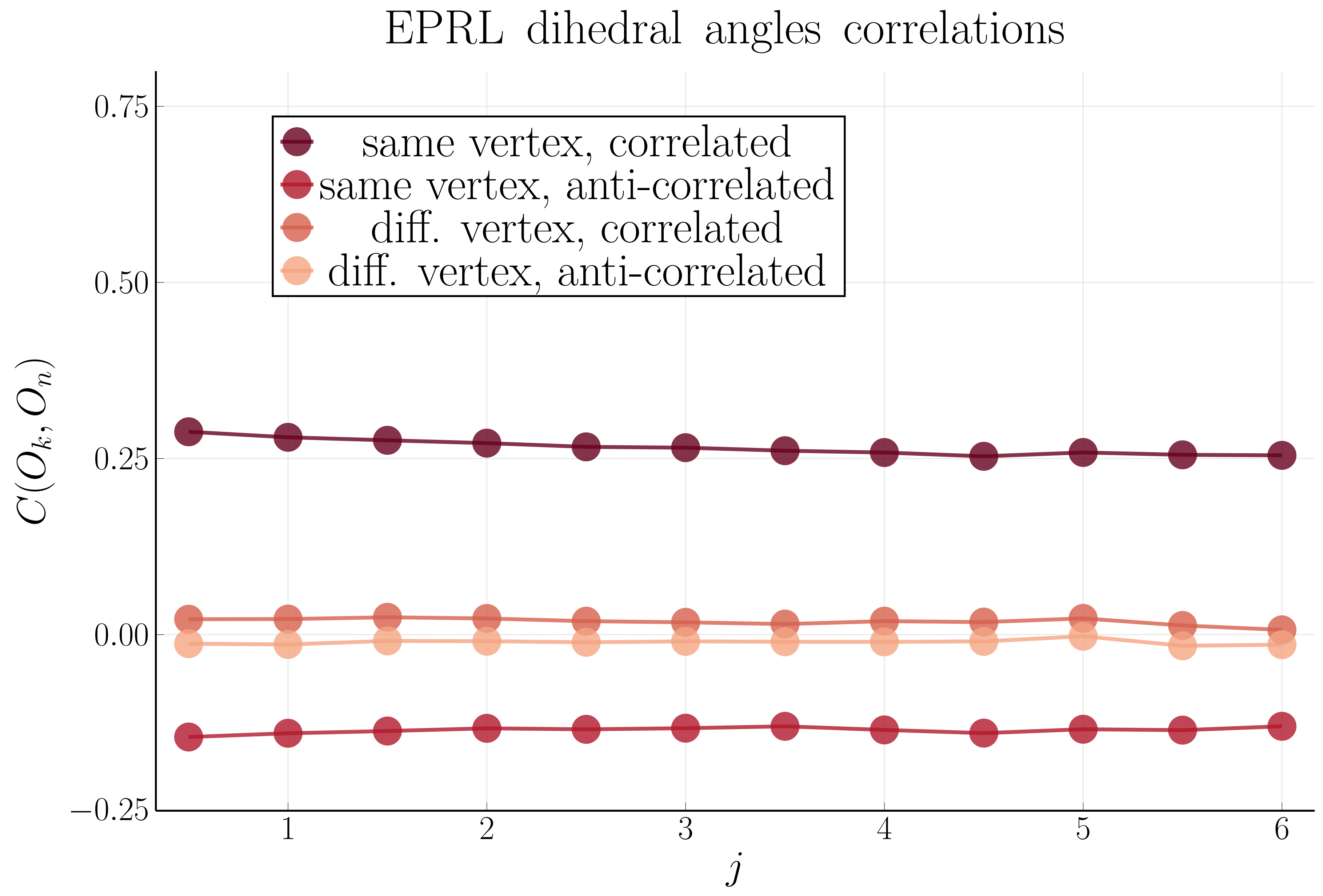
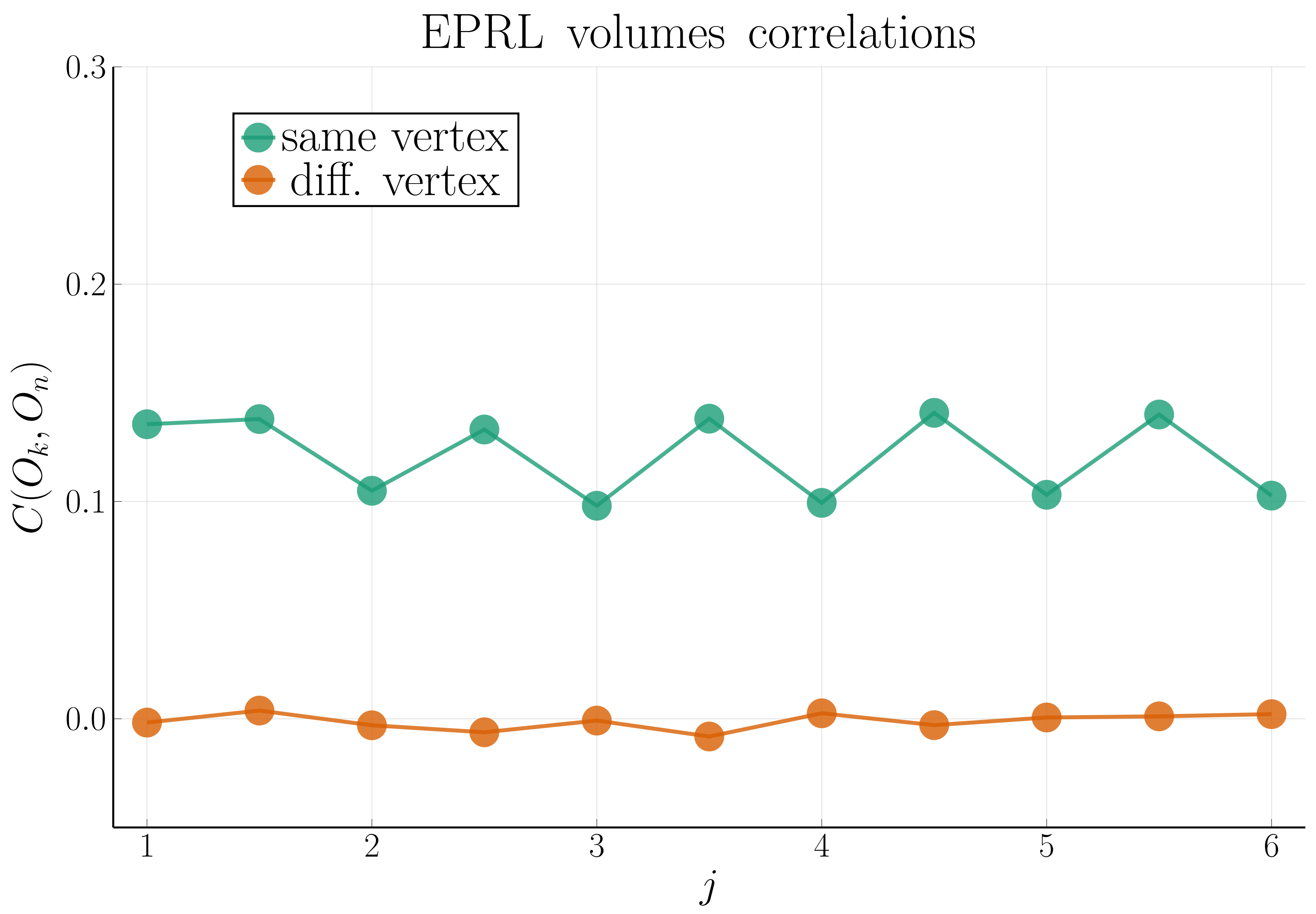


CORRELATIONS

Frisoni, Gozzini, Vidotto 2207.02881





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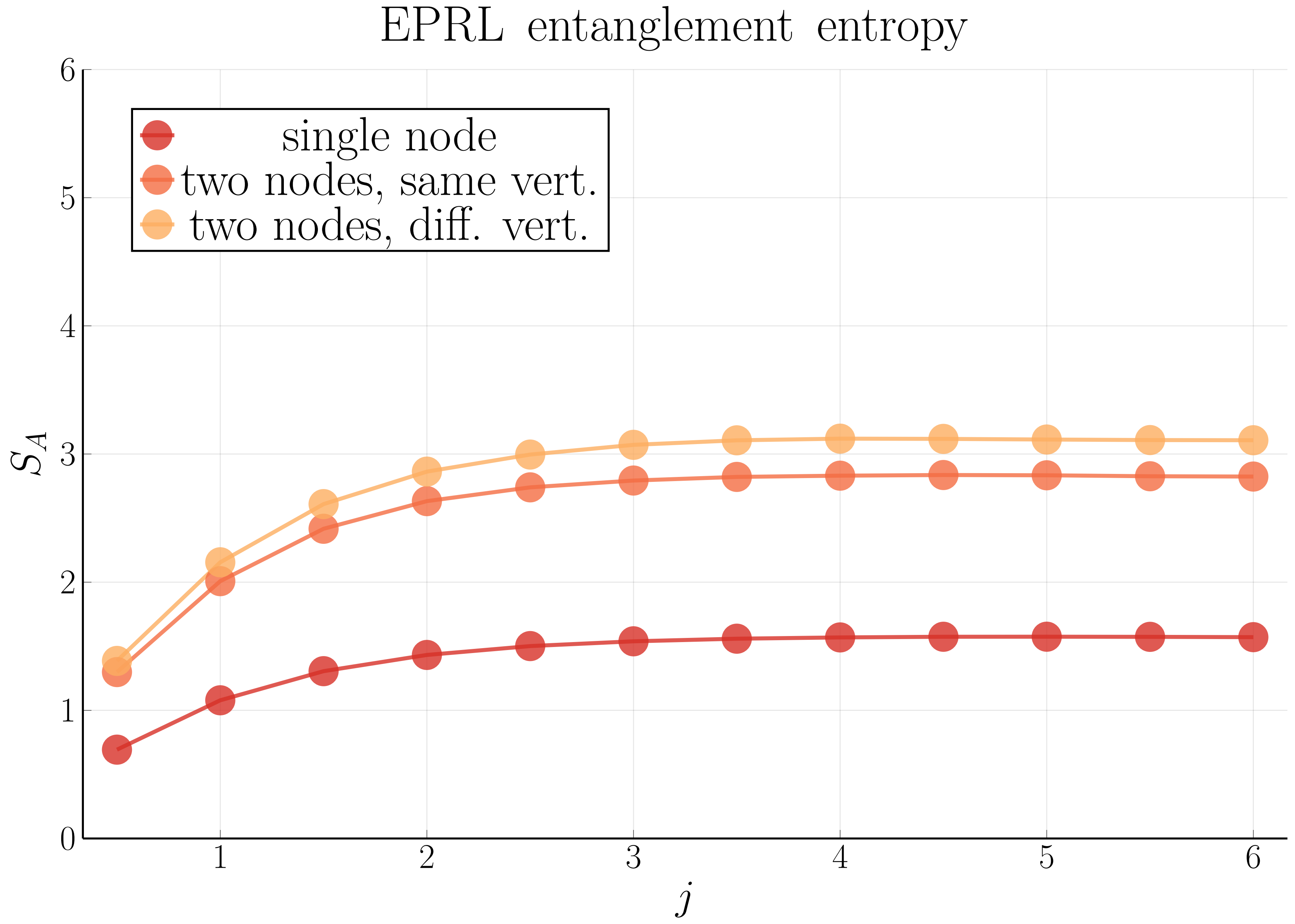

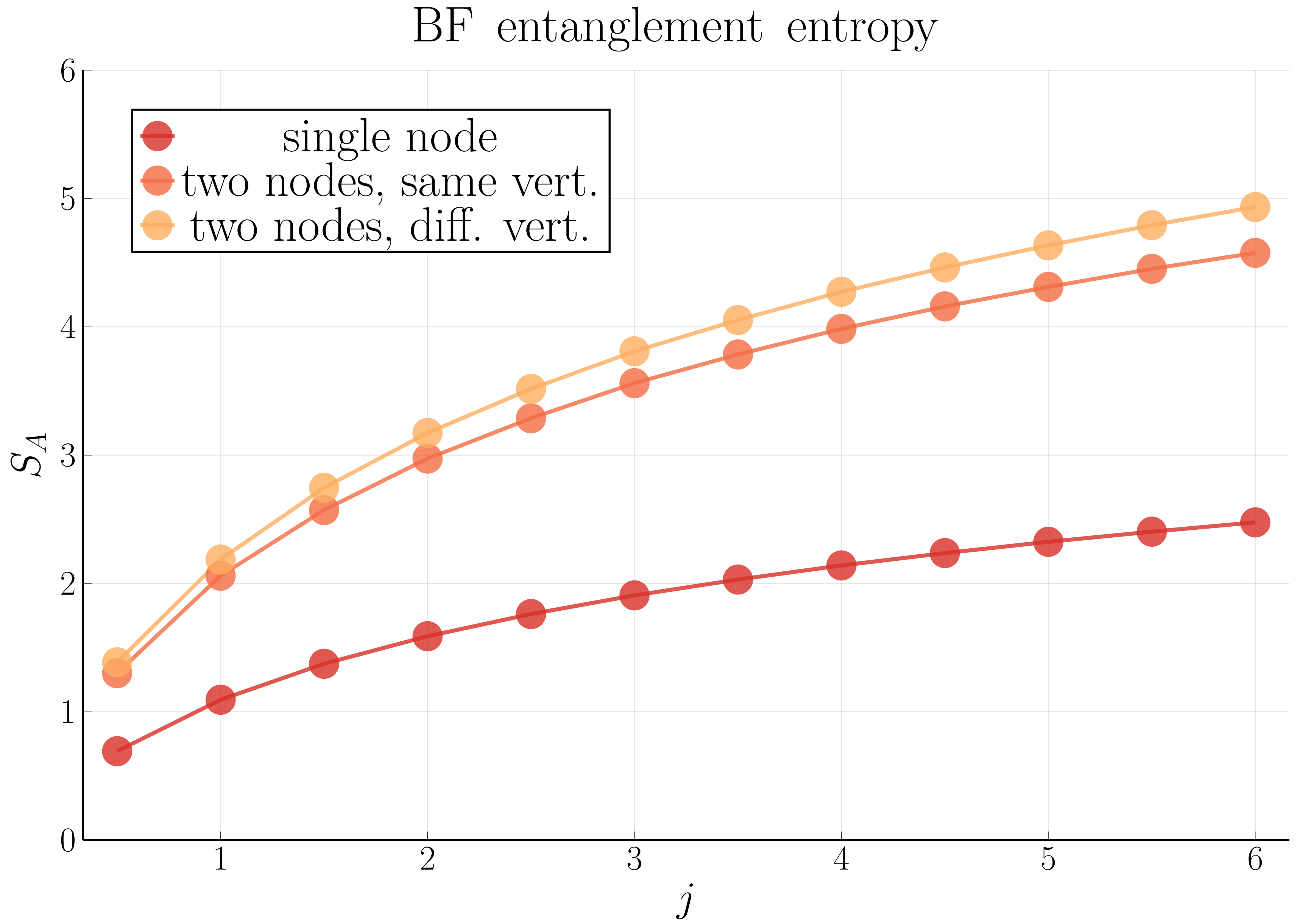

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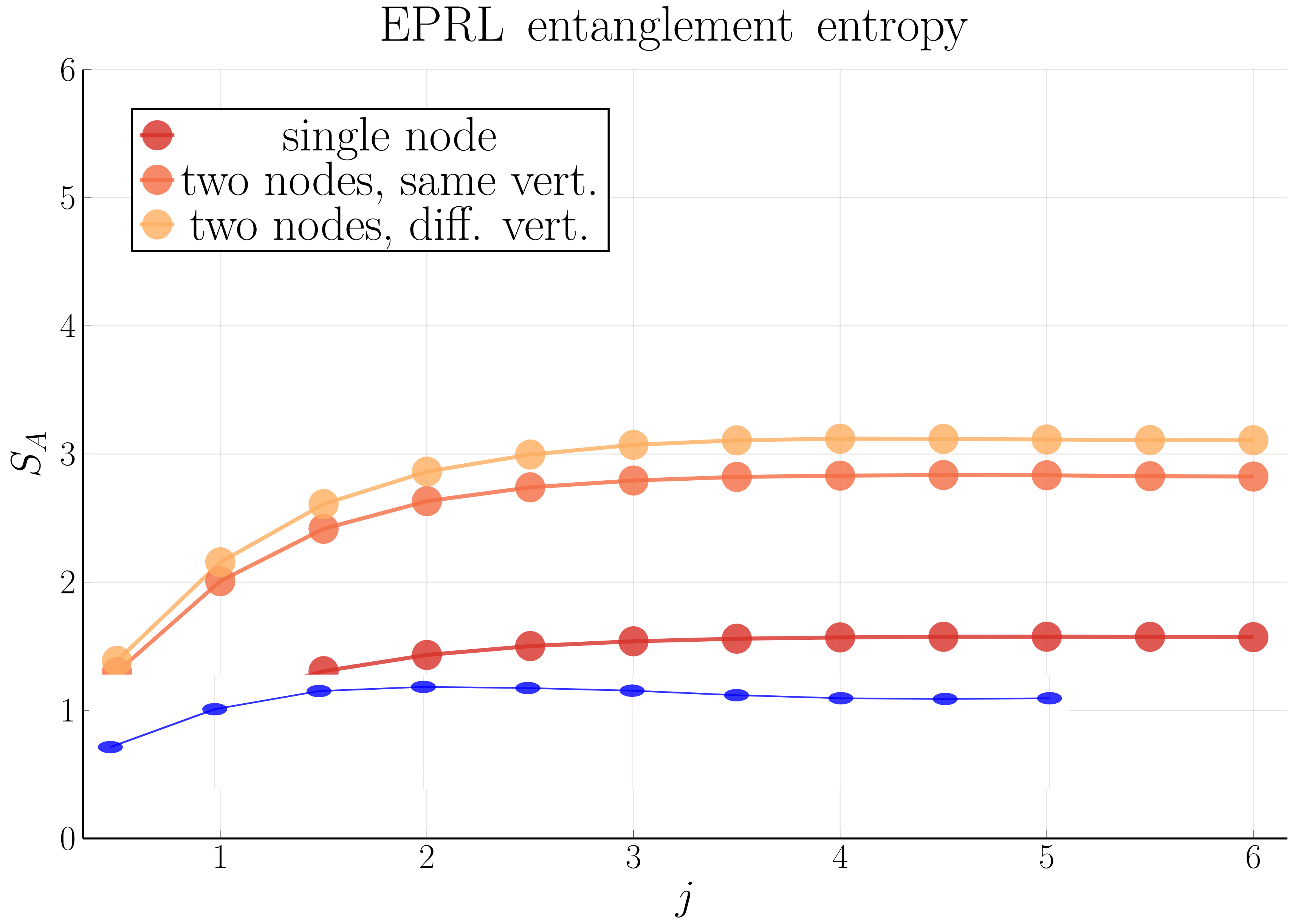
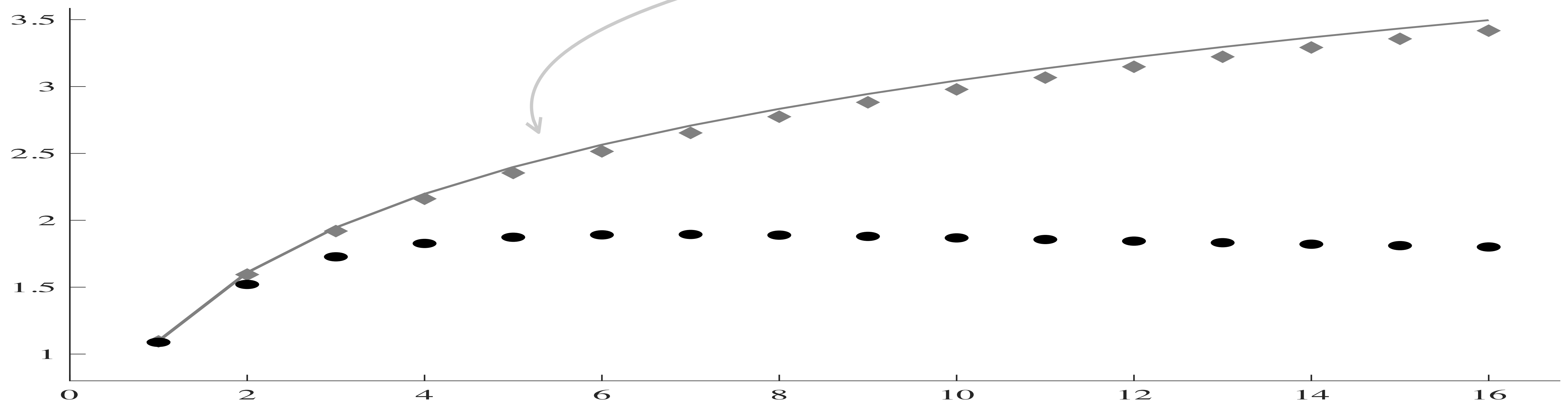
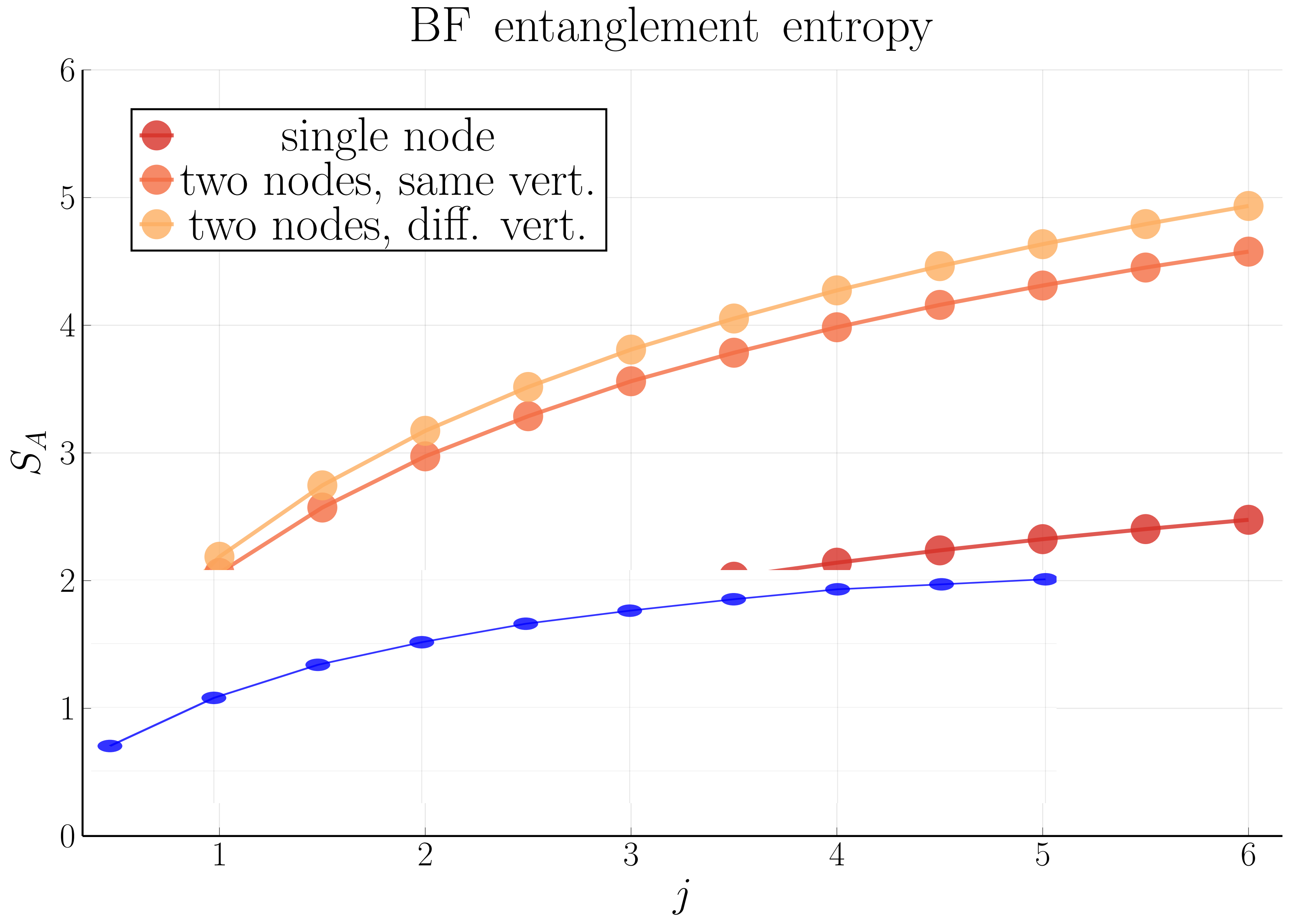


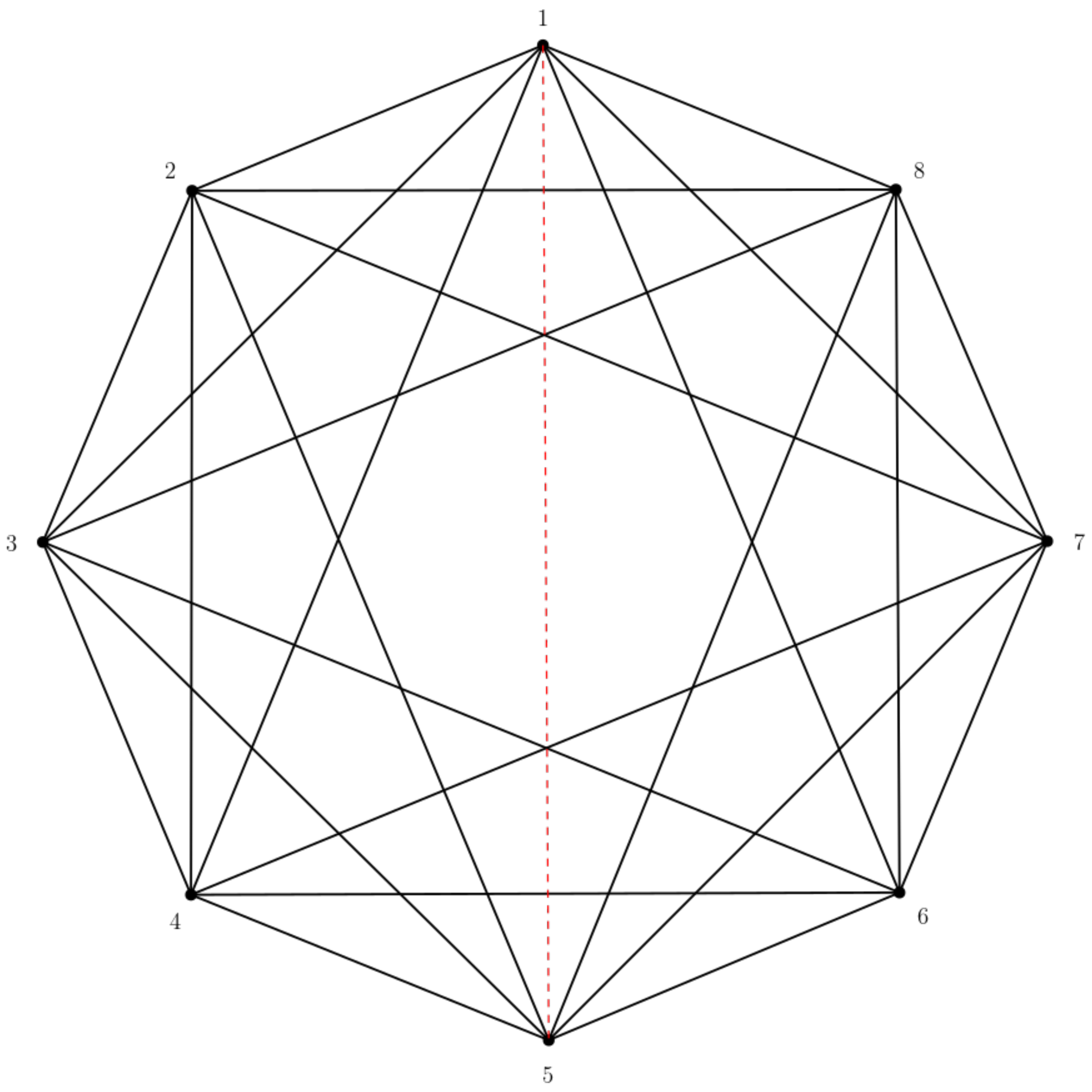
Bianchi, Donà, Vilensky "Entanglement entropy of Bell-network states in LQG"

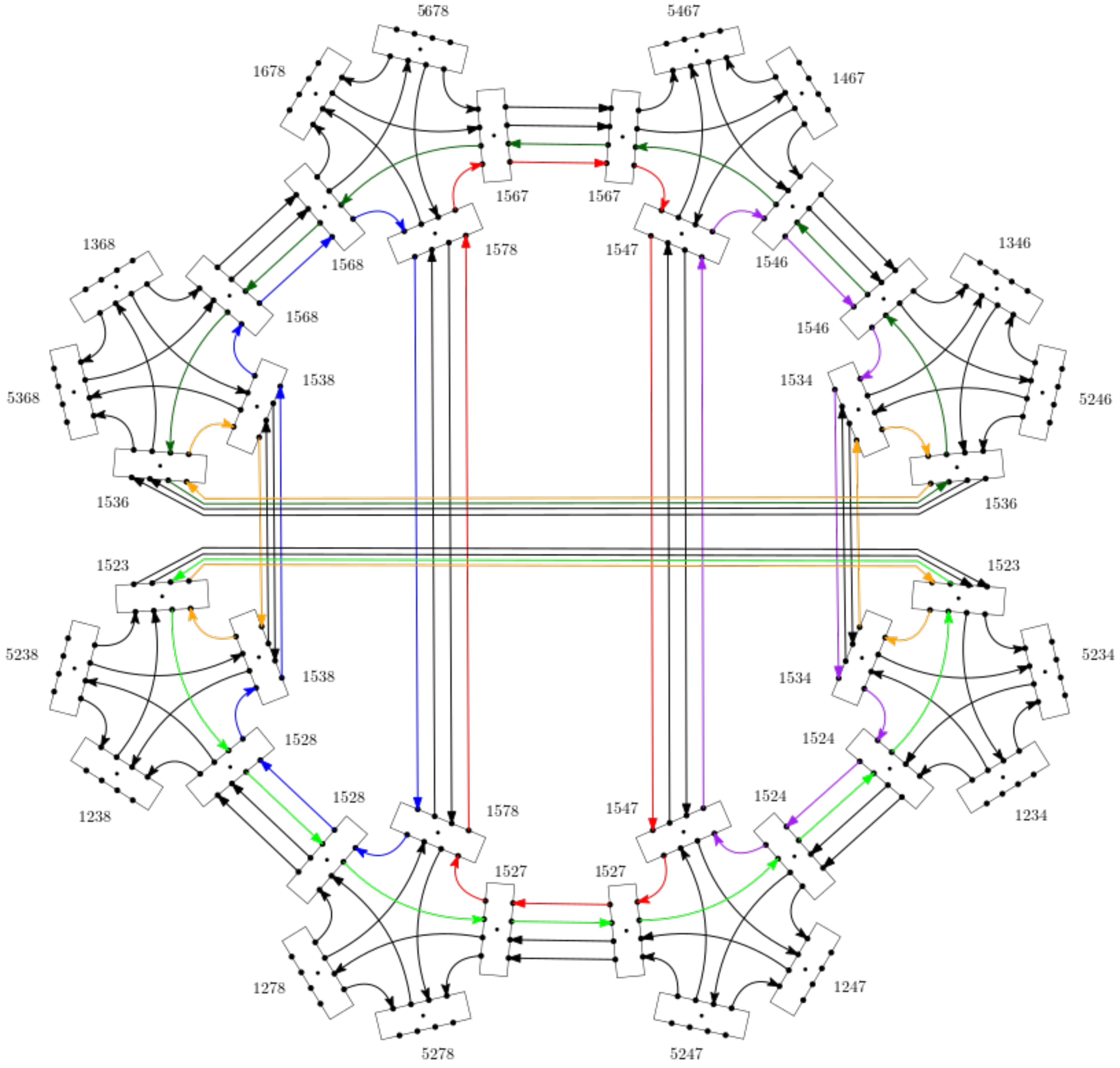




BF 16-CELL MODEL

Frisoni, Gozzini, Vidotto - to appear





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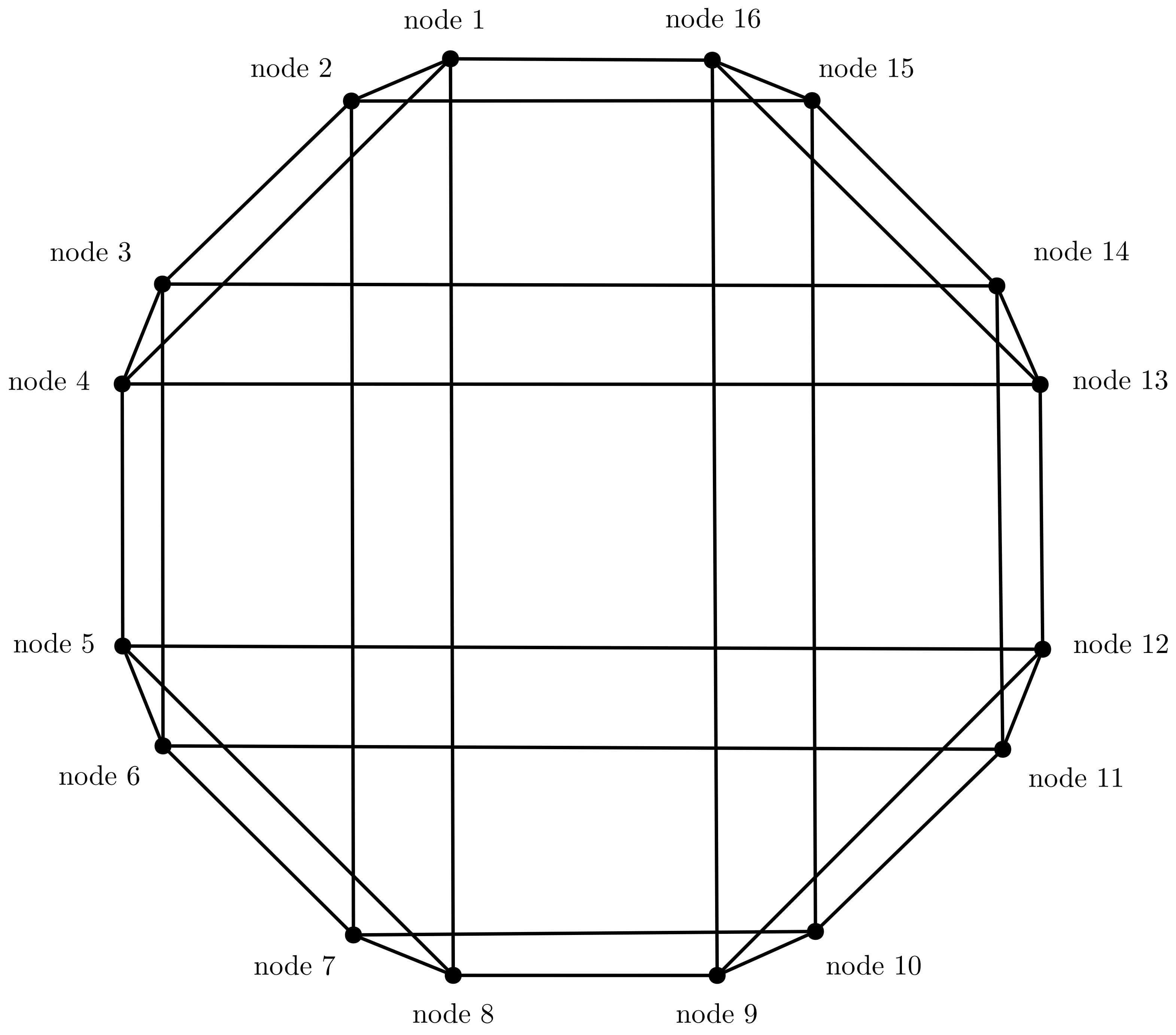
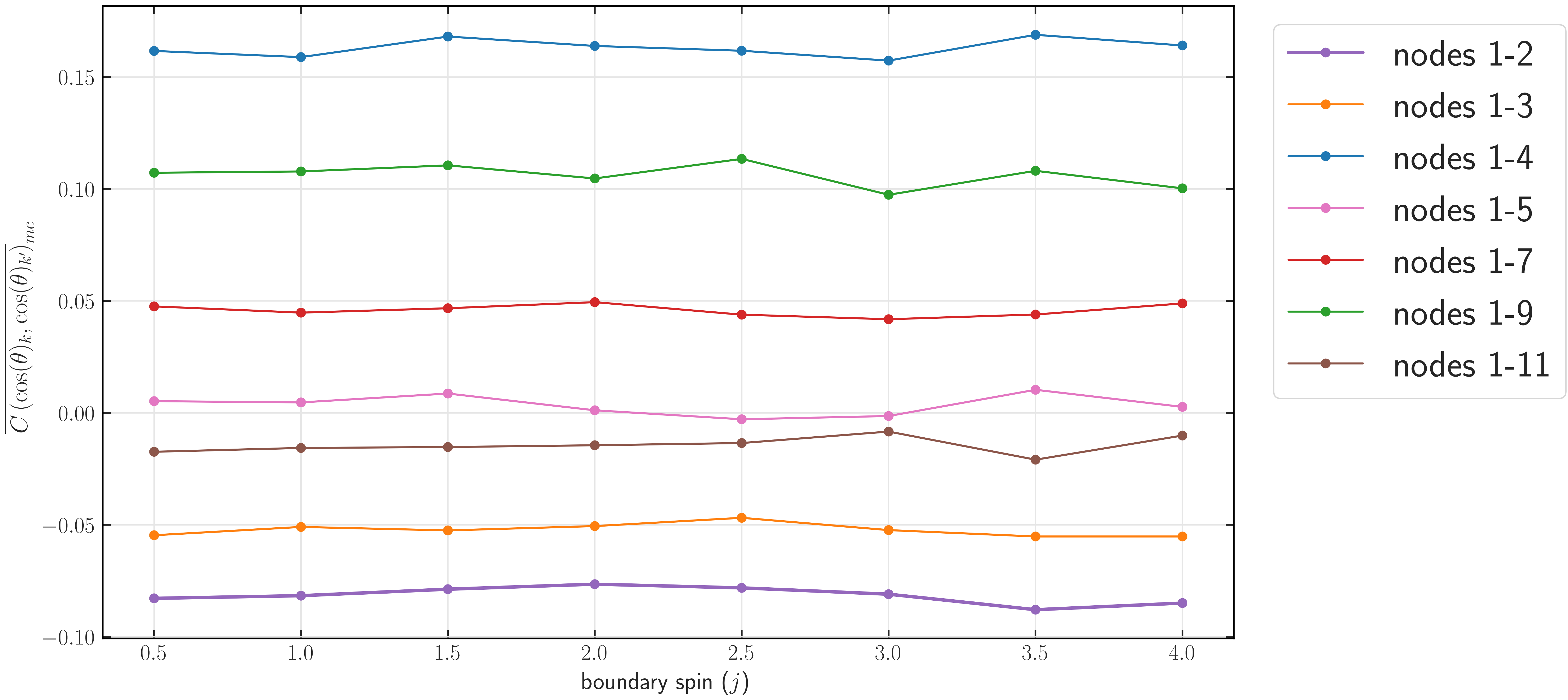


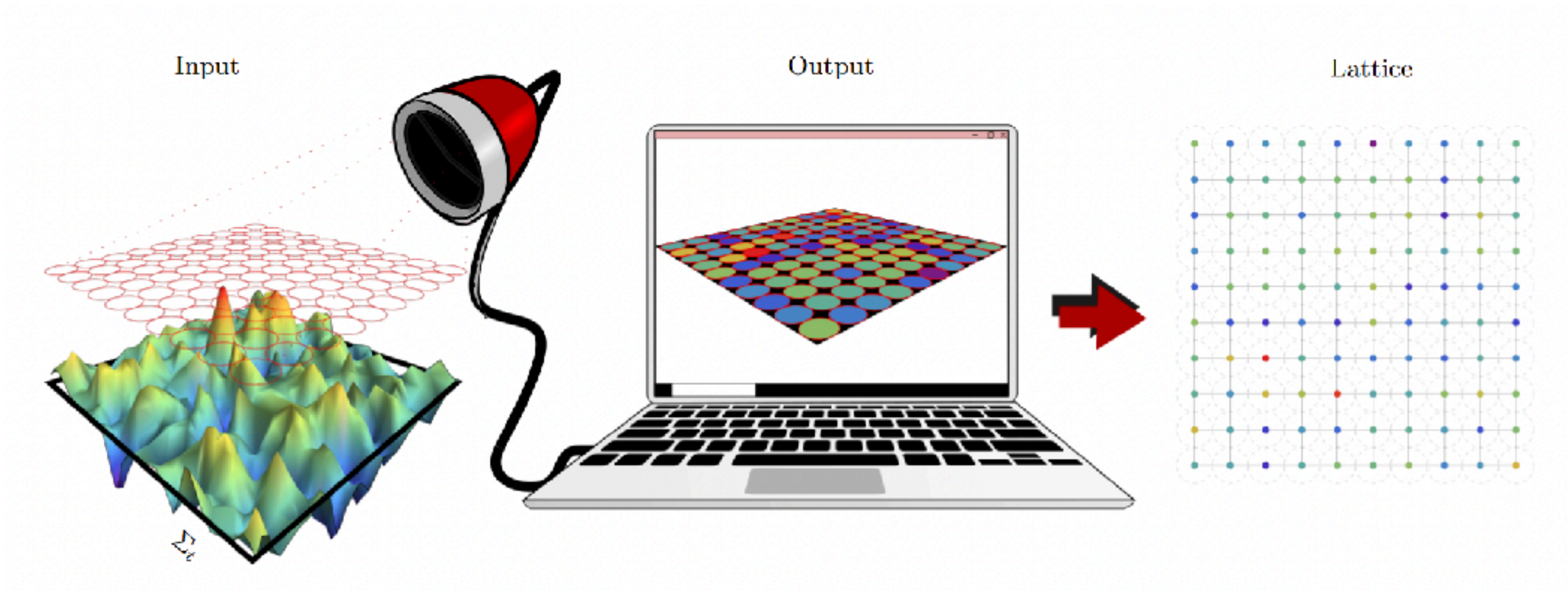


PERTURBATIONS THEORY IN QFT

- QFT correlations and entanglement entropy between specially separated regions

- Strategy:
  - ➤ Smearing of the field on two space-like separated 3-balls

Agullo, Bonga, Ribes-Metidieri, Kranas, Nadal-Gisbert 2023



- Cosmological perturbations on a 3-sphere

- Strategy:
  - ➤ Expansion in spherical harmonics

Ried, Vidotto - in progress

SUMMARY

- Computing primordial quantum fluctuations from the full theory is one of the main goals of a quantum theory of gravity!
- Proposal: use Spinfoam Hartle-Hawking States
- Graph truncation: 5-cell (full) ✓, 20-cell (refinement) ✓, 16-cell (topological) ✓
- Computational challenge: compute expectation values for observables
- Results:
  1. emerging S_3 geometry
  2. large fluctuations
  3. large correlations (for adjacent nodes) $\longrightarrow$ 16-cell needed for richer structure
- Next: effective QFT model to compare with

COLLABORATIONS AND FUTURE DIRECTIONS

- FIRST SIMPLE MODEL
  - 1 vertex
  - 5-cells boundary graph
  - computation of observables
  - high correlations

with Francesco Gozzini

- RELATION TO COSMOLOGICAL VACUUM
  - properties of standard cosmological vacua
  - QFT on a triangulated 3-sphere
  - entanglement entropy

with Sofie Ried

- MORE COMPLEX RELIABLE MODELS
  - 1 vertex, 6 vertices
  - 16-cells and 20-cells boundary graphs
  - MCMC to compute observables
  - rich behaviour of correlations

with Pietropaolo Frisoni

- NON-INFLATIONARY MODELS
  - cosmological perturbations from an effective highly-correlated vacuum states
  - matter bounce as an alternative to the inflationary models

with Mateo Pascual