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Fundamental Physics
and Applications

Istituto Nazionale di Fisica Nucleare

UNIVERSITY
OF TRENTO

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SCALE-INVARIANT INFLATION

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OUTLINE

1. FUNDAMENTAL SCALE INVARIANCE

2. SCALE-INVARIANT QUADRATIC GRAVITY

3. INFLATIONARY PREDICTIONS

UNIFYING PRINCIPLE FOR 3 MAIN ISSUES

INFLATION

How to realise naturally flat inflationary potentials without fine-tuning?

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COSMOLOGICAL CONSTANT PROBLEM

What drives the present Λ so small? $\frac{\rho_{vac}}{\rho_\Lambda} \sim 10^{120}$

UNIFYING PRINCIPLE FOR 3 MAIN ISSUES

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COSMOLOGICAL CONSTANT PROBLEM

What drives the present Λ so small? $\frac{\rho_{vac}}{\rho_{\Lambda}} \sim 10^{120}$

RENORMALIZABILITY

Can we find a new, highly predictive, criterion beyond renormalizability?

FUNDAMENTAL SCALE INVARIANCE

C. Wetterich , Nuclear Physics B, 115326 (2021)
A. Strumia & A. Salvio, J High Energ Phys, 6 (2017)

Basic idea: a fundamental QFT does not involve any intrinsic parameter with dimension of mass or length

Following Wetterich, we can introduce an explicit mass scale k

Canonical field
Dimension of a mass

$$\phi = k\,\tilde{\phi}$$

Scale-invariant field
Dimensionless

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Scale-invariant field
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The corresponding effective actions obey

$$\boxed{k\partial_k \Gamma_k[\phi] = \zeta_k[\phi]}$$

General solution

$$\boxed{k\partial_k \Gamma_k[\tilde{\phi}] = 0}$$

Particular, scaling solution holding
when the canonical fields are expressed
in terms of the scale-invariant ones

FUNDAMENTAL SCALE INVARIANCE

NATURALLY FLAT POTENTIALS FOR INFLATION

Scale-invariant theory non-minimally coupled to gravity

$$\mathcal{L}_J = \sqrt{-g}\left[\xi\,\phi^2 R - \lambda\phi^4 - \frac{1}{2}(\partial\phi)^2\right]$$

Weyl rescaling from the Jordan to the Einstein frame

$$\mathcal{L}_E = \sqrt{-\tilde{g}}\left[\frac{M_{pl}^2}{2}\tilde{R} - M_l^4\frac{\lambda}{\xi^2} - \frac{1}{2}(\partial\tilde{\phi})^2\right]$$

↓

The potential is flat at tree-level: no fine-tuning. Scale symmetry breaking can occur from quantum corrections.

FUNDAMENTAL SCALE INVARIANCE

EXPLICIT SCALE SYMMETRY BREAKING AND DYNAMICAL DARK ENERGY

Introduce a small explicit scale-symmetry breaking term

$$\mathscr{L} = \sqrt{-g}\left[\underbrace{\frac{\phi^2}{2}}_{\text{Dynamical Planck mass}} R + \underbrace{k^4}_{\text{Explicit mass scale}}\right]$$

The effective cosmological “constant” evolves

$$\lambda = \frac{k^4}{\phi^4}$$

For cosmological solutions ϕ increases without bounds such that $\lambda \to 0$ in the infinite future.

* Today $k \sim 10^{-3}\,\text{eV}$ for $\phi \sim M_{pl}$

FUNDAMENTAL SCALE INVARIANCE

A CRITERION BEYOND RENORMALIZABILITY

For general renormalizable theories the effective action remains well defined in the continuum limit if one employs renormalized fields

Renormalized fields ← $\phi_{R,i}(x) = k^{d_i} f_i(k)\, \tilde{\phi}_i(x)$ → **Scale-invariant field**

FUNDAMENTAL SCALE INVARIANCE

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$$\text{Renormalized fields} \longleftarrow \phi_{R,i}(x) = k^{d_i} f_i(k)\, \tilde{\phi}_i(x) \longrightarrow \text{Scale-invariant field}$$

Theories with fundamental scale invariance:

- Renormalizable
- For some choice of the fields $\tilde{\phi}$ the effective action becomes k-independent
- Exact scaling solutions: no free parameters. High predictive power

SCALE-INVARIANT QUADRATIC GRAVITY

THE MODEL

M. Rinaldi and L. Vanzo PR D 94 (2016)

- $\mathscr{L}_{EH} \longrightarrow f(R,\phi)$: scalar-tensor theory of modified gravity
- Most general scale-invariant $\mathscr{L}$ up to R^2

$$\mathscr{L}_J = \sqrt{-g}\left[\underbrace{\frac{\alpha}{36}R^2}_{\text{Higher order term in R}} + \frac{\xi}{6}\phi^2 R \underbrace{- \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4}_{\text{Scalar field}}\right], \qquad \alpha, \lambda, \xi > 0$$

- Scale transformations act as
  - $\bar{g}_{\mu\nu}(x) = g_{\mu\nu}(\ell x)$
  - $\bar{\phi}(x) = \ell\phi(\ell x)$

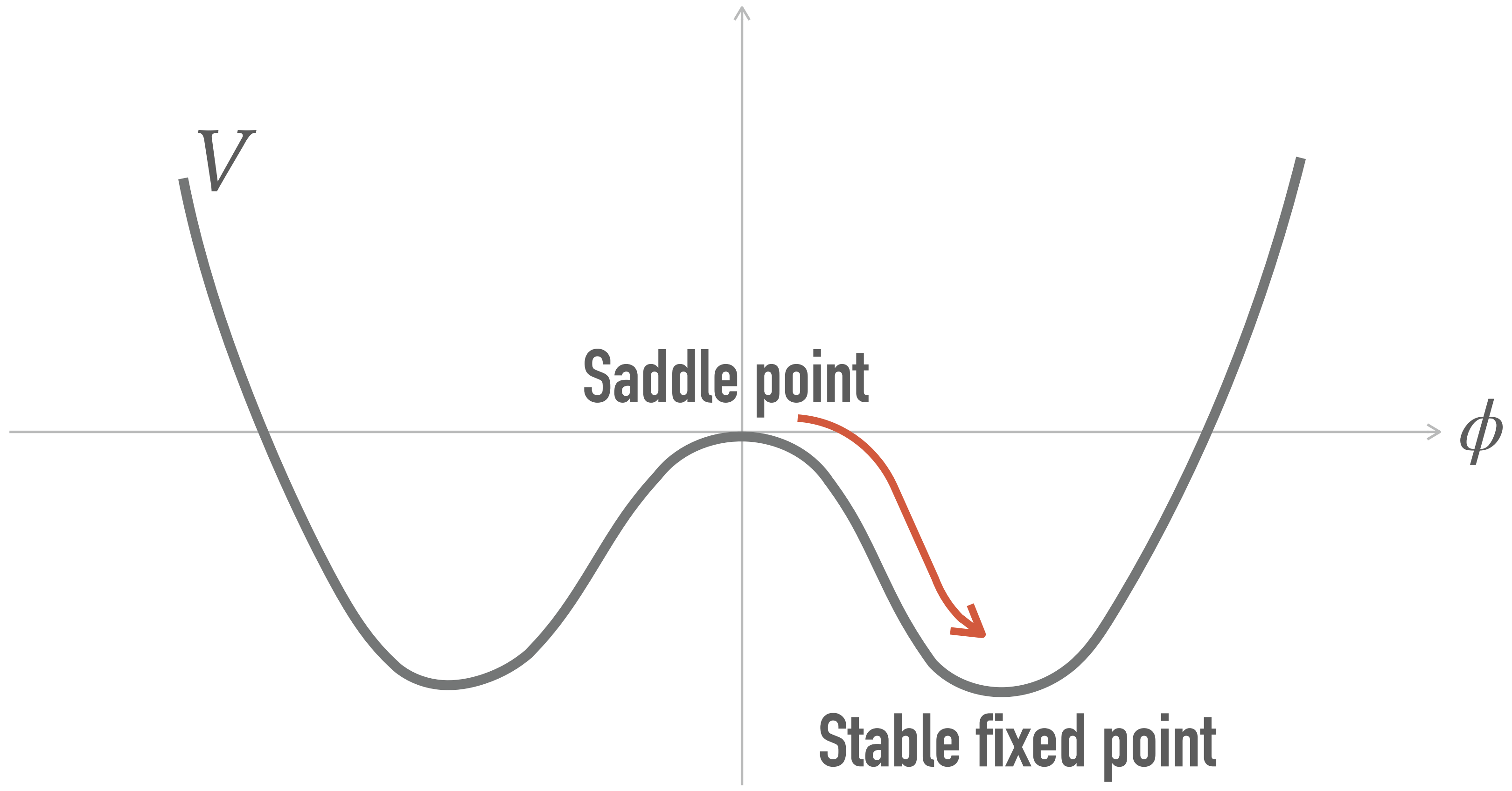
$$\bar{\mathscr{L}} = \mathscr{L}$$

SCALE-INVARIANT QUADRATIC GRAVITY

JORDAN FRAME

The field ϕ is subjected to an effective potential

$$V_{eff}(\phi) = -\frac{\xi}{6}\phi^2 R + \frac{\lambda}{4}\phi^4$$



Classical scale-symmetry breaking

The scalar field takes a non-zero VEV at the minimum

$$\langle\phi_0^2\rangle = \frac{\xi R}{3\lambda}$$

Dynamical generation of a mass scale

Natural identification with the Planck mass

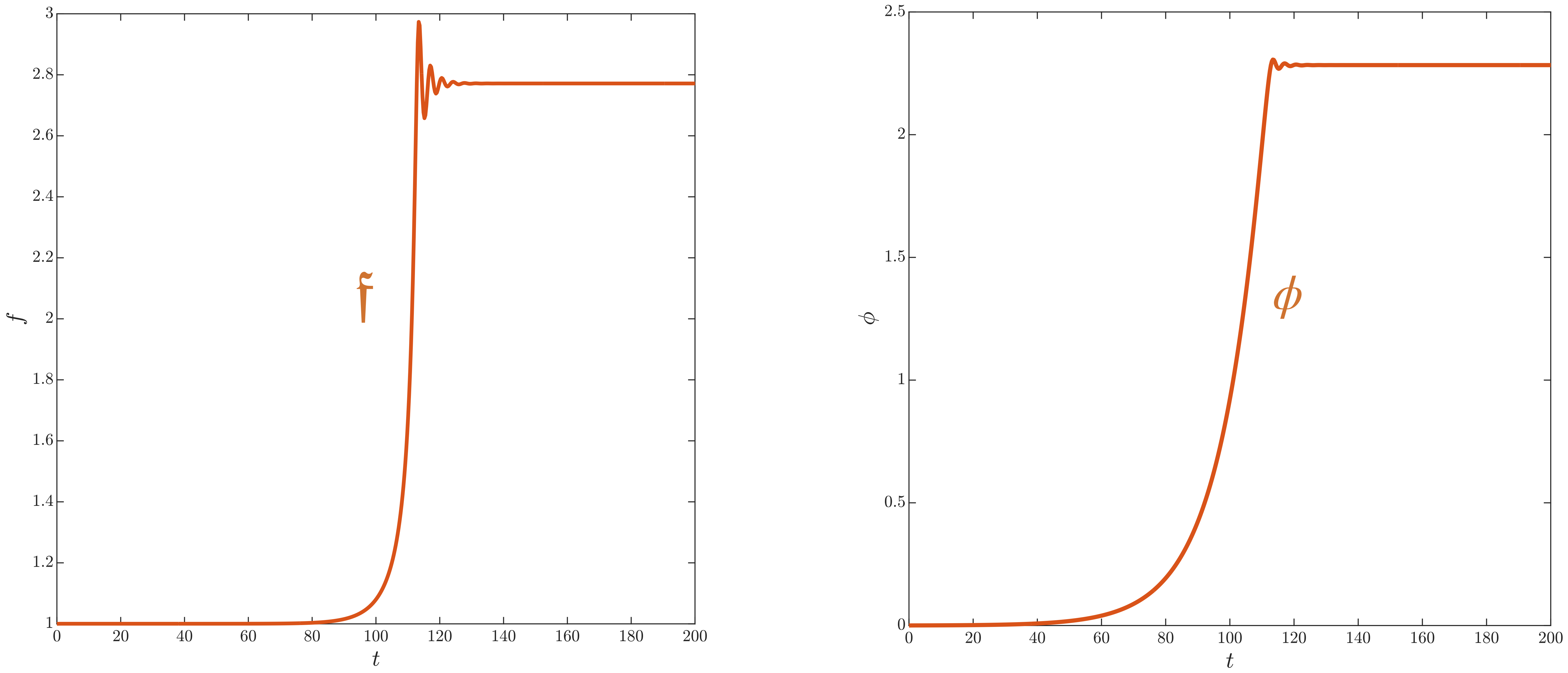
$$\frac{\xi}{6}\phi_0^2 R \equiv \frac{1}{2}M_{pl}^2 R$$

SCALE-INVARIANT QUADRATIC GRAVITY

EINSTEIN FRAME $g^*_{\mu\nu} = \Omega^2 g_{\mu\nu}$

Two dynamical degrees of freedom: are we in multi-field inflation?

$$\mathscr{L}_E = \sqrt{-g}\left[\frac{M^2}{2}R - \frac{3M^2}{f^2}(\partial f)^2 - \frac{f^2}{2M^2}(\partial\phi)^2 - V(f,\phi)\right]$$



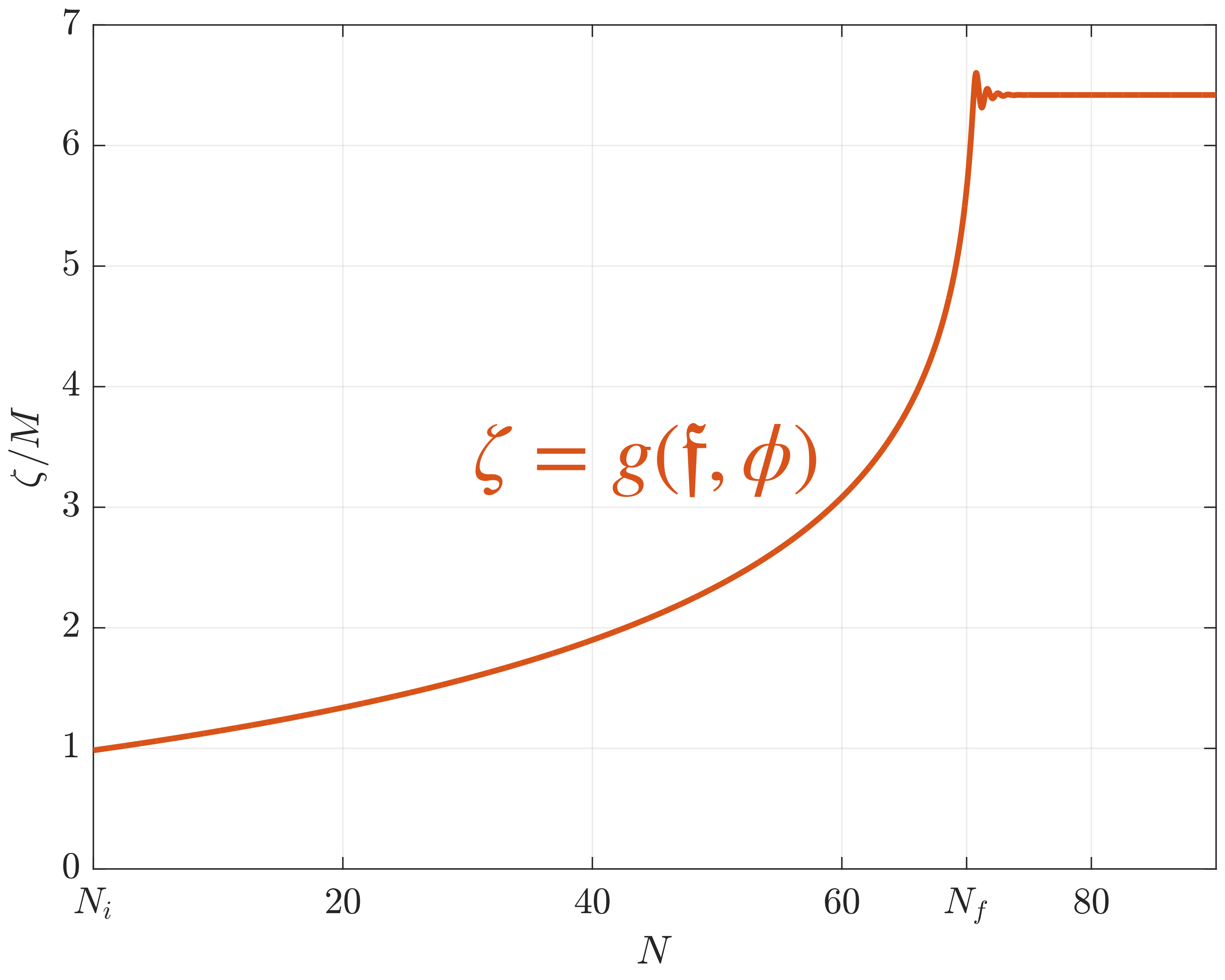
SCALE-INVARIANT QUADRATIC GRAVITY

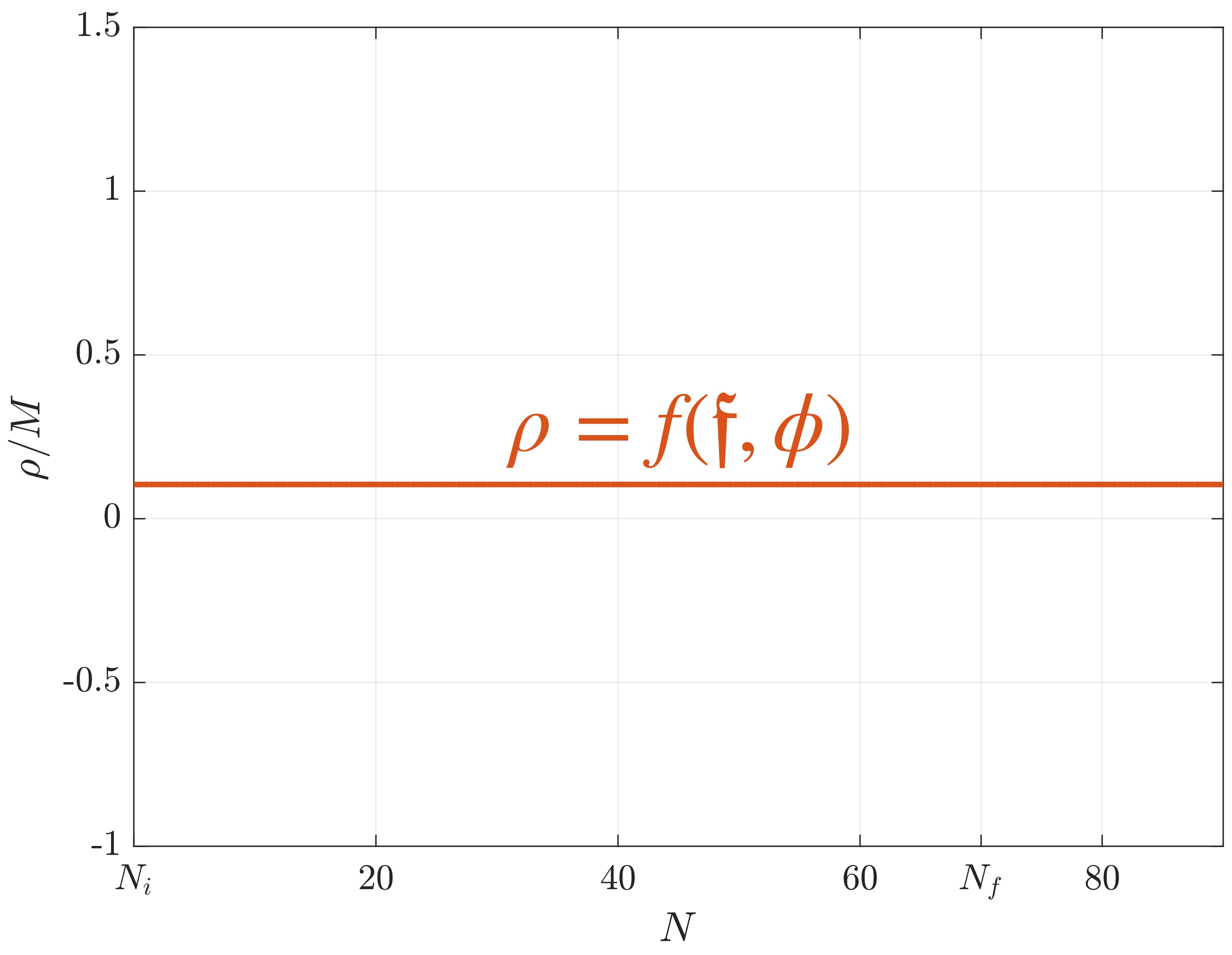
EINSTEIN FRAME: FIELDS' REDEFINITION

G. Tambalo & M. Rinaldi Gen Relativ Gravit 49 (2017)

Noether's current conservation can be employed to shift all the dynamics on one field

$$\mathscr{L}_E = \sqrt{-g}\left(\frac{M^2}{2}R - \frac{1}{2}\partial_\mu\zeta\,\partial^\mu\zeta - 3\mathrm{Cosh}\left[\frac{\zeta}{\sqrt{6}M}\right]^2\partial_\mu\rho\partial^\mu\rho - U(\zeta)\right)$$



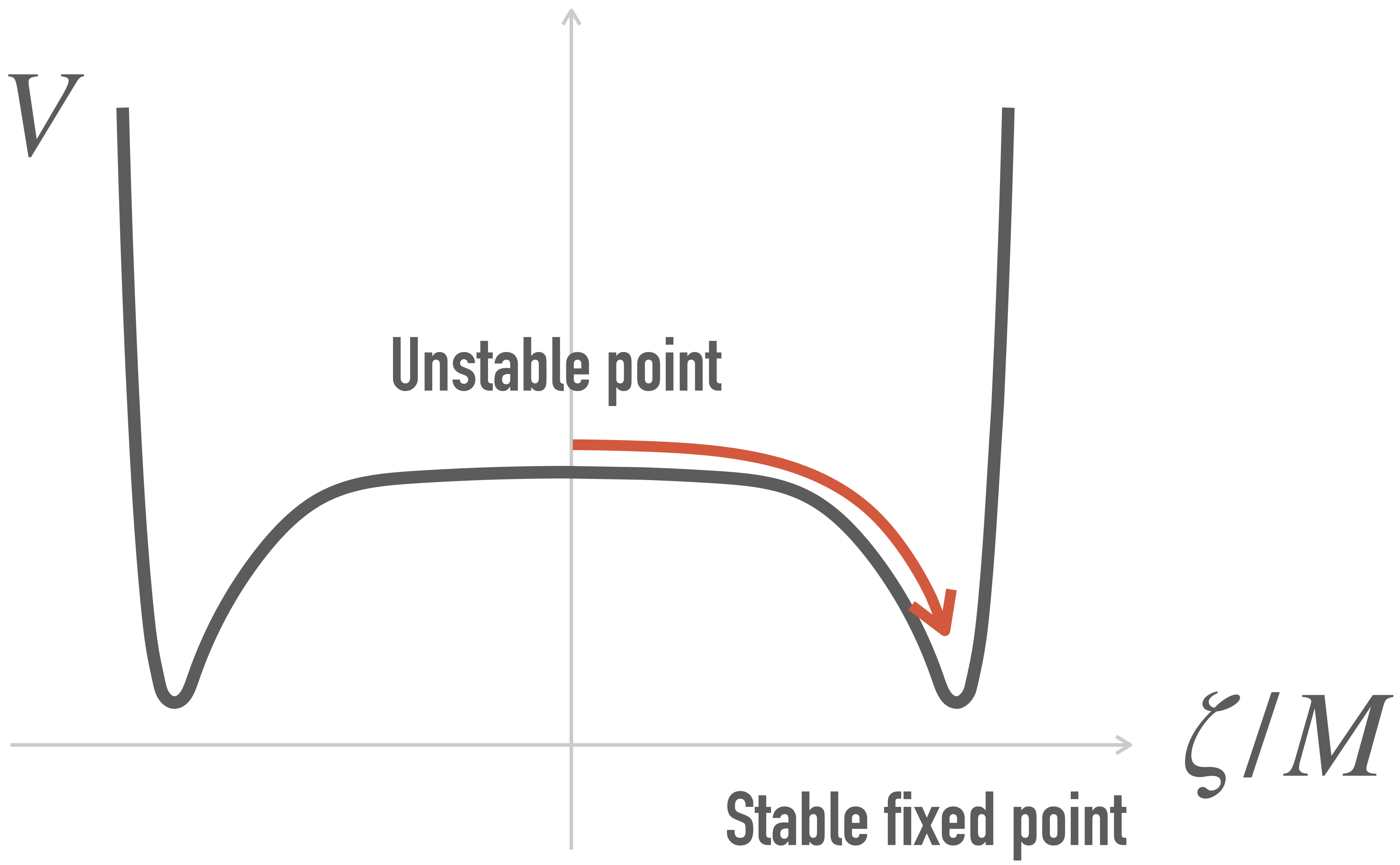


SCALE-INVARIANT QUADRATIC GRAVITY

EINSTEIN FRAME: SINGLE-FIELD POTENTIAL

- Naturally flat plateau: no fine-tuning
- Non-vanishing at the minima

ρ	GOLDSTONE BOSON
ζ	INFLATON

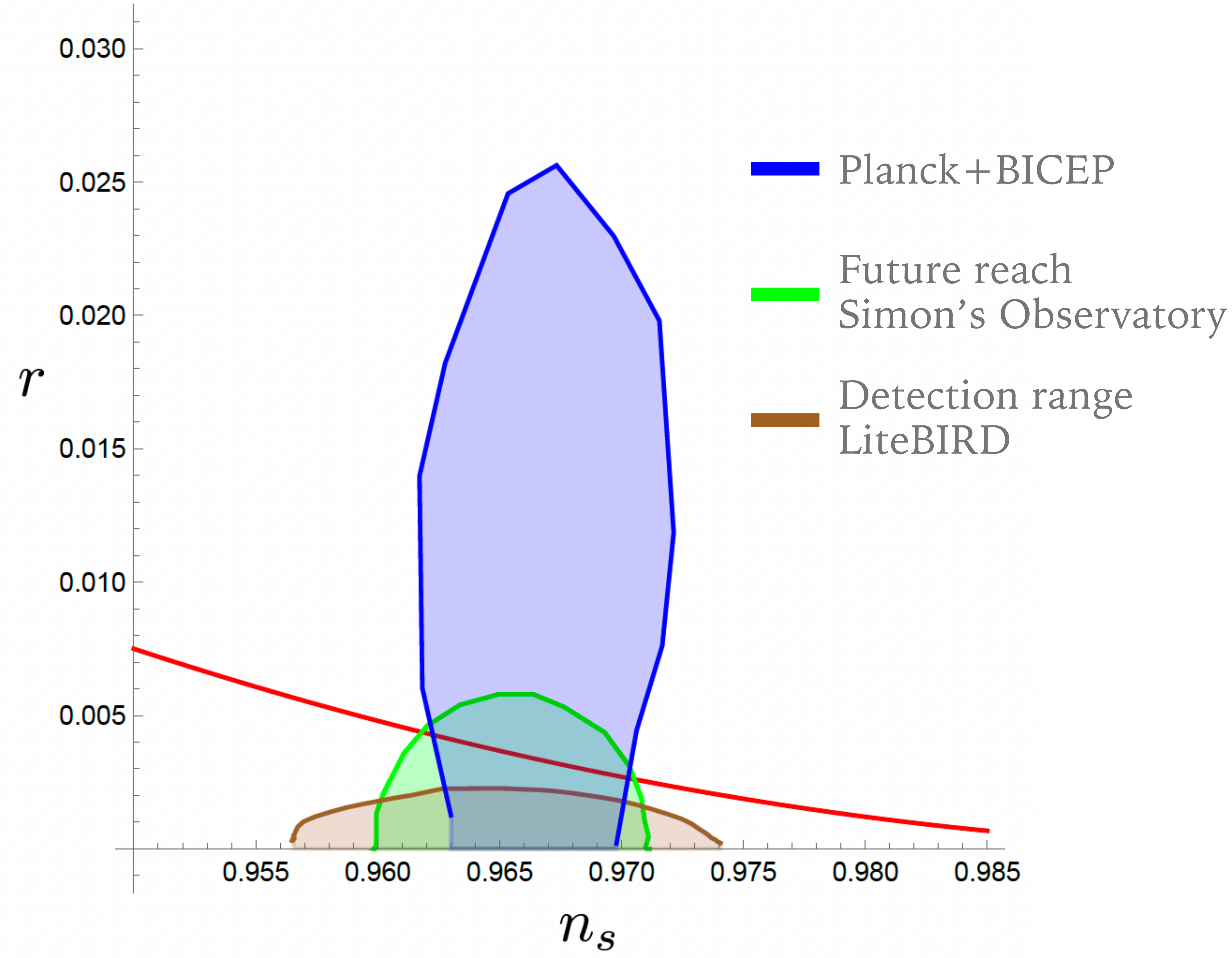


INFLATIONARY PREDICTIONS

PRIMORDIAL SPECTRA

A. Ghoshal, D. Mukherjee, & M. Rinaldi JHEP **5** *(2023)*

Single-field predictions are recovered, both in the Jordan and the Einstein frame



Scalar perturbations

$$\Delta_s^2(k) = \frac{1}{2M_{pl}^2\,\epsilon}\left(\frac{H}{2\pi}\right)^2\Bigg|_{k=aH}$$

Tensor perturbations

$$\Delta_t^2(k) = \frac{2}{\pi^2}\left(\frac{H}{M_{pl}}\right)^2\Bigg|_{k=aH}$$

- $\Omega = \alpha\lambda + \xi^2 \lesssim 1.15\,\xi^2$
- $\alpha \gtrsim 2 \times 10^{10}$
- $\xi \lesssim 1.3 \times 10^{-2}$
- $\Delta N \gtrsim 55$

SUMMARY

- Fundamental scale invariance as a new theoretical principle beyond renormalizability
- Dynamical dark energy and naturally flat potentials for inflation
- Modify gravity adopting scale invariance as a guiding principle
- Noether’s current conservation for single-field dynamics

BACKUP SLIDES

FUNDAMENTAL SCALE INVARIANCE

COSMOLOGY AS AN UV-IR CROSSOVER

Fundamental scale invariance → **Quantum scale invariance**

The effective action only depends on one scale k which can be reabsorbed into a fields' redefinition

The effective action carries no scale at all

- $k \to \infty$: UV fixed point
- $k \to 0$: IR fixed point

All couplings $g(\tilde{\phi})$ become constant and dimensionless

Cosmology as a crossover between an UV (inflation) and an IR fixed point (we are already close to it)

INFLATIONARY PREDICTIONS

PRIMORDIAL SPECTRA

Even with non-zero initial velocity the Goldstone boson does not contribute

$$\rho'(N) \sim e^{-3N} \to 0$$

Single-field predictions are recovered, both in the Jordan and the Einstein frame

Scalar perturbations

C. Cecchini & M. Rinaldi in preparation

$$\Delta_s^2(k) = \frac{1}{2M_{pl}^2\,\epsilon}\left(\frac{H}{2\pi}\right)^2\Bigg|_{k=aH}$$

$$n_s - 1 \approx -\,6\,\epsilon(\zeta_*) + 2\eta(\zeta_*)$$

Tensor perturbations

A. Ghoshal, D. Mukherjee, & M. Rinaldi JHEP ***5*** *(2023)*

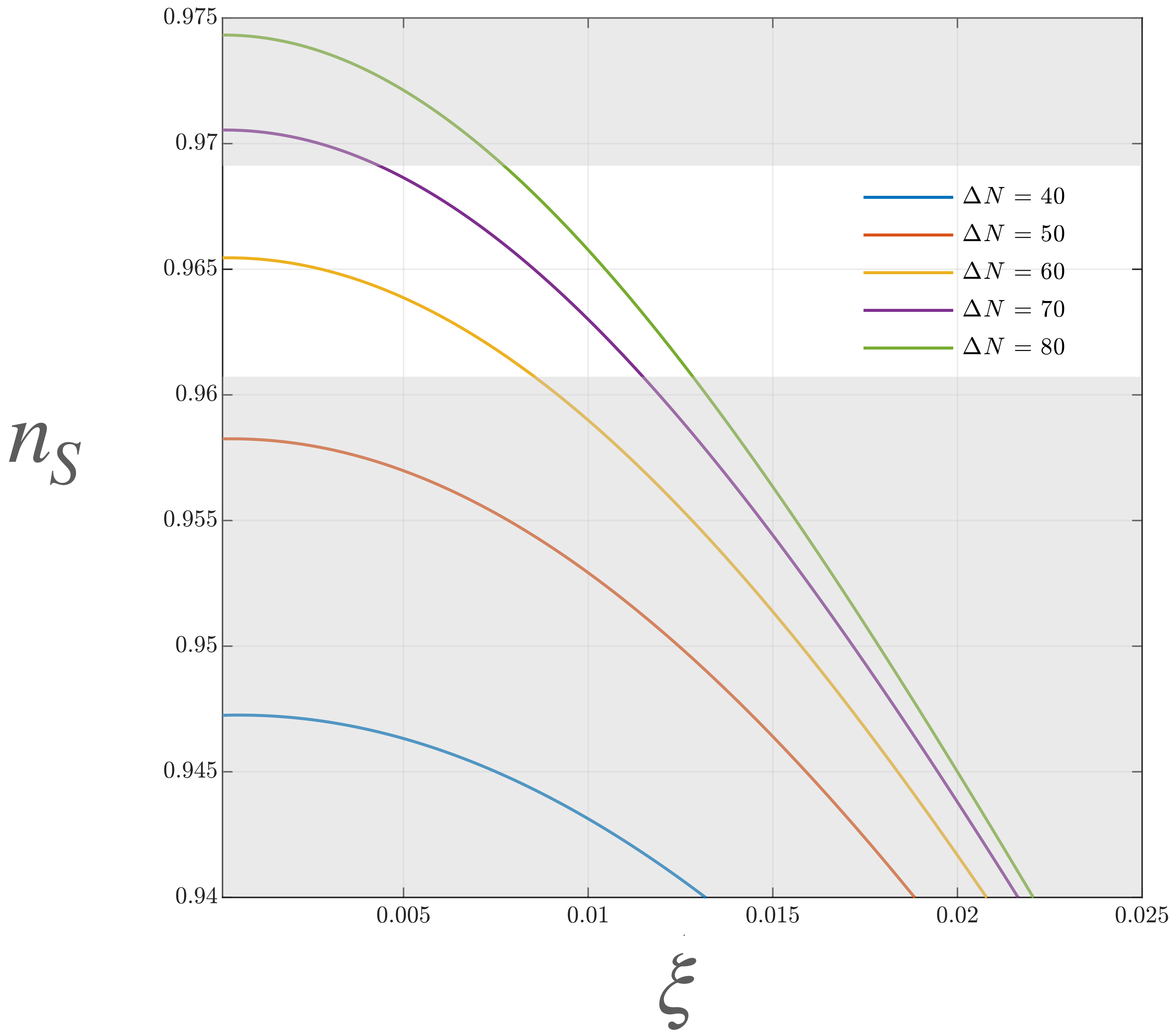
$$\Delta_t^2(k) = \frac{2}{\pi^2}\left(\frac{H}{M_{pl}}\right)^2\Bigg|_{k=aH}$$

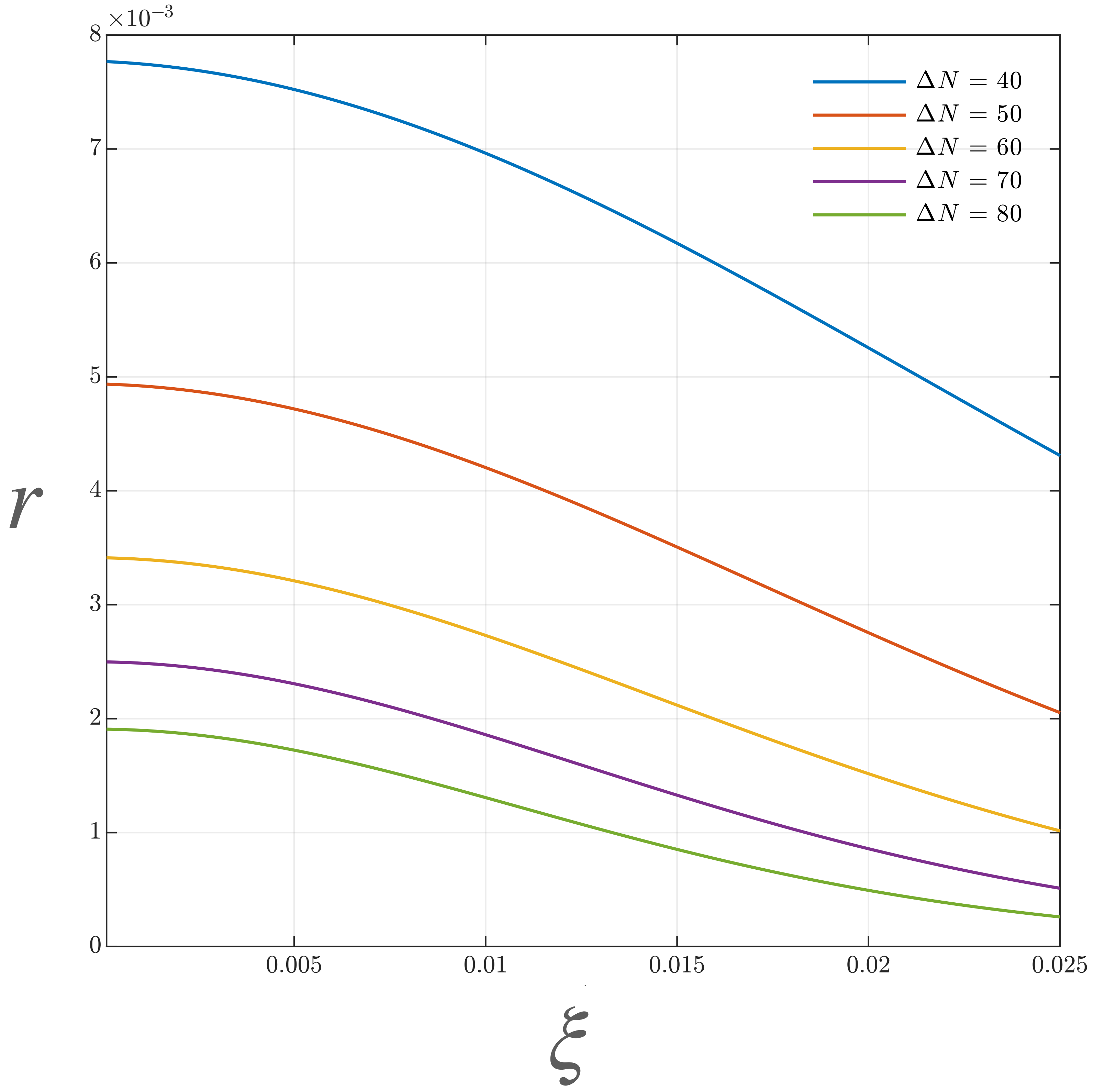
$$r \approx -\,16\,\epsilon(\zeta_*)$$

INFLATIONARY PREDICTIONS

SPECTRAL INDICES

- $\Omega = \alpha\lambda + \xi^2 \lesssim 1.15\,\xi^2$
- $\alpha \gtrsim 2 \times 10^{10}$
- $\xi \lesssim 1.3 \times 10^{-2}$
- $\Delta N \gtrsim 55$





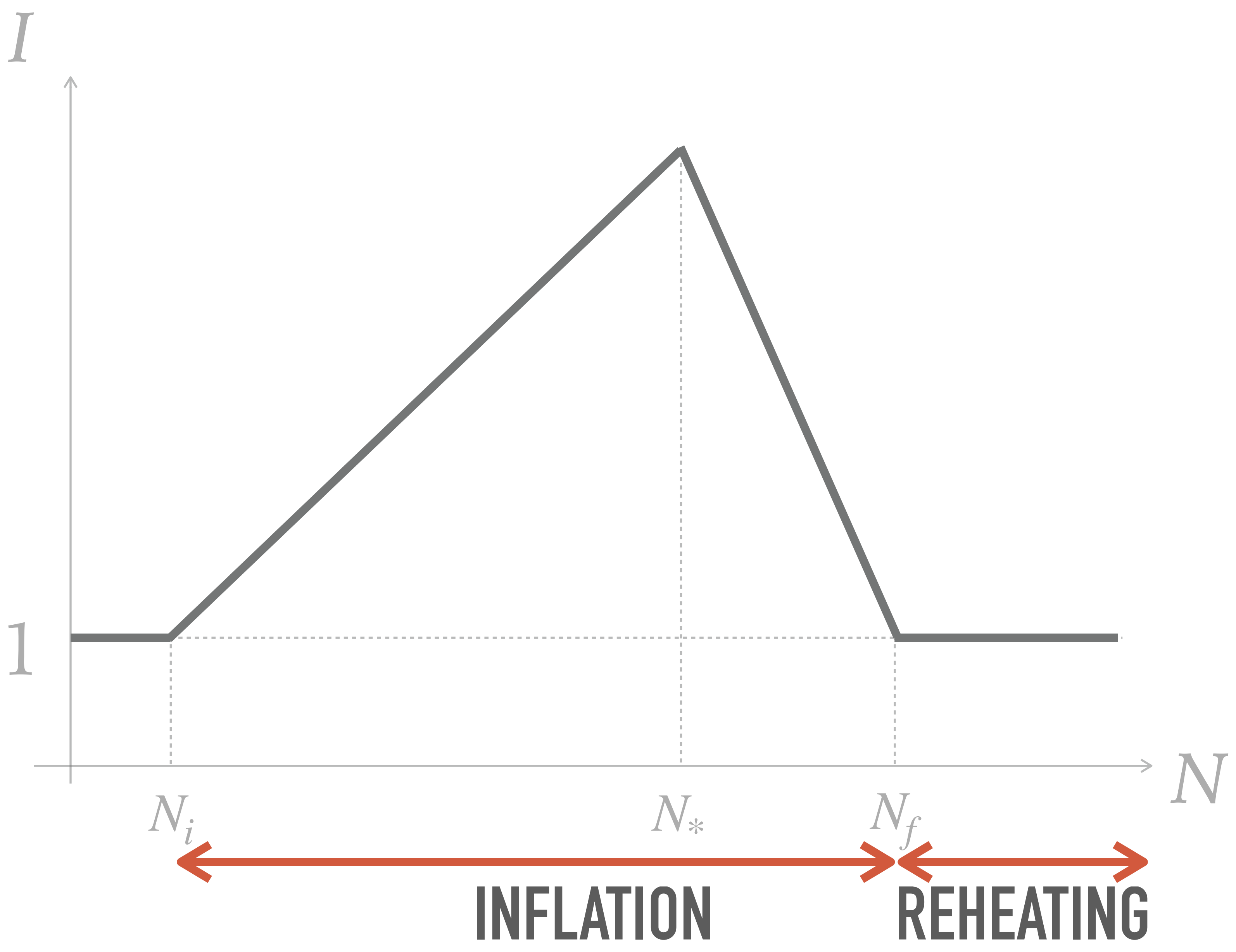
INFLATIONARY PREDICTIONS

MAGNETOGENESIS

C. Cecchini & M. Rinaldi Phys Dar Univ ***40*** *(2023)*

Modify the Maxwell's action and add helicity to generate primordial magnetic fields through a sawtooth coupling to the inflaton: EM conformal invariance is broken only during inflation → amplification of vector perturbations

$$S = -\frac{1}{16\pi}\int d^4x\sqrt{-g}\; I^2[\zeta(t)]\left[F_{\mu\nu}F^{\mu\nu} - \gamma F_{\mu\nu}\tilde{F}^{\mu\nu}\right] + \int d^4x\sqrt{-g}\,\mathscr{L}_E$$



$$I = \begin{cases} \mathscr{C}\left(\dfrac{a}{a_*}\right)^{\nu_1} & a_i > a > a_* \\[2ex] \mathscr{C}\left(\dfrac{a}{a_*}\right)^{-\nu_2} & a_* > a > a_f \end{cases}$$

INFLATIONARY PREDICTIONS

MAGNETOGENESIS

C. Cecchini & M. Rinaldi Phys Dar Univ ***40*** *(2023)*

Present-day magnetic field's amplitude and coherence length compatible with bounds on the IGM fields