Subdominant spin effects in black-hole binaries

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Outline

Overview of two subdominant spin effects in current and future LIGO/Virgo data:

I. Detecting the signature imprinted by **two precessing spins** on the emitted GW signal using a carefully designed estimator

[De Renzis+, PRD, 2022]

II. Prospects for **detecting unstable binaries** which formed with aligned spins but enter the detector sensitivity window in precessing configurations

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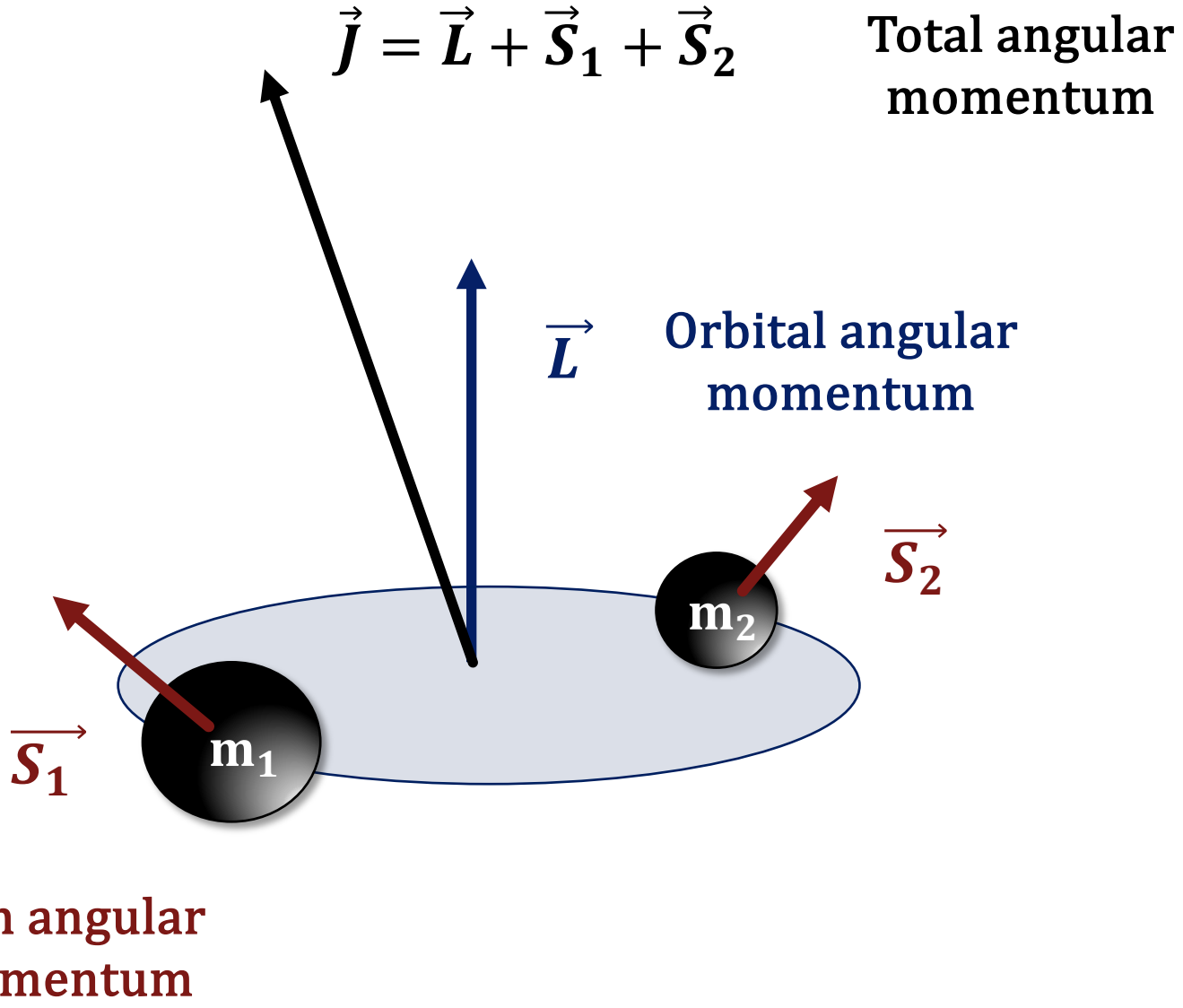
Generic binary black holes

- Mass ratio: $q = \frac{m_1}{m_2} \leq 1$
- Orbital separation: r
- Total mass: $M = m_1 + m_2$
- Dimensionless spins: $-1 \leq \chi_1, \chi_2 \leq 1$

$$\vec{\chi}_i = \frac{\vec{S}_i}{m_i^2}, \quad (G = c = 1)$$



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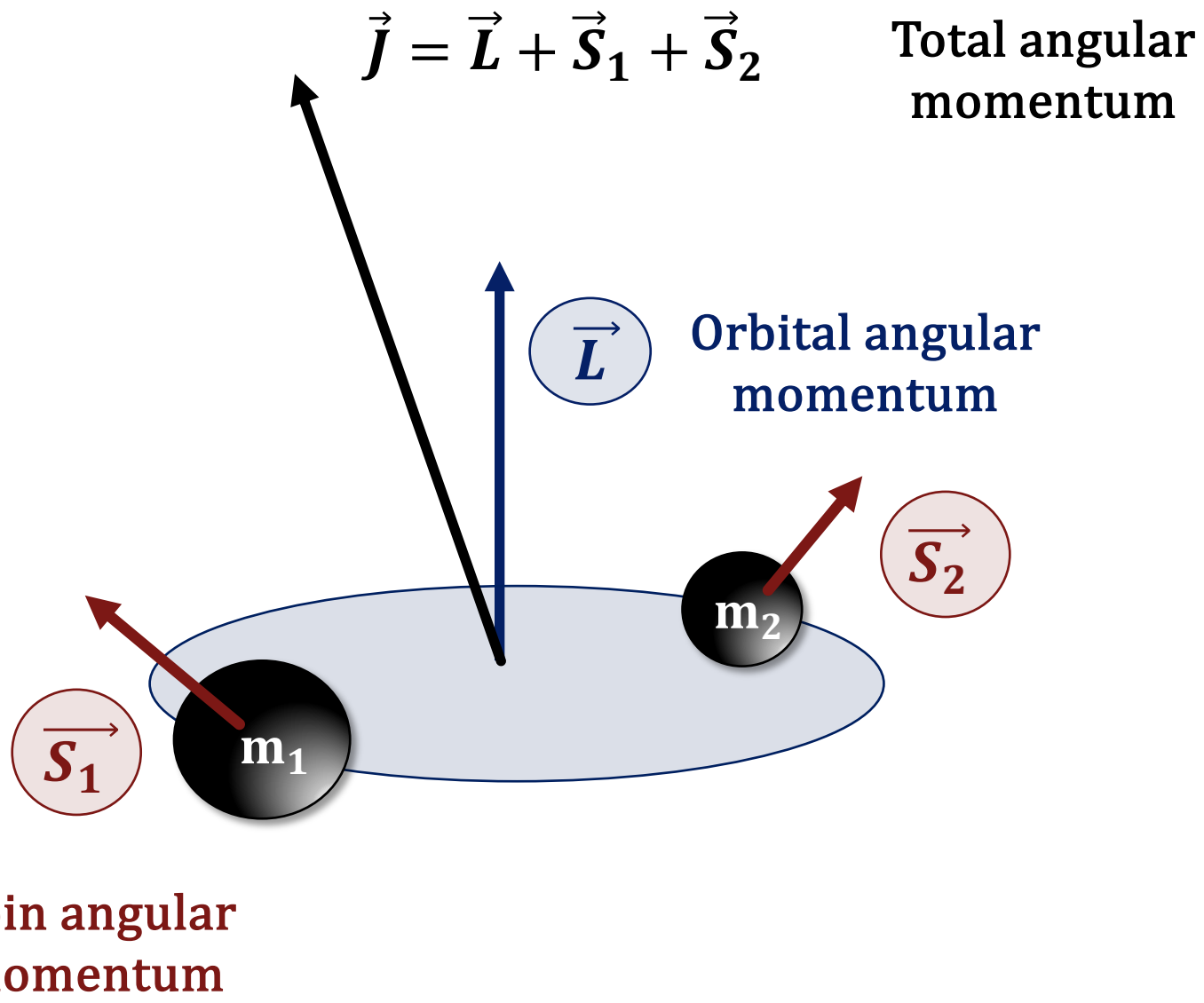

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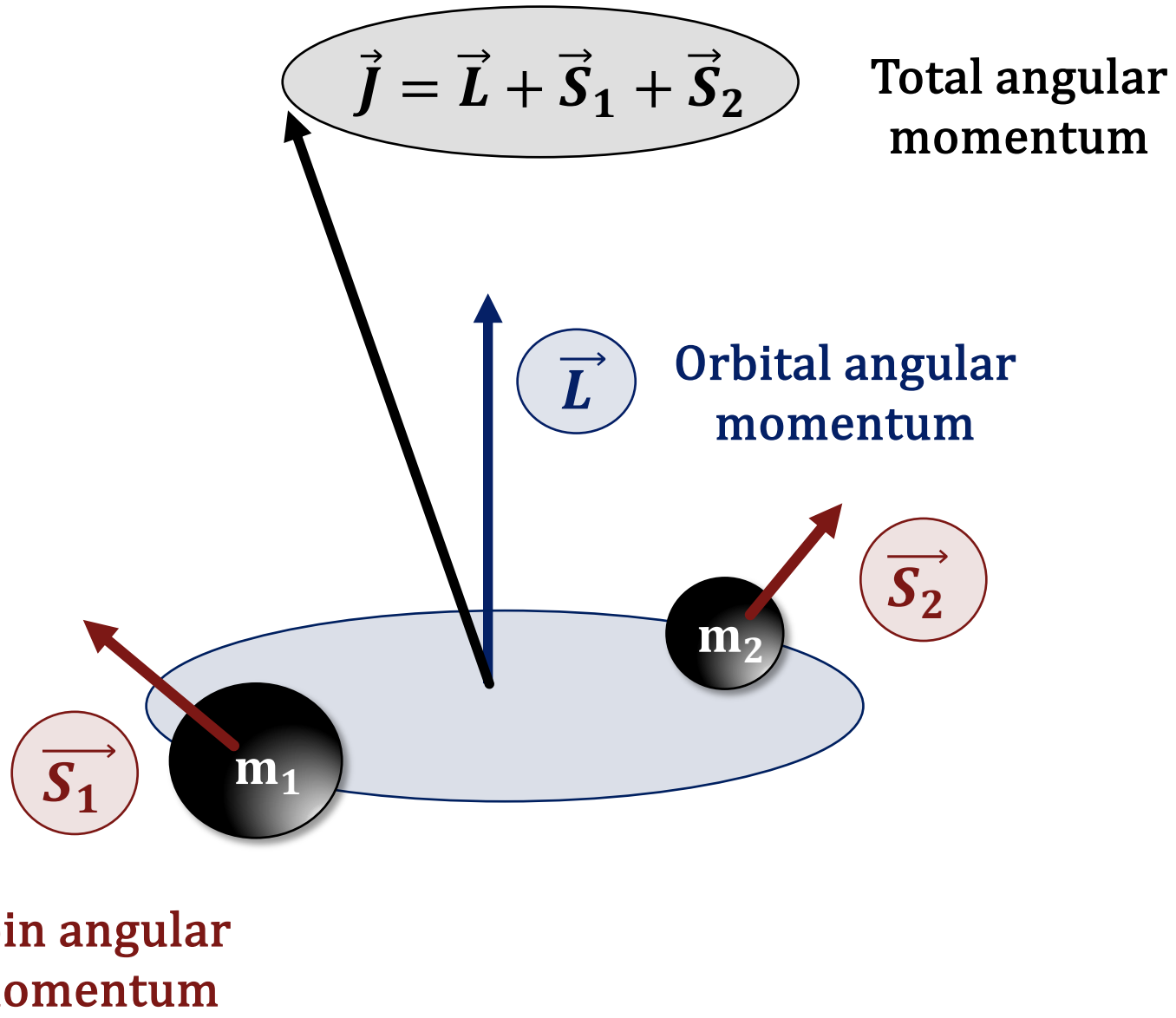
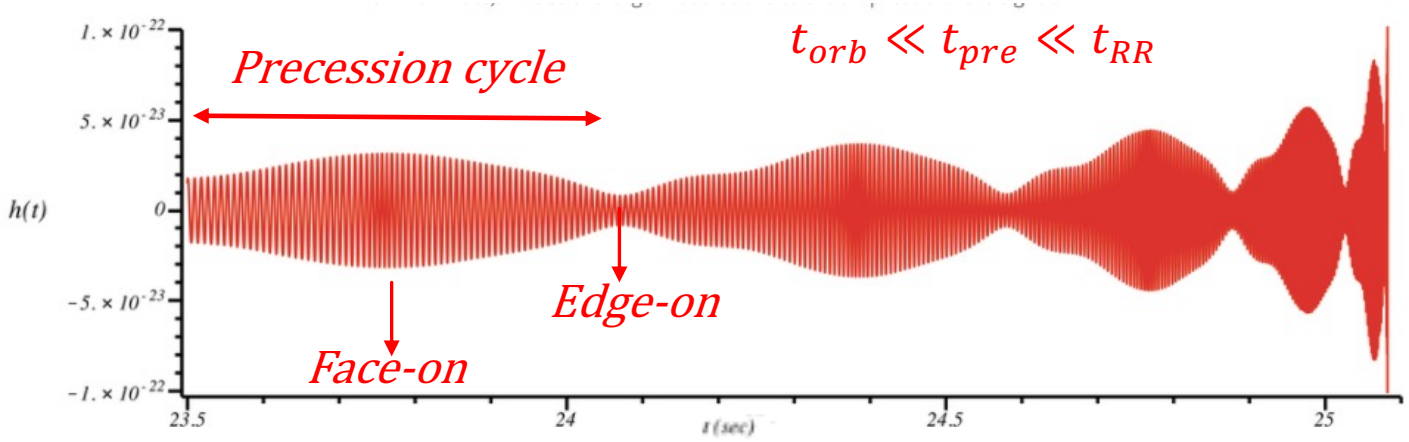
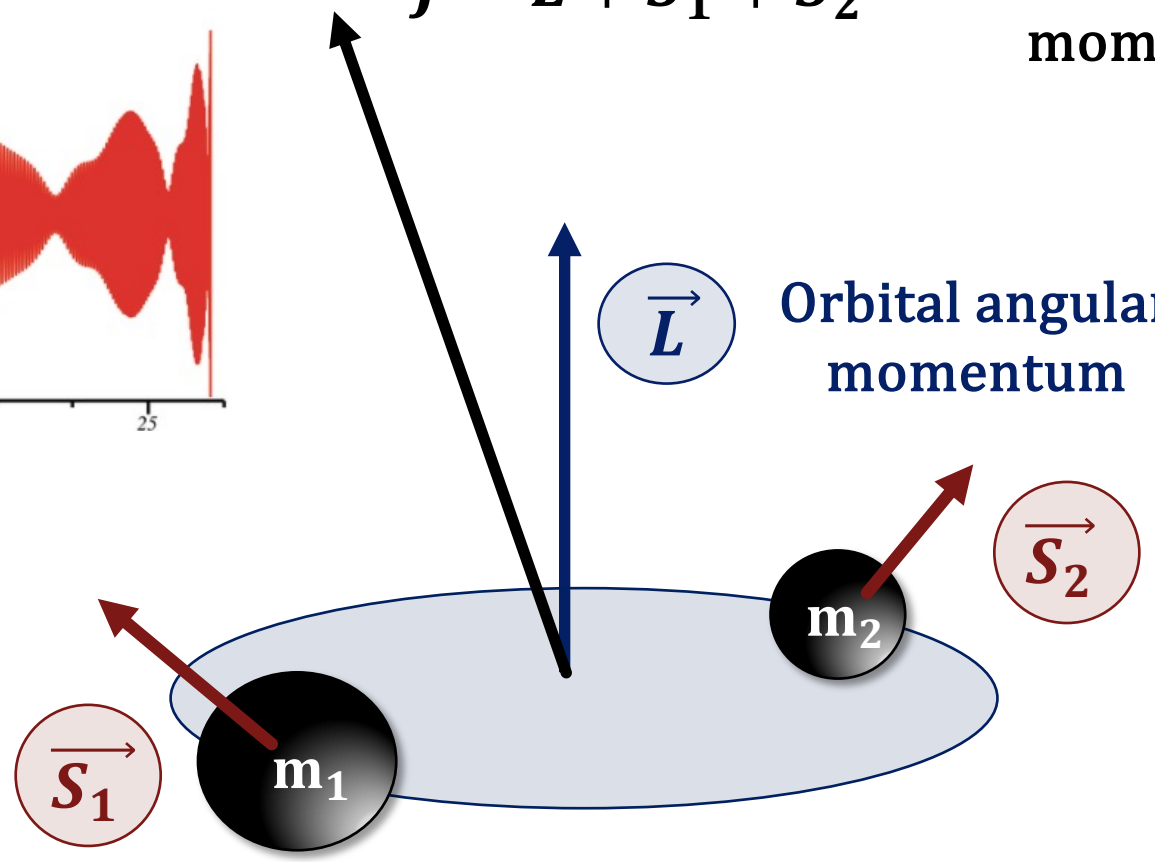


Misaligned spins→ spin precession



$$\vec{J} = \vec{L} + \vec{S}_1 + \vec{S}_2$$

Total angular momentum

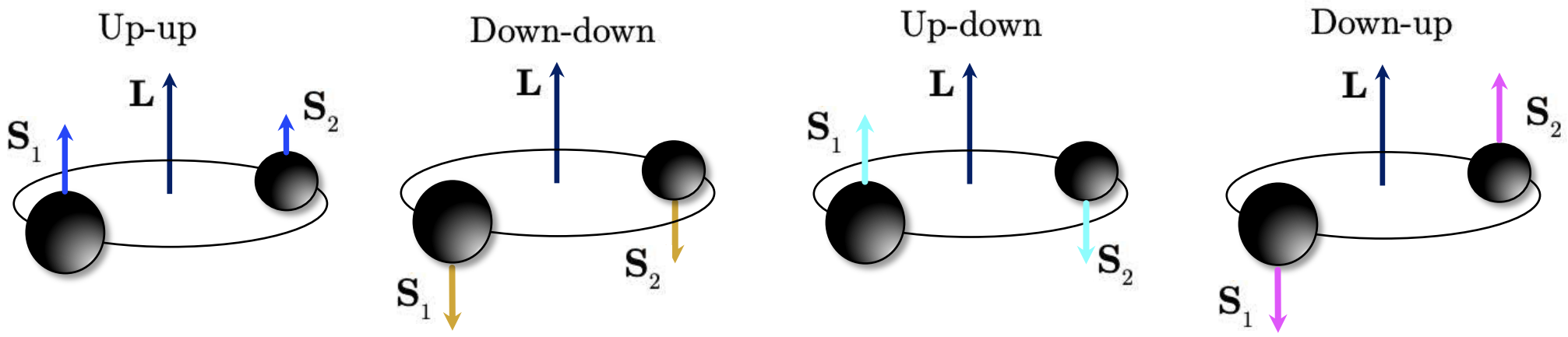


Spin angular momentum

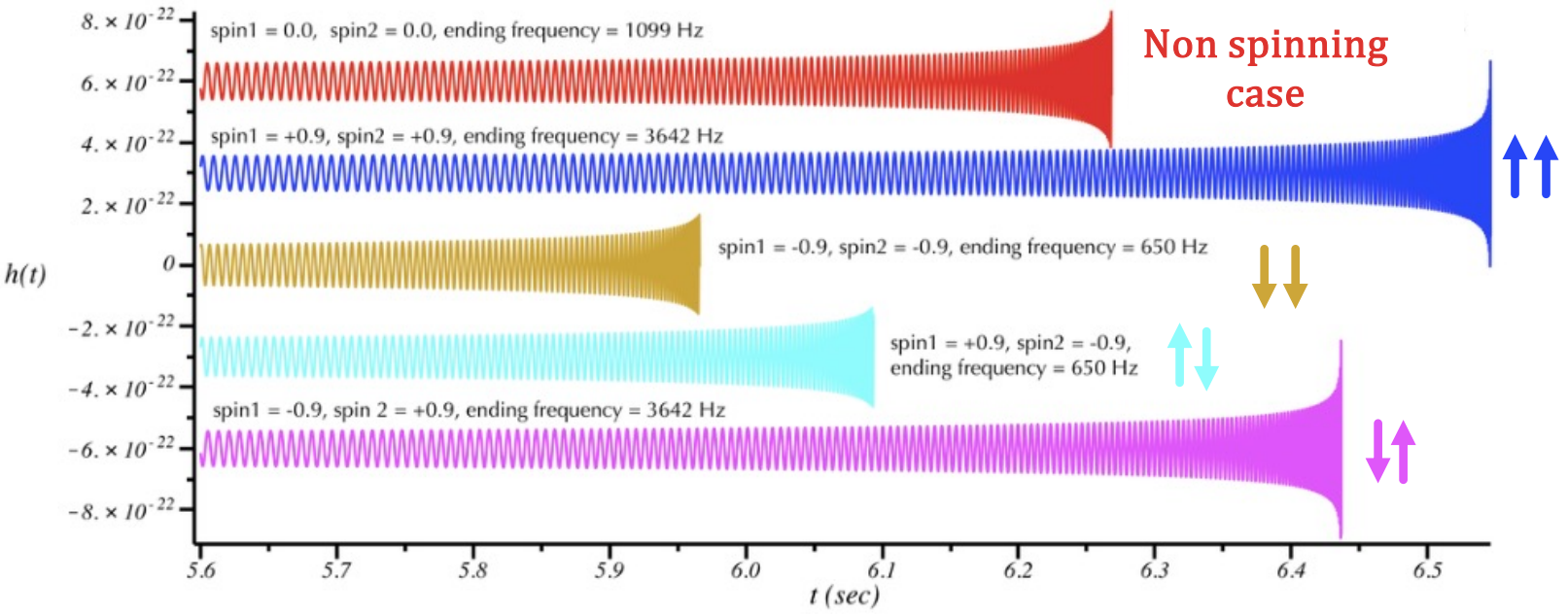
- Modulation in amplitude and phase
- Change in direction of the GW emission

Aligned spins → no precession

Four equilibrium solution to the relativistic spin-precession equation:



[M. Favata]



- Similar morphologies
- Different merger frequencies and signal durations

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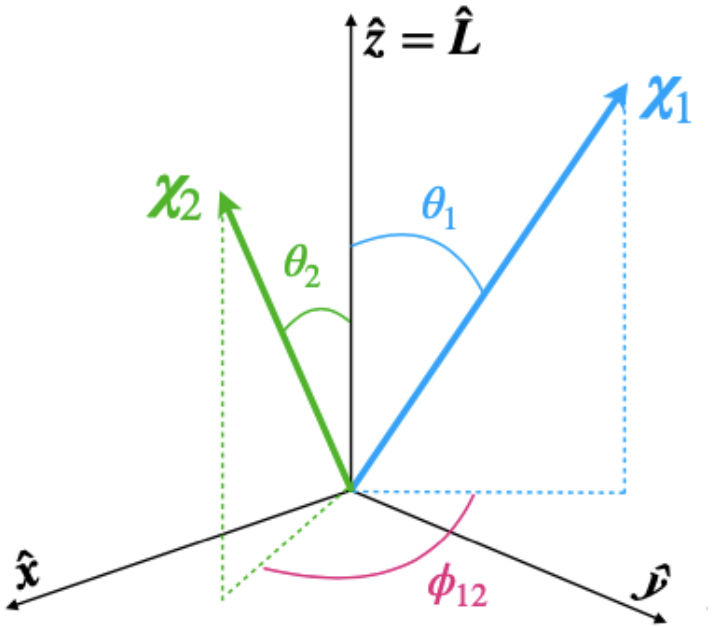
Effective spin parameters

(Biscoveanu+ 2021)



$$\overrightarrow{\chi_{12}} = \overrightarrow{S_{12}}/m_{12}^2$$
$$q = m_2/m_1$$

Alignment information:

$$\chi_{\text{eff}} = \frac{m_1\overrightarrow{\chi_1} + m_2\overrightarrow{\chi_2}}{m_1 + m_2} \cdot \hat{L}$$

- Only aligned spin components (Damour 2001)
- Constant of motion at 2PN (Racine 2006)

Precessing spin parameter

$$\chi_{\text{p}} = \max\left(\chi_1 \sin\theta_1 , q\,\frac{4q+3}{(4+3q)}\chi_2 \sin\theta_2 \right)$$

- In-plane spin components (Schmidt+ 2015)
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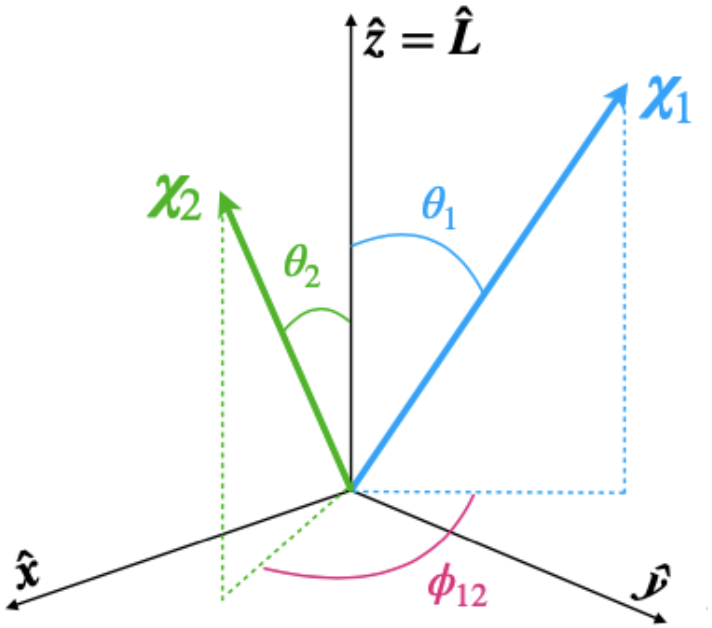

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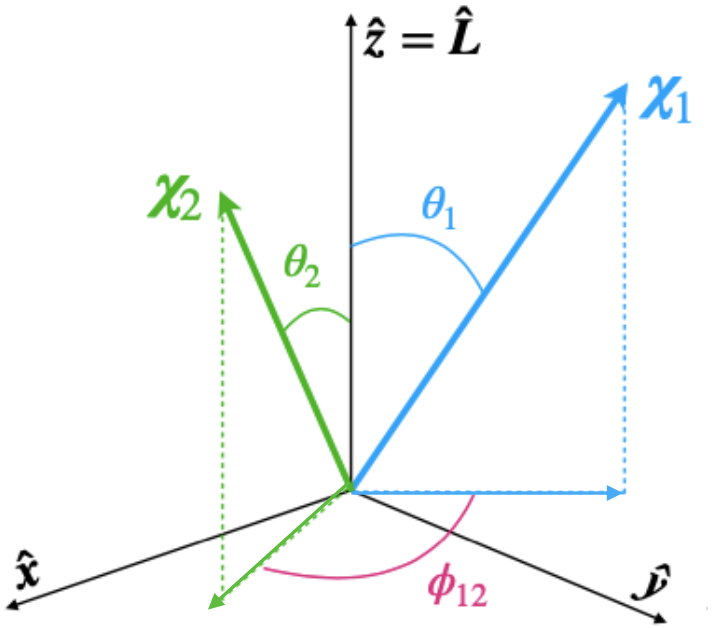

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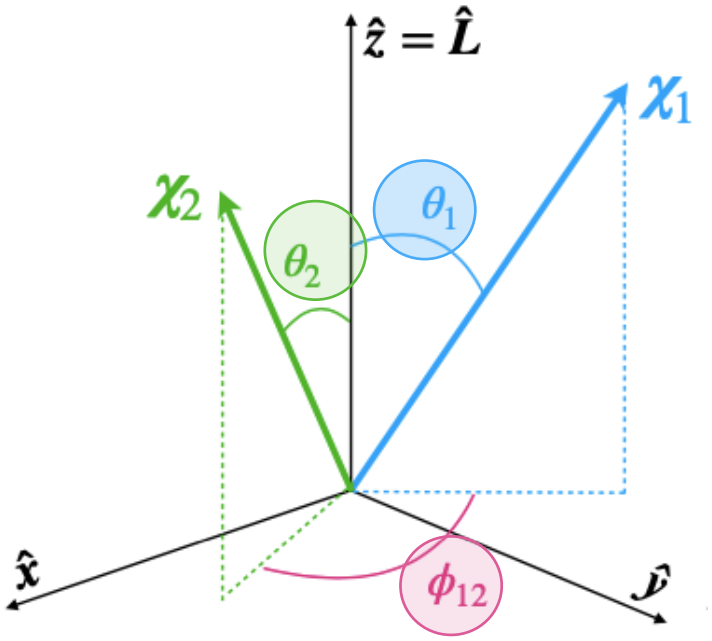
Averaged vs Heuristic χ_p



$\theta_1, \theta_2, \phi_{12}$ all vary on the precession timescale!

(Kesden+ 2015,Gerosa+ 2015)

Heuristic

$$\chi_{\rm p}^{\rm heu} = \max\left(\chi_1 \sin\theta_1 \,, q\,\frac{4q+3}{(4+3q)}\chi_2 \sin\theta_2\right)$$

- Takes information from only one of the two objects.
- Retains some, but not all the variations taking place on the precession timescale.
- Domain: $0 < \chi_{\rm p}^{\rm heu} < 1$

Averaged

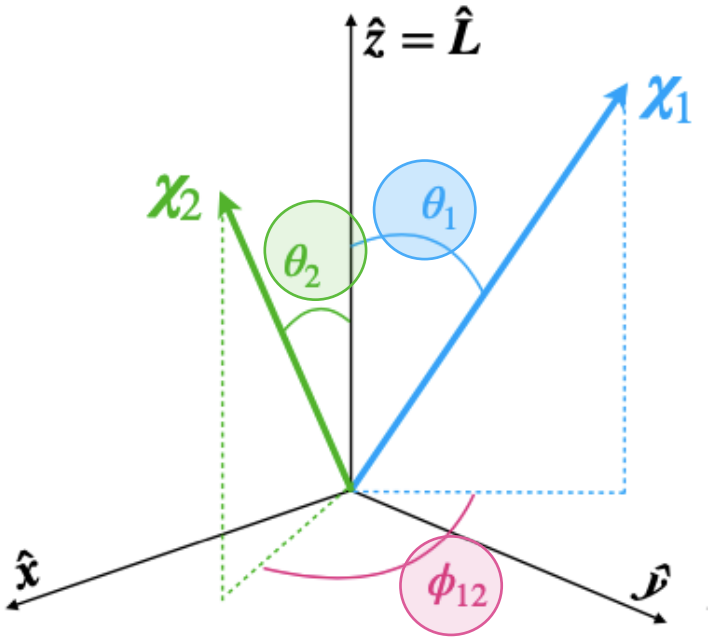
$$\langle \chi_{\rm p}^{\rm av} \rangle = \frac{\int \chi_p(\psi) \left(\frac{d\psi}{dt}\right)^{-1} d\psi}{\int \left(\frac{d\psi}{dt}\right)^{-1} d\psi}$$

(Gerosa+ 2020

$$(\chi_1 \sin\theta_1)^2 + \left(q\,\frac{4q+3}{(4+3q)}\chi_2 \sin\theta_2\right)^2 + 2q\,\frac{4q+3}{(4+3q)}\chi_1\chi_2 \sin\theta_1 \sin\theta_2 \cos\phi_{12}$$

- Averages over all the variations occurring on the precession timescale.

Averaged vs Heuristic χ_p



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(Gerosa+ 2020

- Take information from both objects.
- Averages over all the variations occurring on the precession timescale.
- Domain: $0 < \chi_{\rm p}^{\rm av} < 2$

Two spin-effects: $1 < \chi_{\rm p}^{\rm av} < 2$

Is it possible to detect two-spin effects with the new averaged definition of $\chi_{\mathrm{p}}^{\mathrm{av}}$?

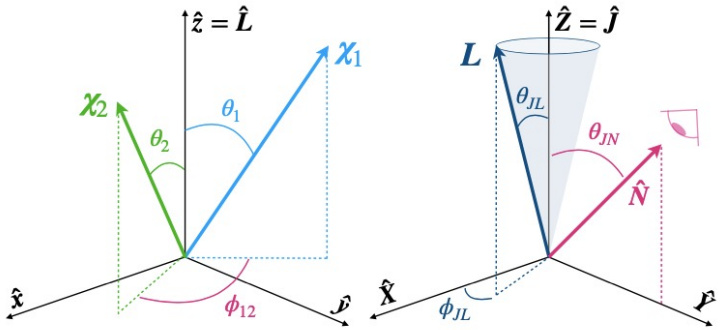
Full parameter estimation with BILBY

Bayesian inference framework for parameter estimation

Parameter describing a BBH merger

8 intrinsic parameters:
$m_1, m_2, \chi_1, \chi_2, \theta_1, \theta_2, \phi_{12}, \phi_{JL}$

7 extrinsic parameters:
$d_L, ra, dec, \psi, \phi, \theta_{JN}, t_C$



Settings for the injections

Detectors: *H,L,V (O4)*
Gaussian noise: no
Detector arguments: $f_{ref} = 20.0$ Hz
$f_{min} = 20.0$ Hz
$f_{sampling} = 2048.0$ Hz
$duration = 4$s
Injection and recovery waveform: *IMRPhenomXPHM*
Priors: *Standard BBH priors*
Sampler: *dynesty*

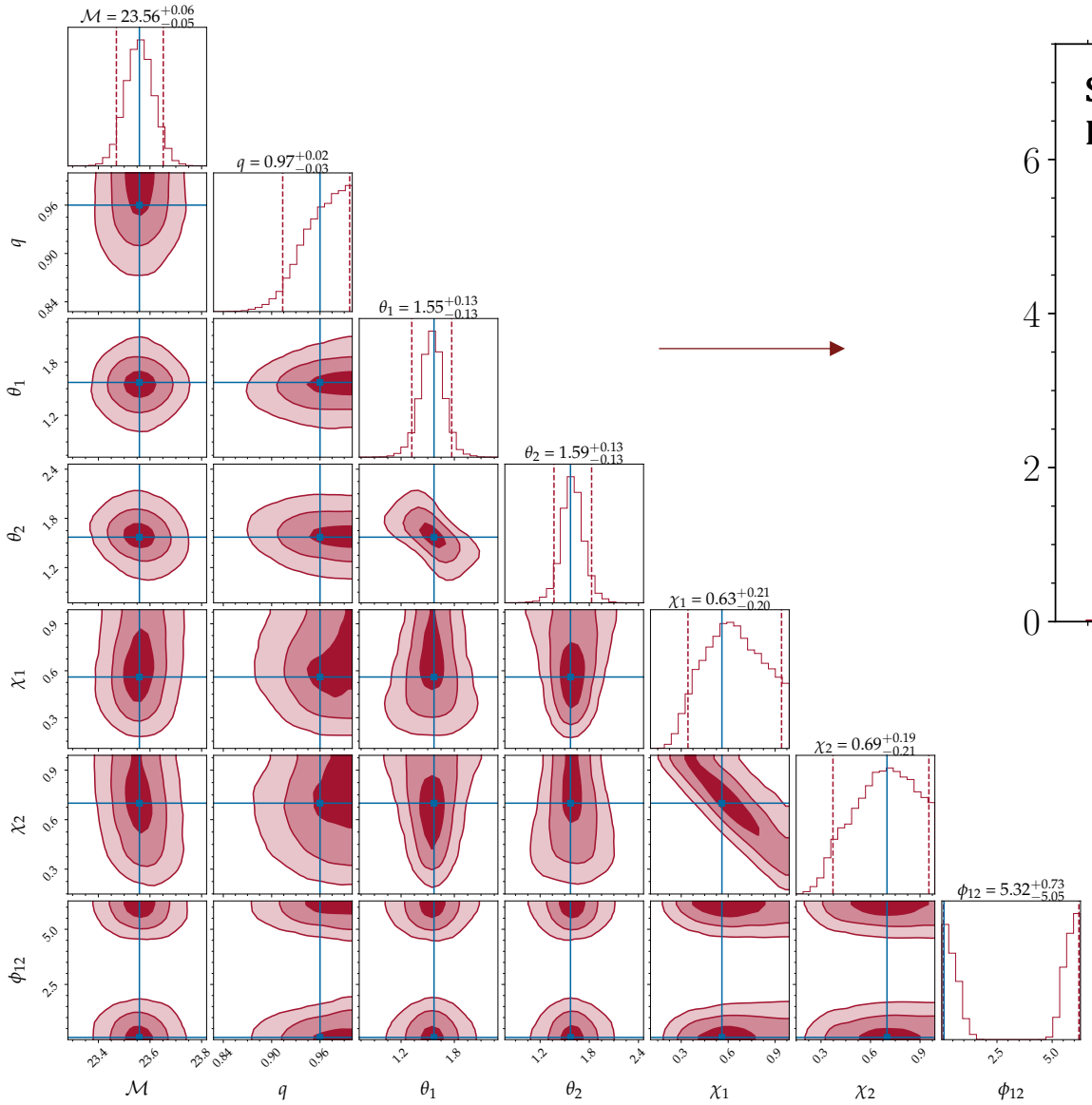
Recovery of χ_p

15 injected parameters

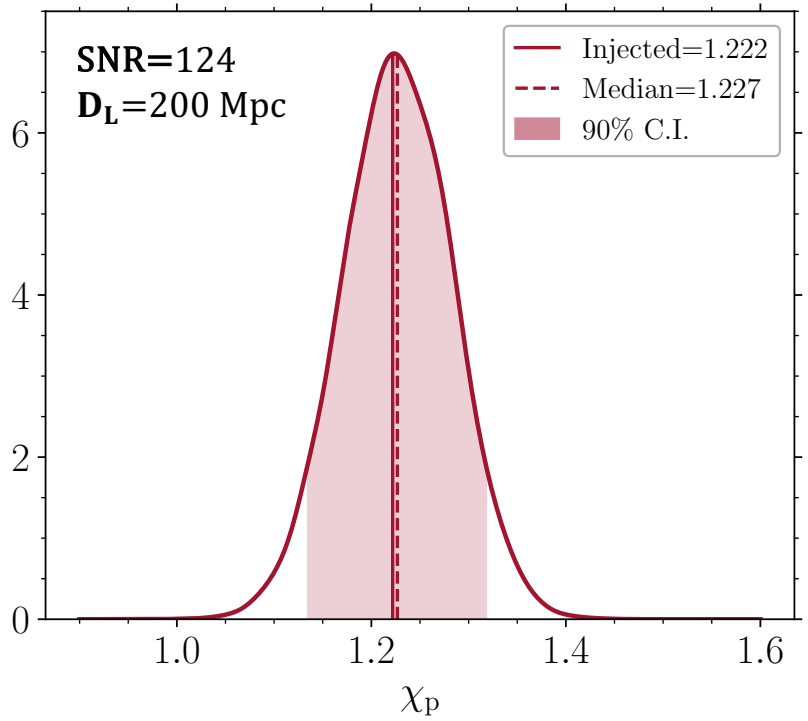
$M_{1,detector} = 27.6$
$M_{1,detector} = 26.5$
$\boldsymbol{\theta_1 = \pi/2}$
$\boldsymbol{\theta_2 = \pi/2}$
$\phi_{12} = 0.1$
$\chi_1 = 0.56$
$\chi_1 = 0.7$
$d_L = 200$ Mpc
$\theta_{JN} = 1.0$
$\phi_{JL} = 1.0$
$dec = 0.75$
$ra = 0.5$
$\psi = 1.0$
$t_{geocent} = 0.0$
$\phi = \pi/4$

q=0.96

IMRPhenomXPHM
Standard BBH priors



Average χ_p



Two-spin effects!

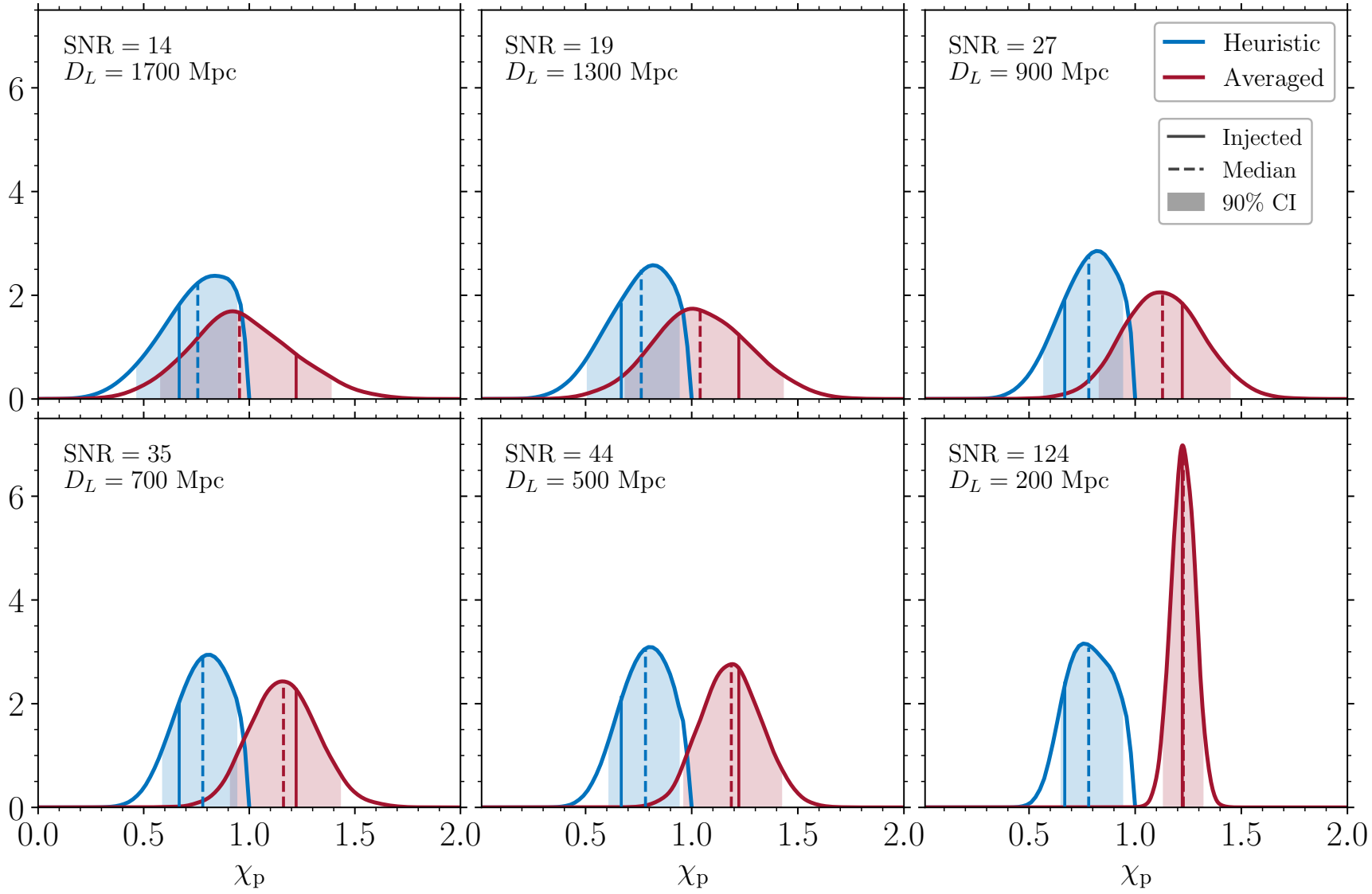
Recovery at different SNR

VARIABLE PARAMETERS:

- Source frame masses $M_{1,2}^{source}$
- D_L = [200, 500, 700, 900, 1300, 1700] Mpc → SNR=[124, 44, 35, 27, 19, 14]

FIXED PARAMETERS:

- Detector-frame masses. $M_{1,2}^{det} = [27.6, 26.5]$
  → Average $\chi_p^{inj} = 1.22$
  → Heuristic $\chi_p^{inj} = 0.67$
- All the other parameters are fixed



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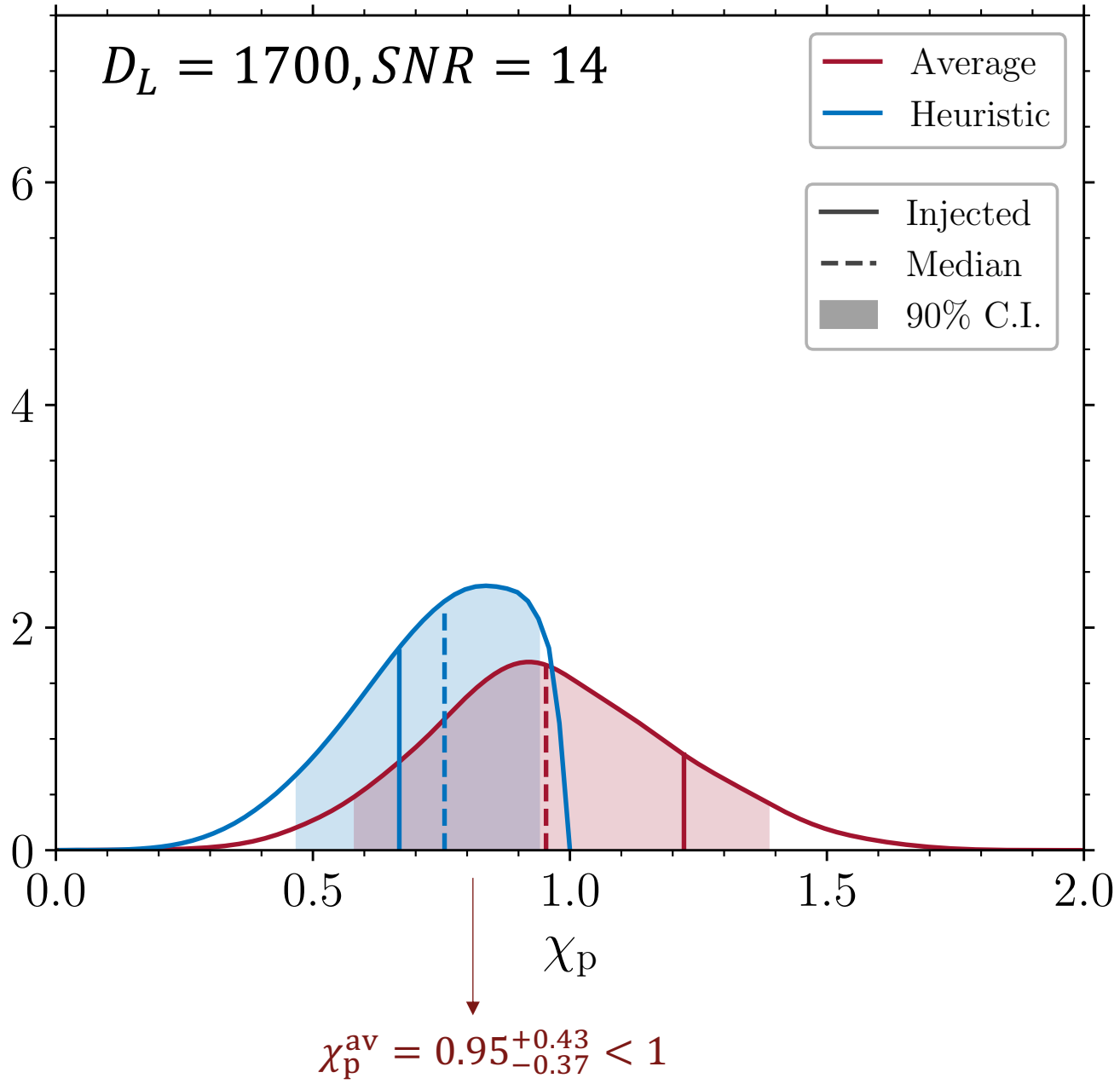

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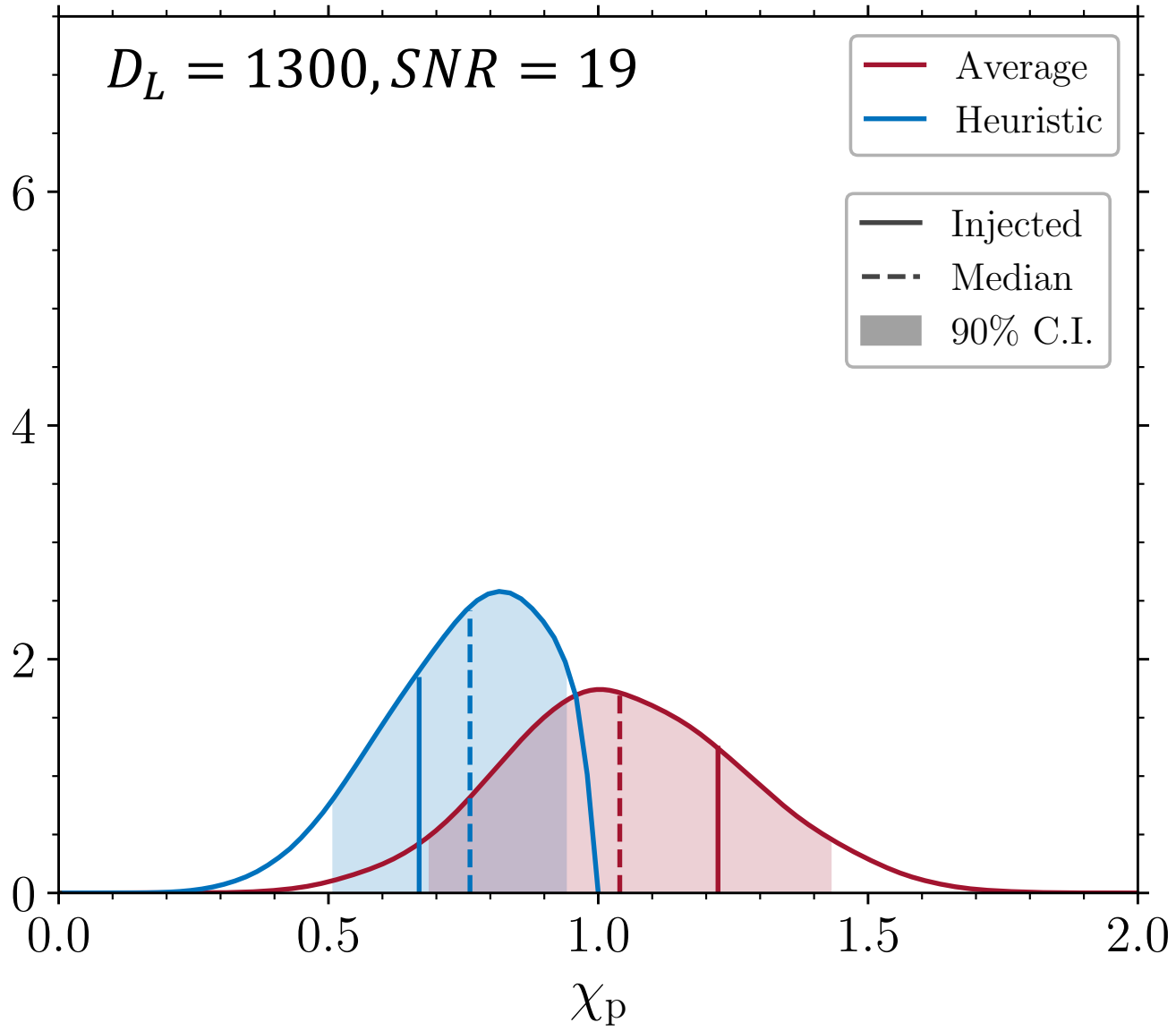

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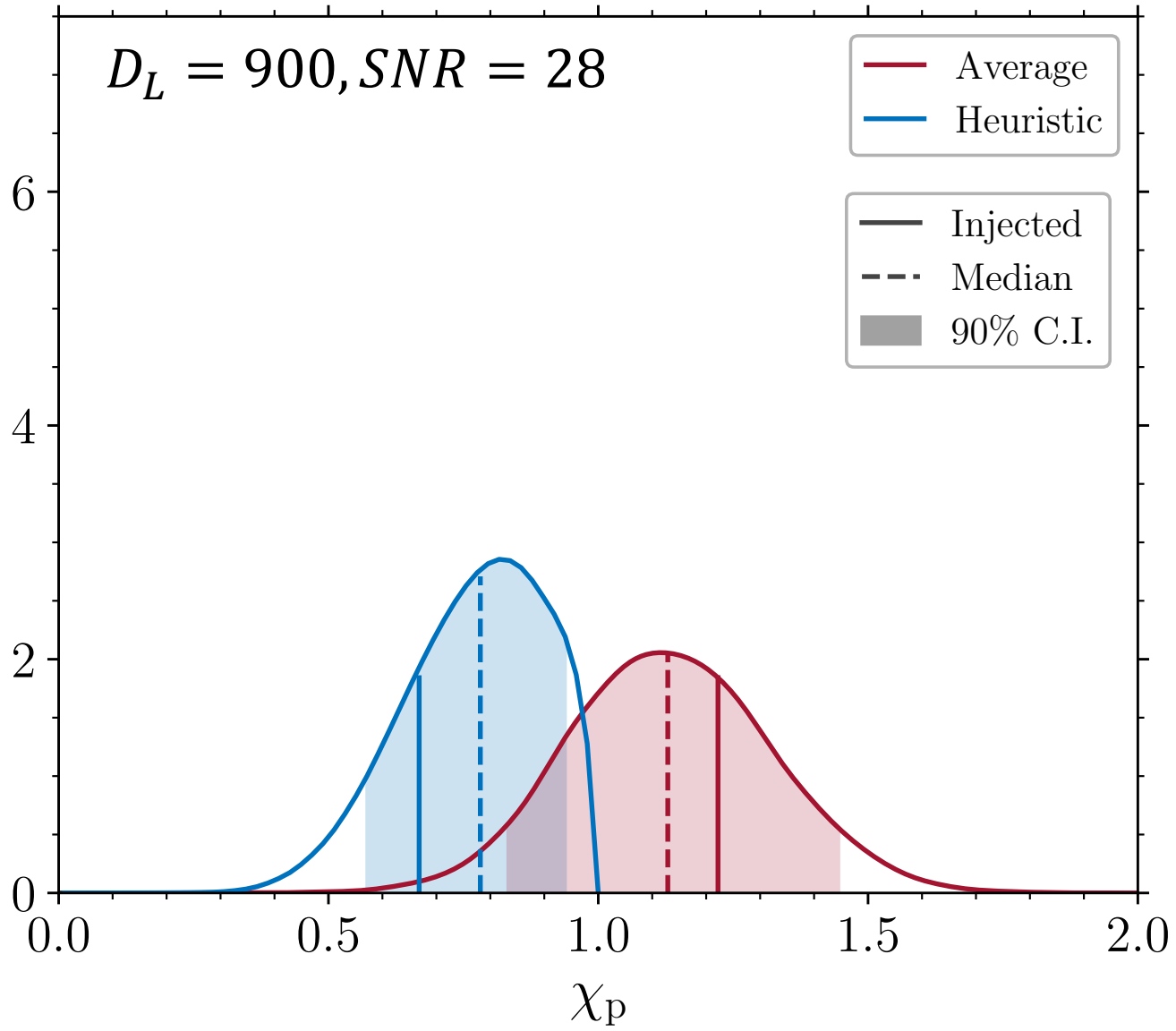

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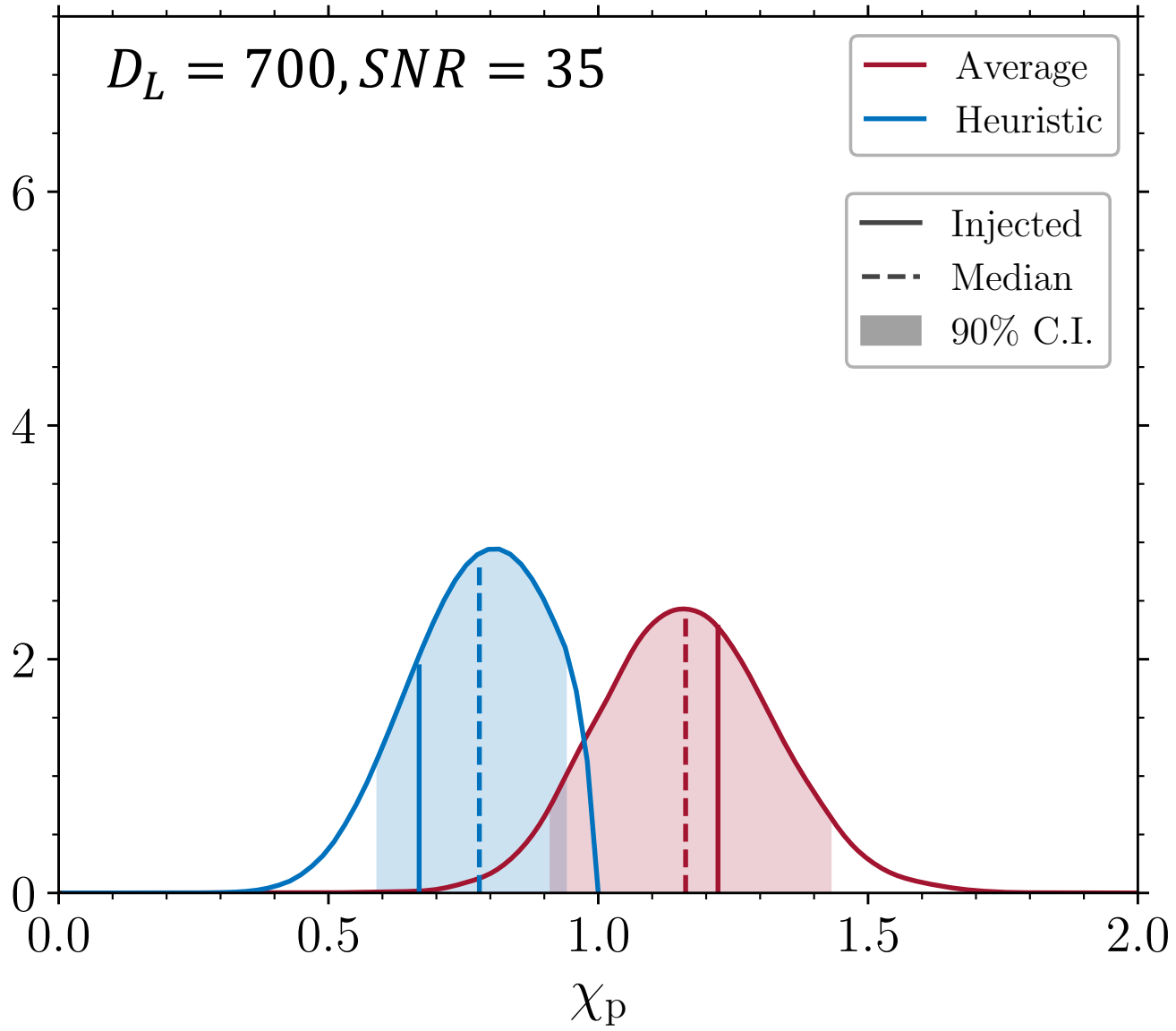

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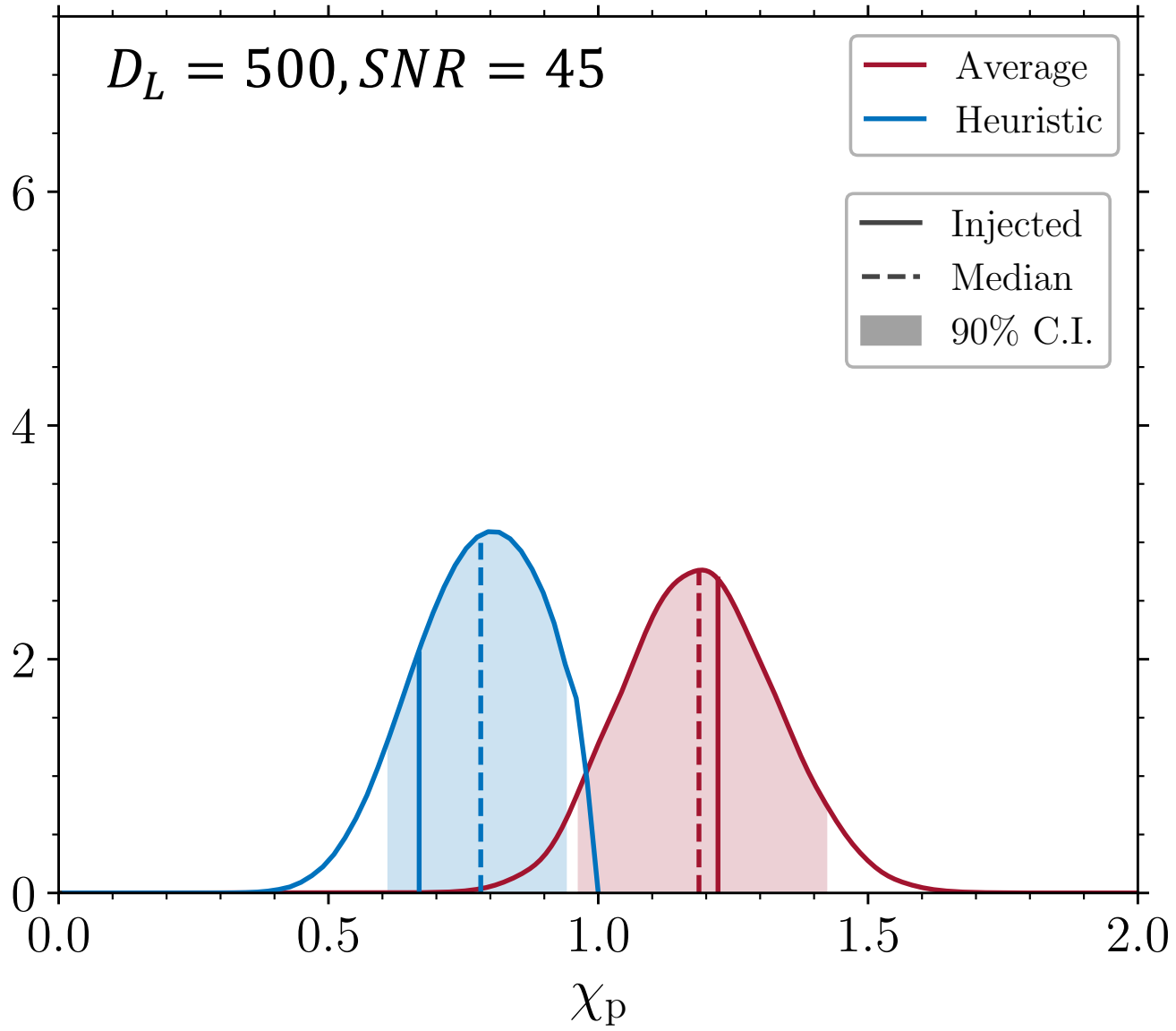

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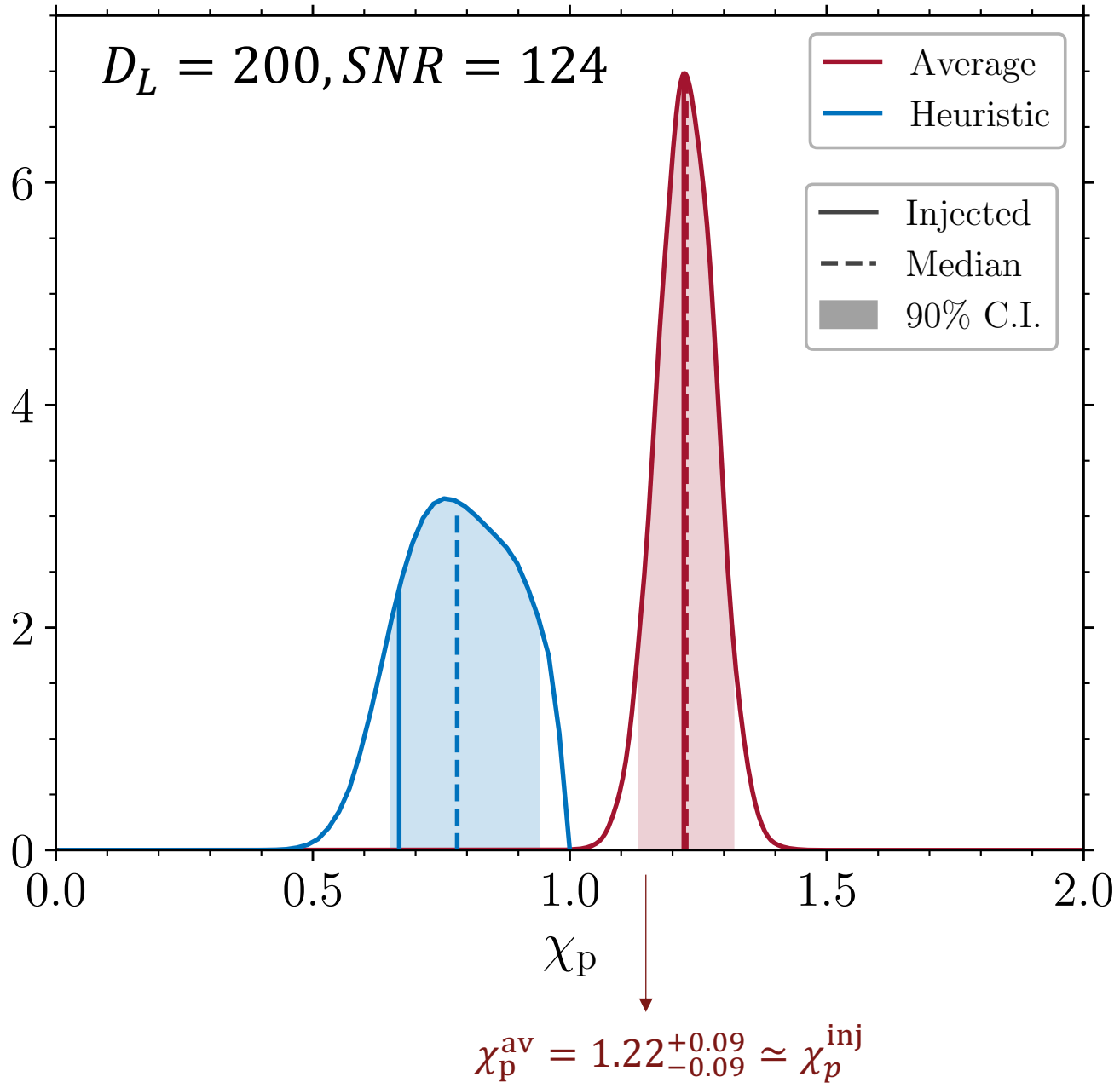

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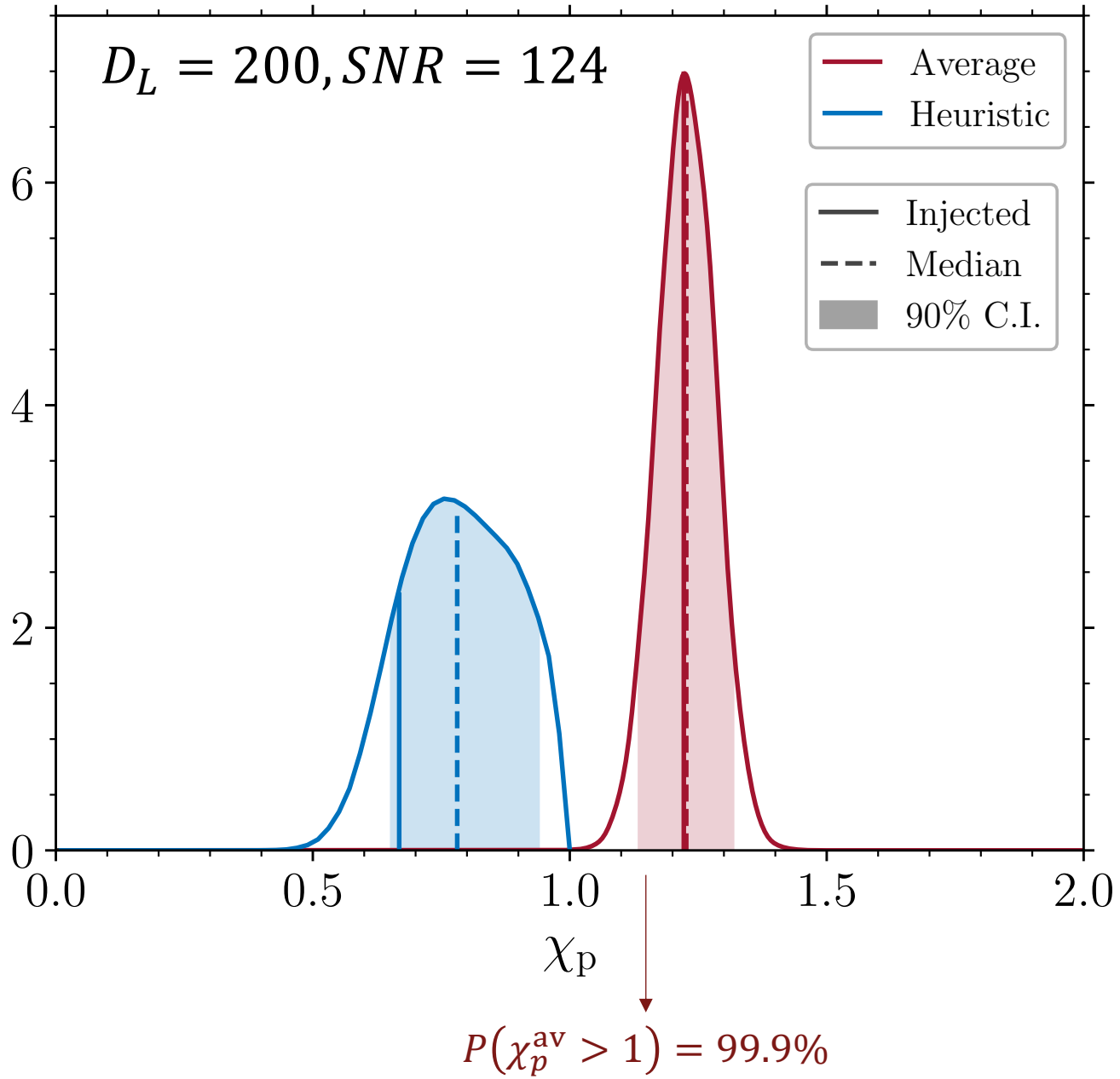
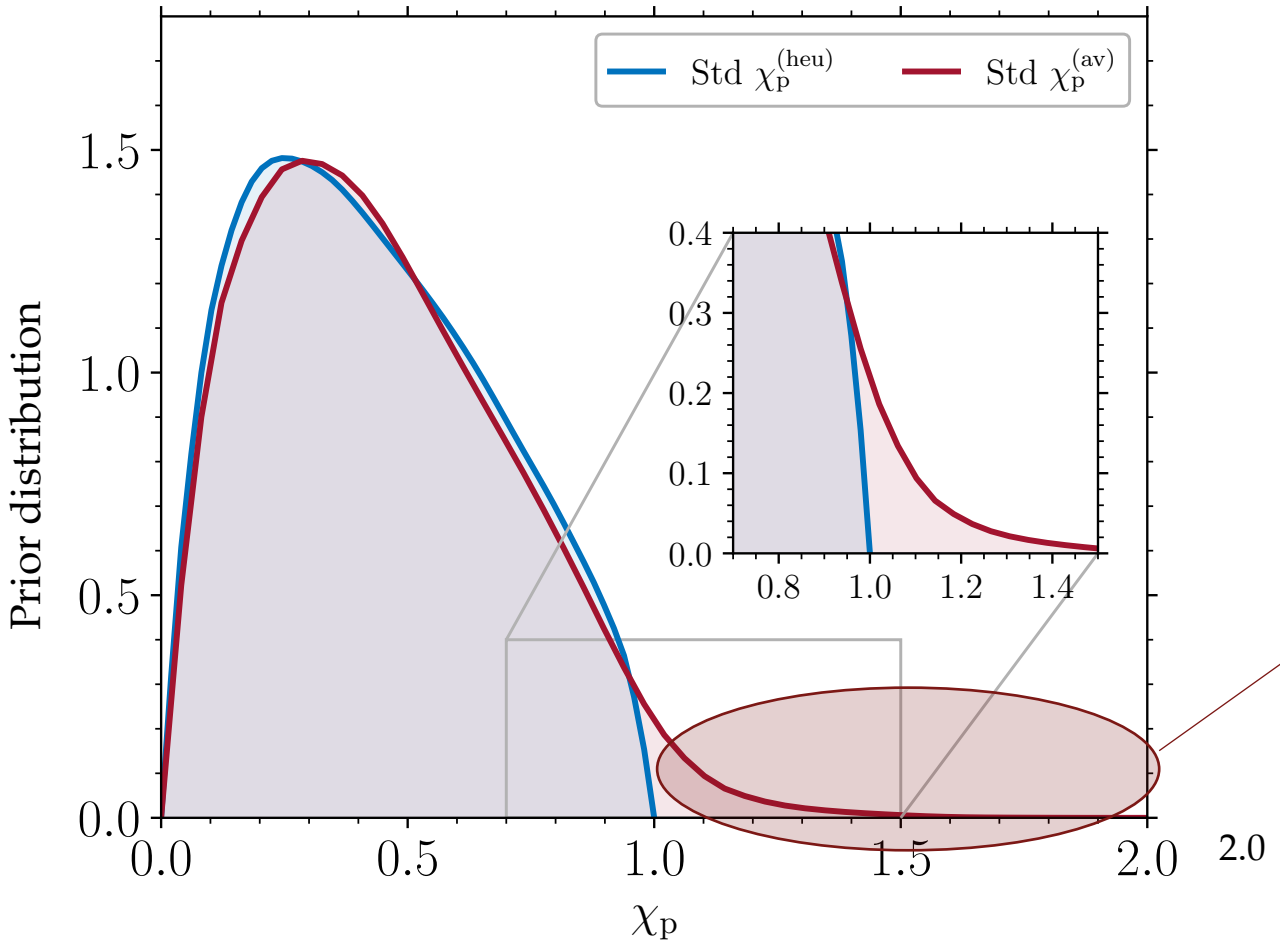


Parameter estimation analysis for 100 injections

METHOD I: take samples for all the 15 parameters from the standard BBH priors



Highly prior-suppressed region: few events in the domain of interest

$P(\chi_p > 1) = 2\%$

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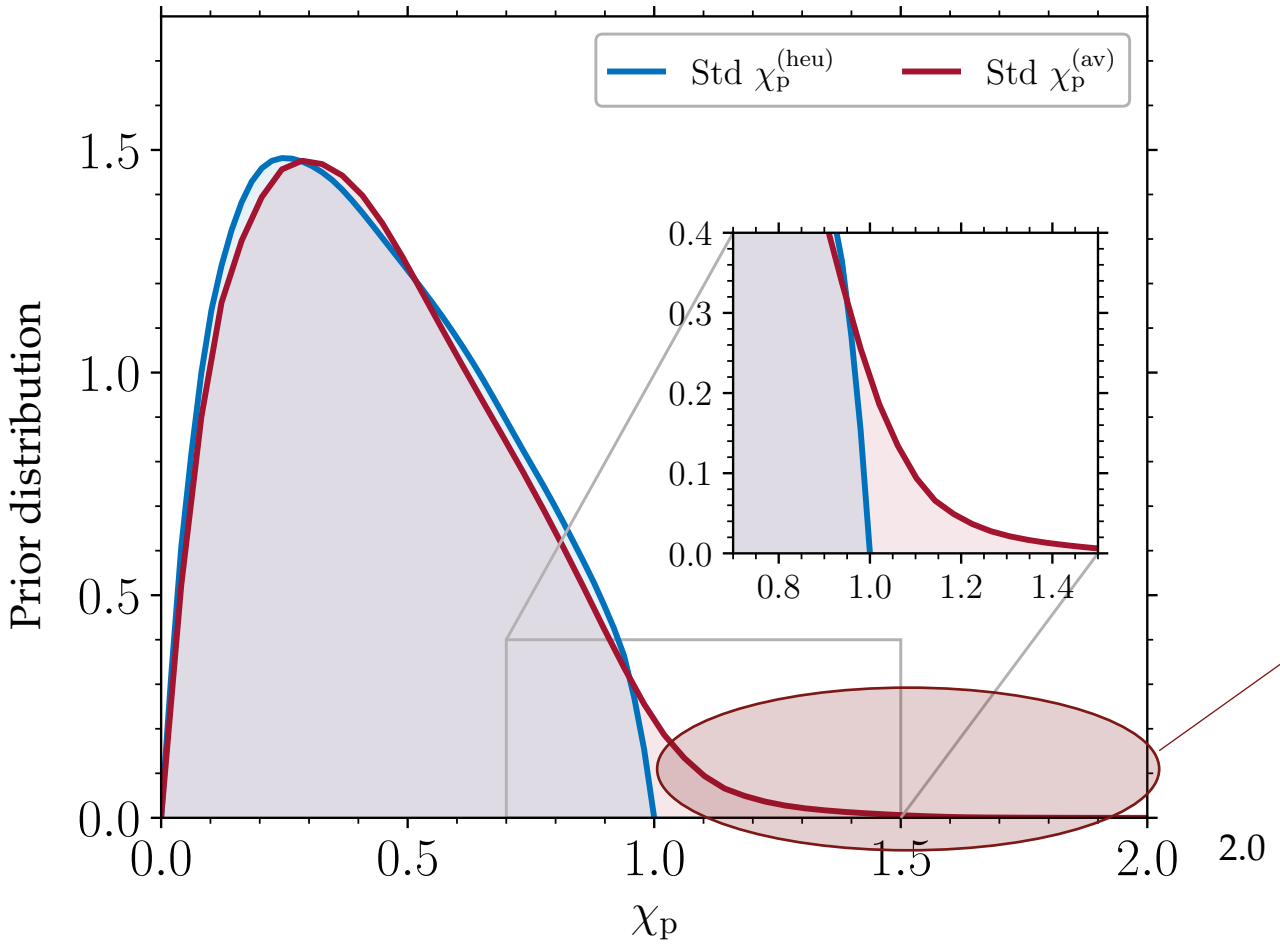

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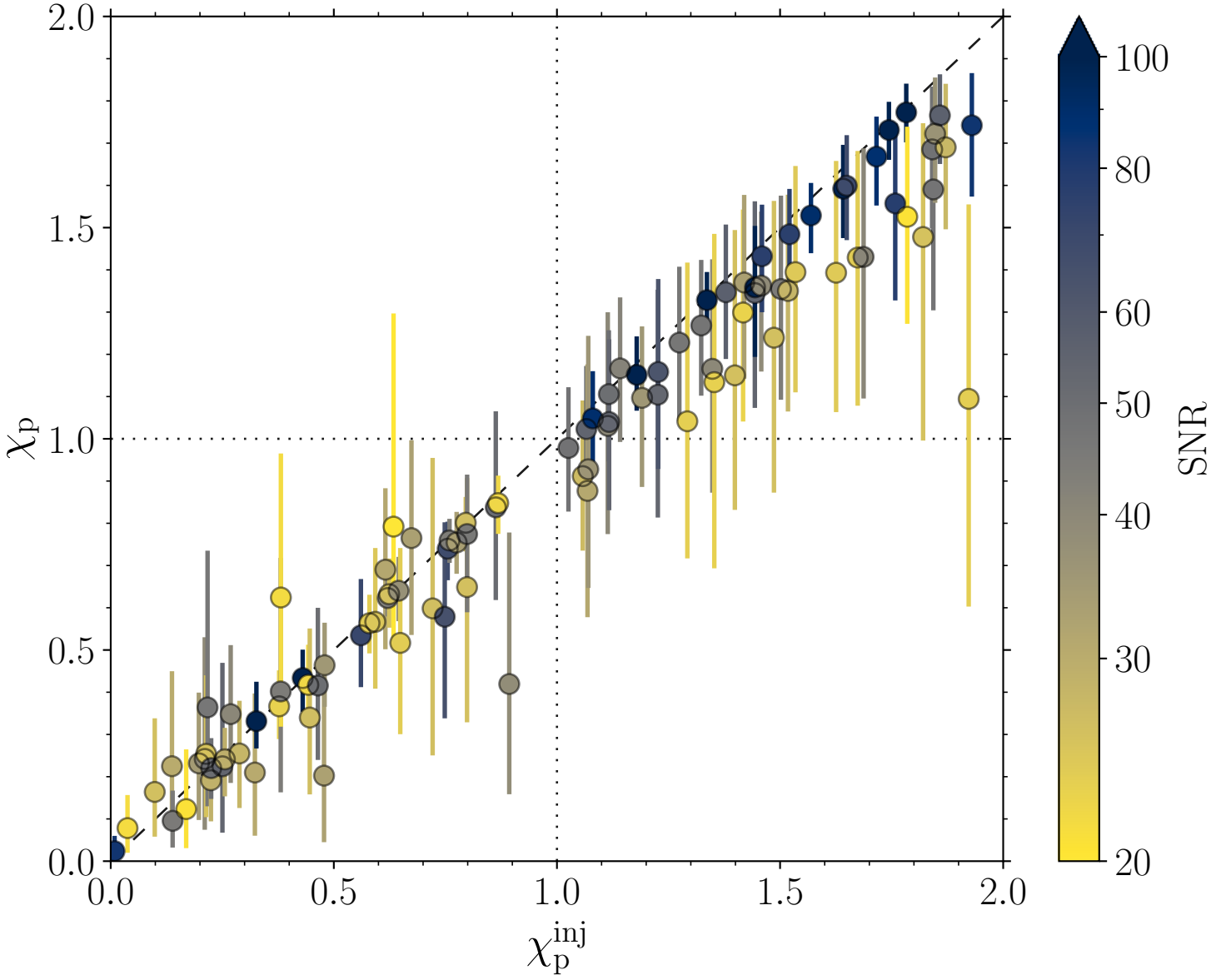
METHOD II:
1. reweight the samples for the intrinsic parameters drawn from the standard BBH priors in order to obtain a uniform distribution of $\chi_p \in [\,0\,,2\,]$
2. select the extrinsic parameters such that SNR>20

Can we recover two spin-effects with χ_p^{av} ?

If there is an event with two misaligned spins, can we tell?

- For the majority of the injections we are able to detect two precessing spins.
- Even for the most extreme case, the 90% C.I. is over the horizontal line $\chi_{\mathrm{p}}=1$
- The bias at $\chi_{\mathrm{p}}\sim 2$ is due to the low prior tail at $\chi_{\mathrm{p}}> 1$.

LVK will potentially be able to detect two-spin effects in the next observing runs!



From now on we will deal only with the averaged χ_{p} estimator

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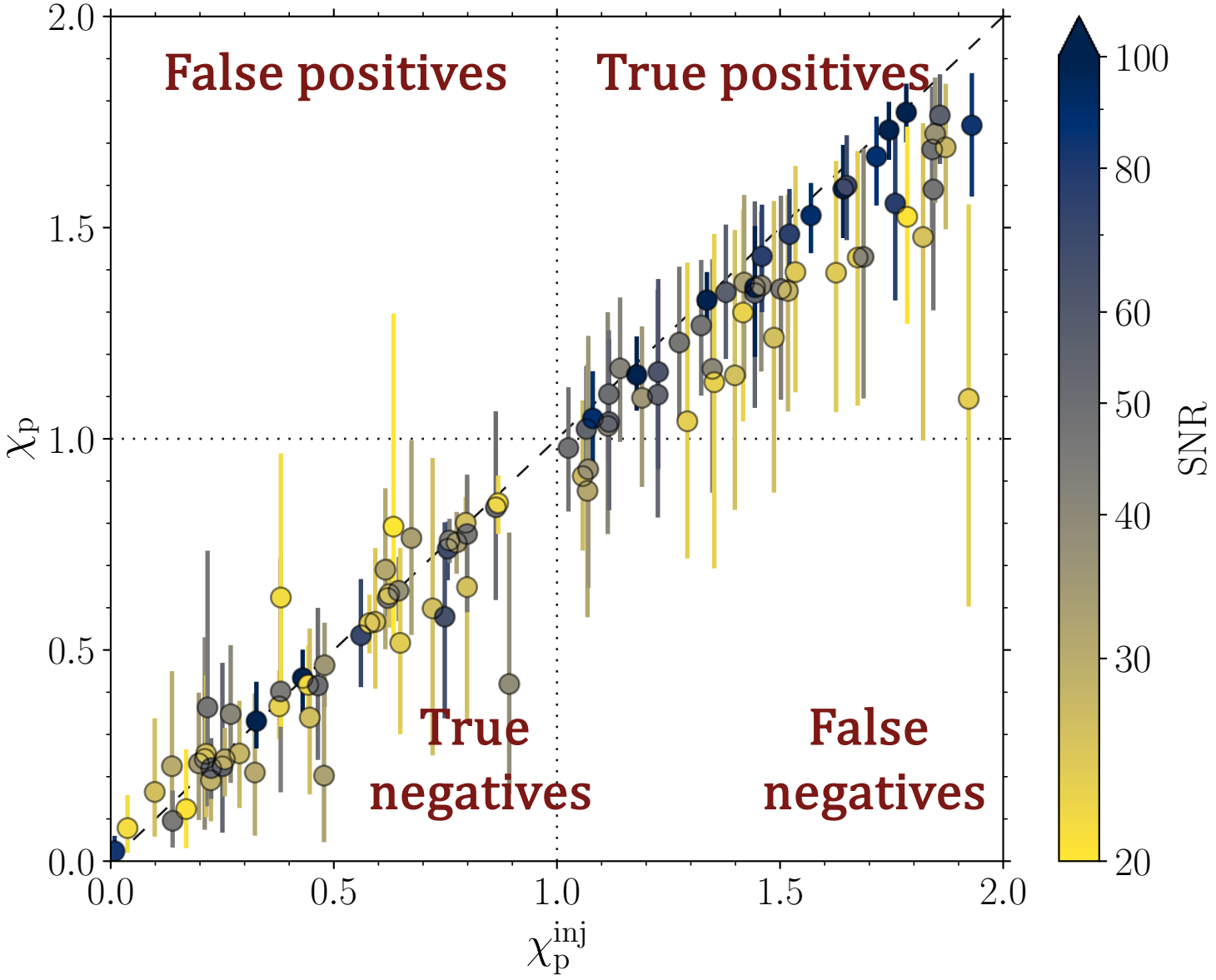

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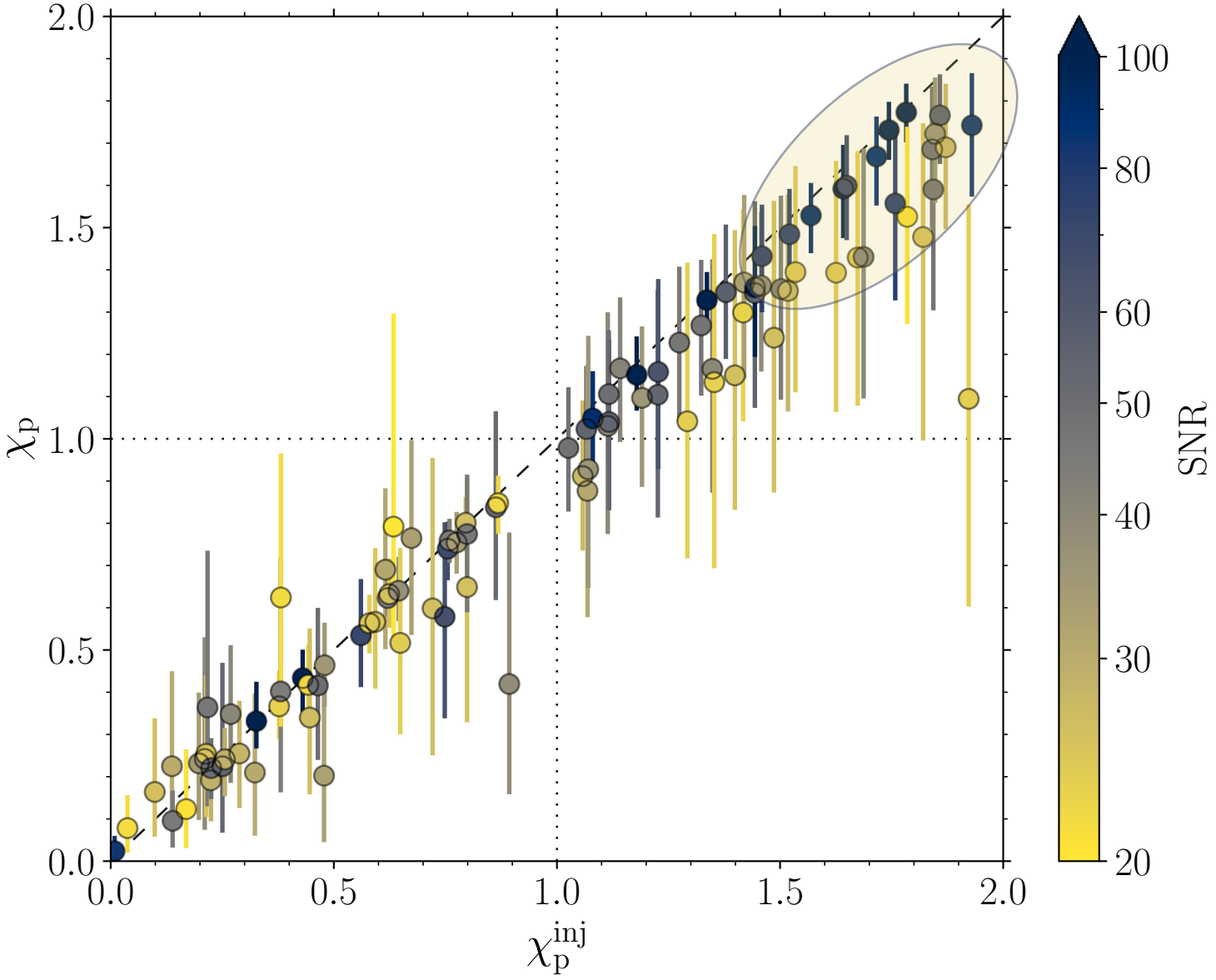
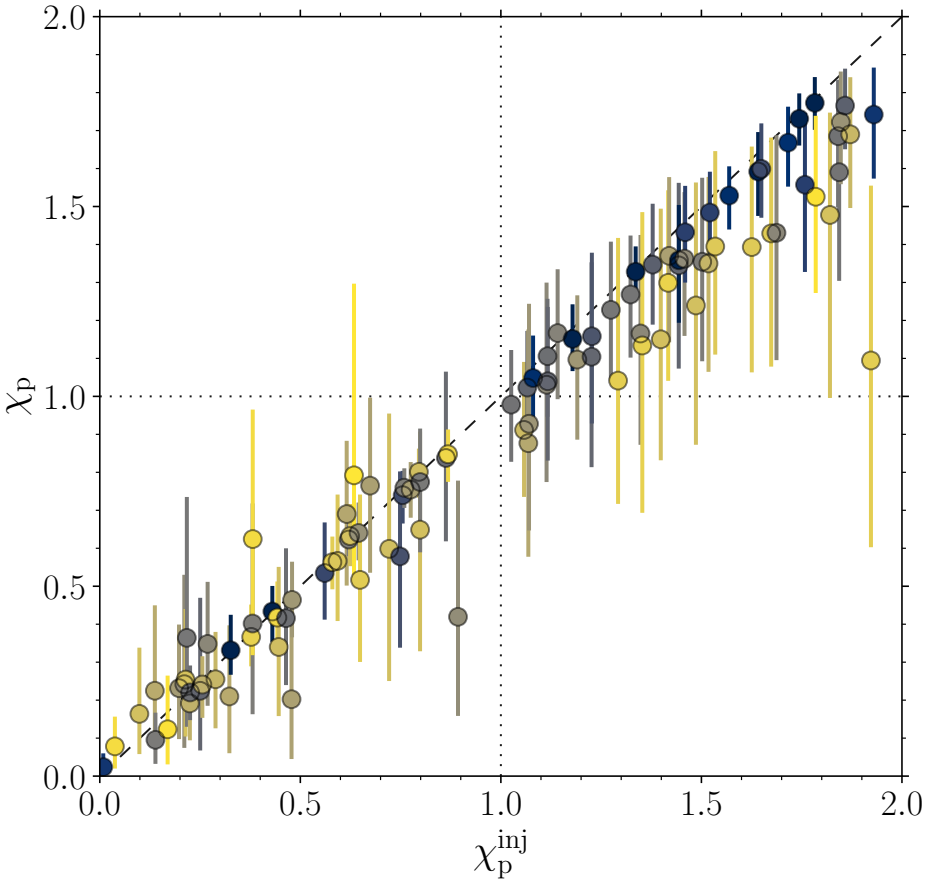
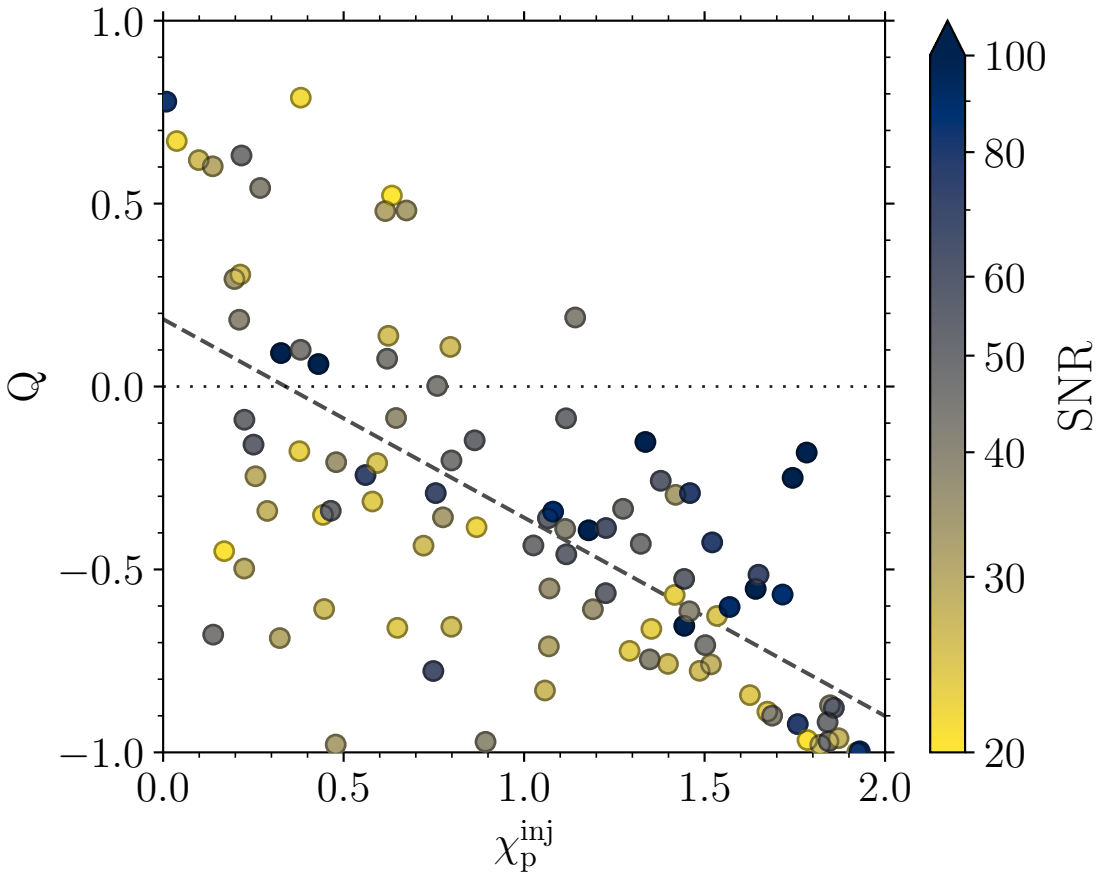


Statistical bias at $\chi_p > 1$





$Q > 0$:
overestimation

$Q < 0$:
underestimation

Statistical errors cause an underestimation of χ_p whenever $\chi_p > 1$

$$Q \equiv 2 \int_{-\infty}^{\chi_p^{inj}} P(\chi_p)\, d\chi_p - 1 \in [-1, 1]$$

Outline

Overview of two subdominant spin effects in current and future LIGO/Virgo data:

I. Detecting the signature imprinted by **two precessing spins** on the emitted GW signal using a carefully designed estimator

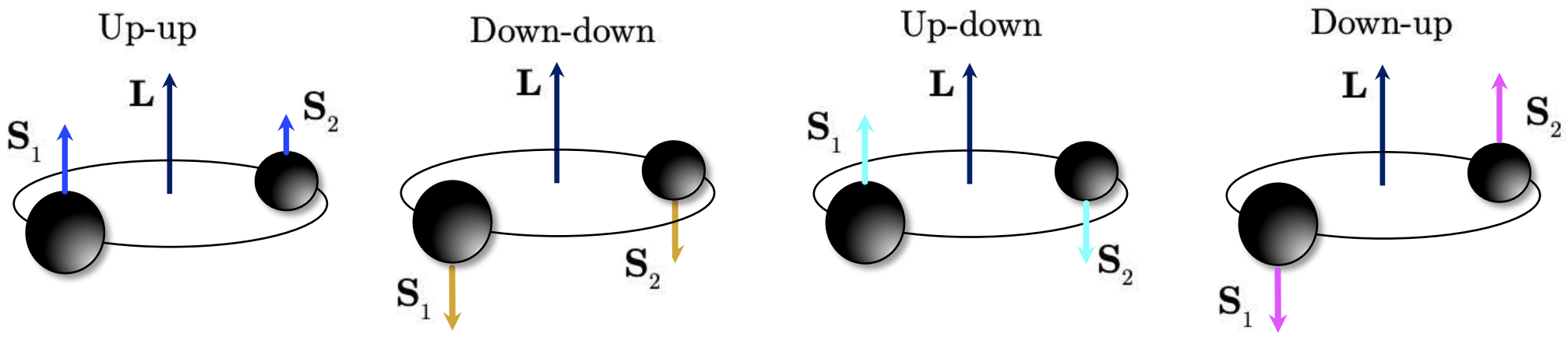
[De Renzis+, PRD, 2022]

II. Prospects for **detecting unstable binaries** which formed with aligned spins but enter the detector sensitivity window in precessing configurations

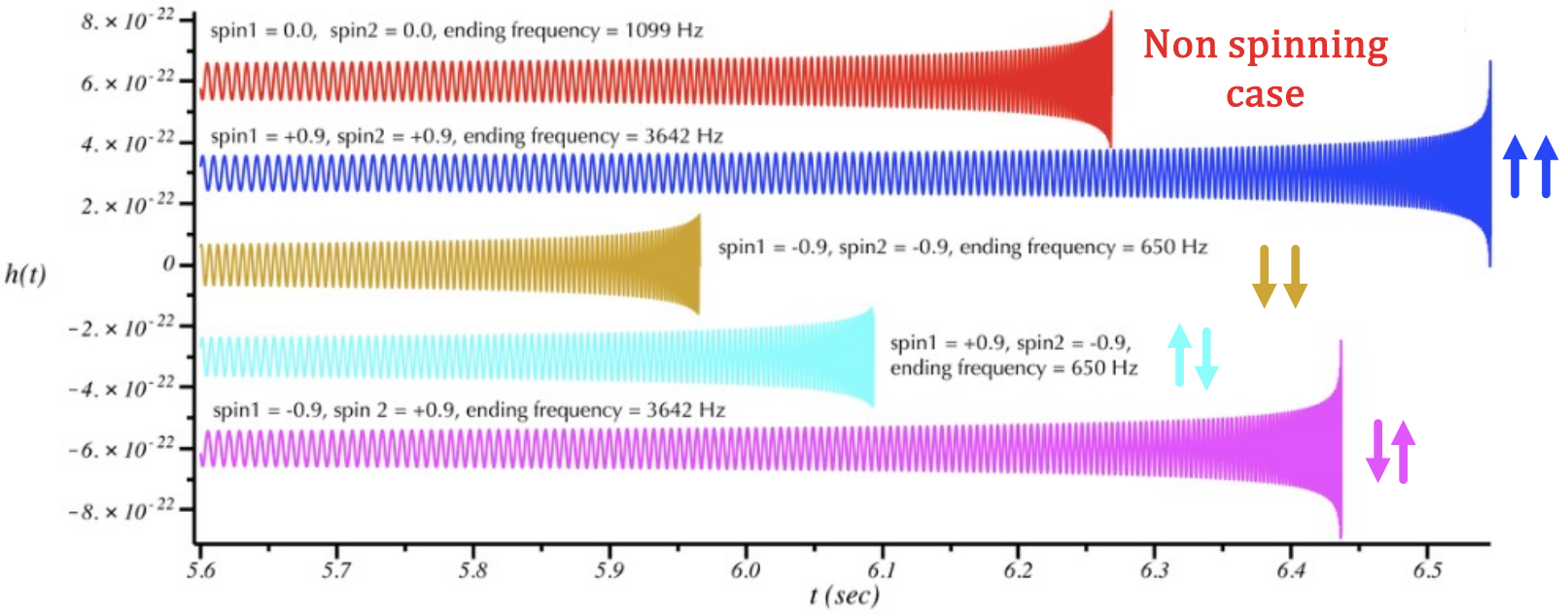
[De Renzis+, PRD, 2023]

Aligned spins → no precession

Four equilibrium solution to the relativistic spin-precession equation:

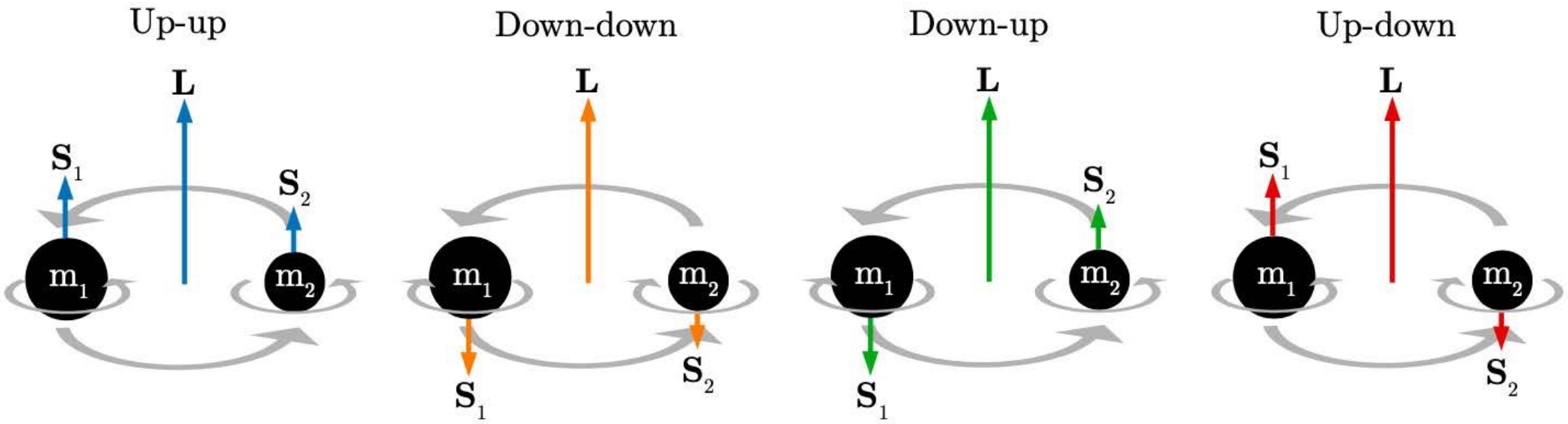


[M. Favata]



- Similar morphologies
- Different merger frequencies and signal durations

Do aligned binaries remain aligned?



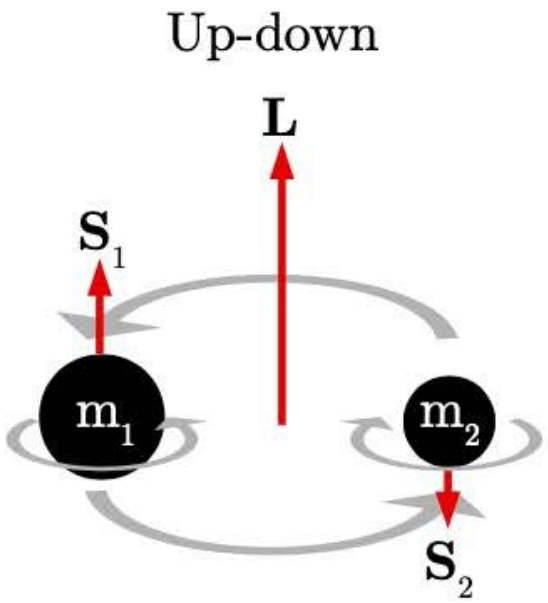
Yes | Yes | Yes | No

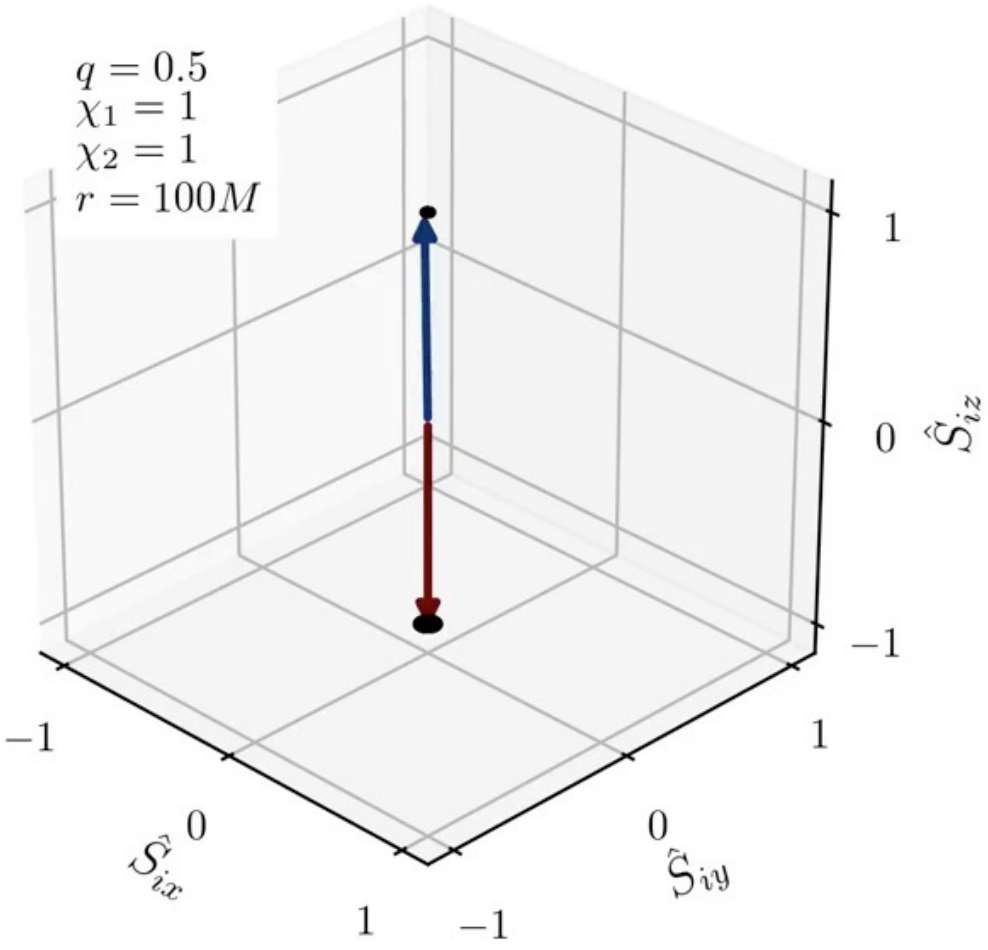
Aligned **up-down** binaries are **unstable** to spin precession and can enter the LIGO band with misaligned spins

Kidder, 1995
Gerosa+, 2015
Mould and Gerosa, 2020

Up-down precessional instability

Only **up-down** binaries are **unstable**



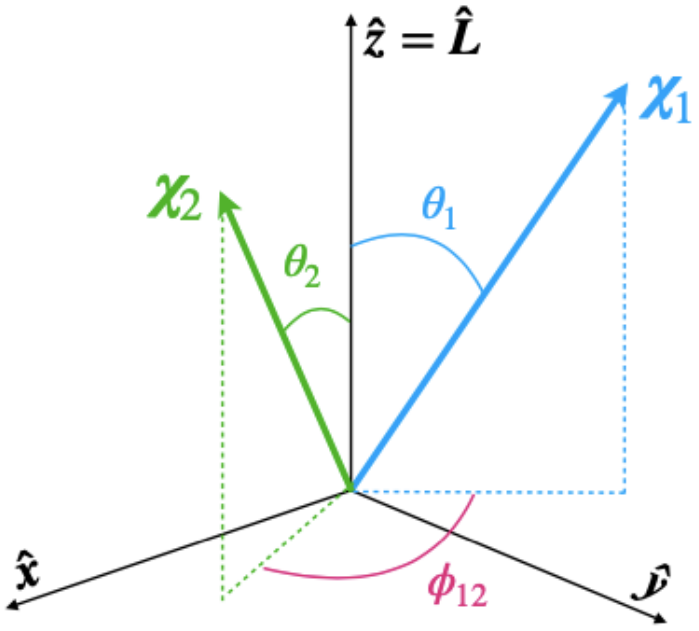


The **critical radius** that define the onset of the instability is

$$r_+ = \frac{(\sqrt{\chi_1} + \sqrt{q\chi_2})^4}{(1-q)^2} M$$

Mould and Gerosa, 2020

What is the endpoint of the up-down instability?



$$\vec{\chi}_i = \frac{\vec{S}_i}{m_i^2}, \quad (G = c = 1)$$

Unstable up-down binaries do not disperse in the available parameter space but tend to **cluster to a specific (and analytical) endpoint:**

I. $$\cos\theta_1 = \frac{\chi_1 - q\chi_2}{\chi_1 + q\chi_2}$$

II. $$\cos\theta_2 = \frac{\chi_1 - q\chi_2}{\chi_1 + q\chi_2}$$

III. $$\phi_{12} = 0$$

Mould and Gerosa, 2020

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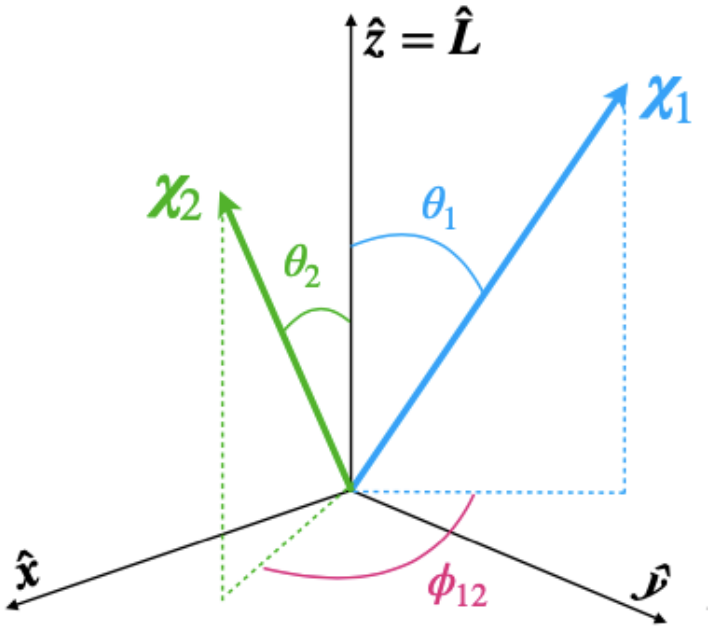

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$\rightarrow$ $\cos\theta_{ud}$

III. $\phi_{12} = 0$

Mould and Gerosa, 2020

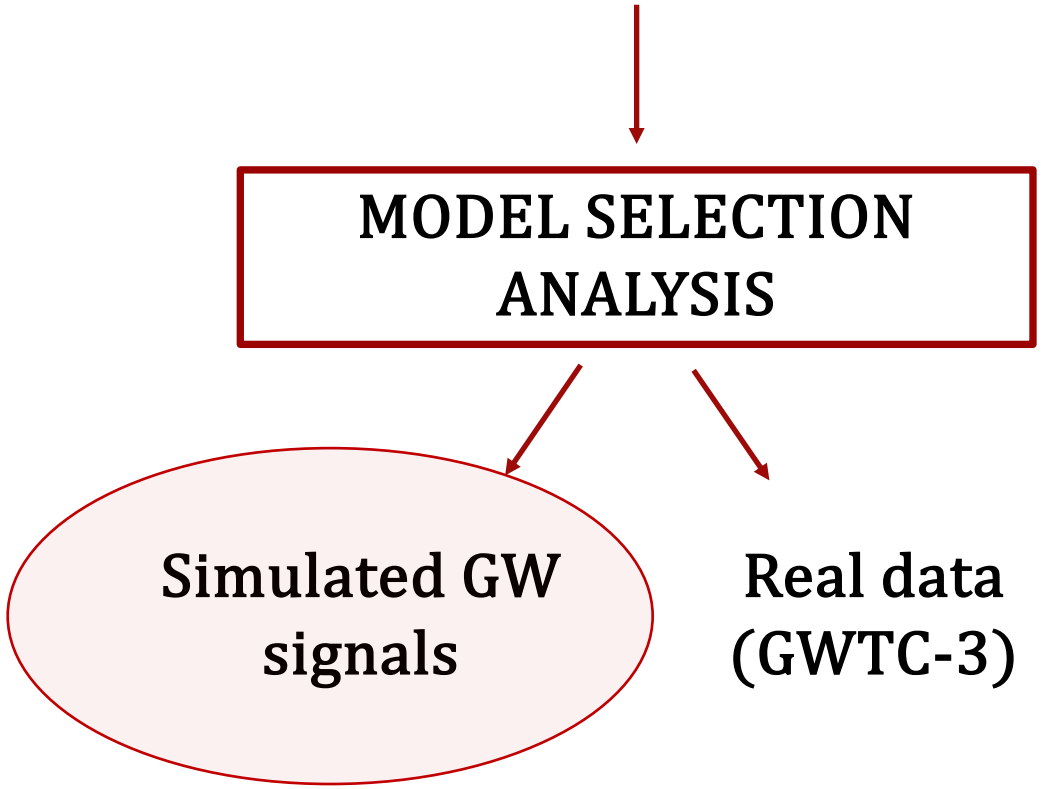
Say an unstable up-down binary enters the LIGO band in its endpoint … can we tell this binary used to be aligned?

MODEL SELECTION ANALYSIS

Simulated GW signals

Real data (GWTC-3)

Say an unstable up-down binary enters the LIGO band in its endpoint … can we tell this binary used to be aligned?



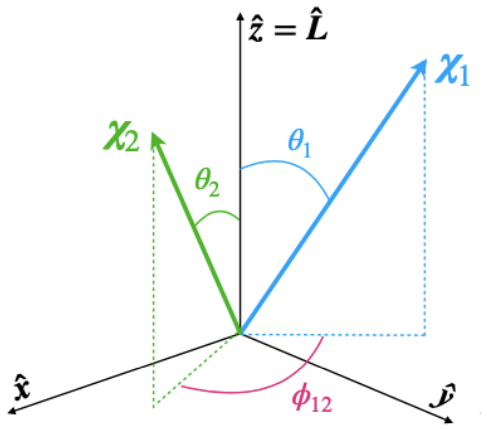
Full parameter estimation with BILBY

Bayesian inference framework for parameter estimation

Parameters $\vec{\theta}$ describing a BBH merger

$$\vec{\theta} = \{m_1, m_2, \chi_1, \chi_2, \theta_1 = \theta_{\mathrm{ud}}, \theta_2 = \theta_{\mathrm{ud}}, \phi_{12} = 0, \phi_{JL}, d_L, ra, dec, \psi, \phi, \theta_{JN}, t_C\}$$

The signals are injected in the endpoint of the up-down instability



Settings for the injections

Detectors: *H, L, V (O4)*
Gaussian noise: no
Detector arguments: $f_{ref} = 20.0$ Hz
$f_{min} = 20.0$ Hz
$f_{sampling} = 2048.0$ Hz
$duration = 4$s
Injection and recovery waveform: *IMRPhenomXPHM*
Priors: *Standard BBH priors*
Sampler: *dynesty*

Model selection

Given the observed data d and two competing models M_A and M_B and assuming equal model priors, the **Bayes' factor** is:

$$\boxed{\mathcal{B} = \frac{Z(d|M_B)}{Z(d|M_A)}}$$

Jeffrey scale

$\ln B_{1,2}$	Strength of evidence
<1.0	inconclusive
1.0 – 2.5	weak
2.5 – 5.0	moderate
> 5.0	strong

Evidence (or marginalized likelihood)

$$Z(d|M_i) = \int d\theta P(d|\vec{\theta}, M_i)\pi(\vec{\theta}|M_i)$$

Bayes' Theorem

$$P(d|\vec{\theta}, M_i) = \frac{\mathcal{L}(d|\vec{\theta}, M_i)\pi(\vec{\theta}|M_i)}{Z(d|M_i)}$$

Model selection

Broad model $\mathcal{H}_B$: 15 parameters $\rightarrow \overrightarrow{\theta_A} = \{\phi, \bar{\gamma}\}$

Precessing BBH

ϕ = common parameters with $\mathcal{H}_B$ $\{m_1, m_2, \chi_1, \chi_2, \phi_{JL}, d_L, ra, dec, \psi, \phi, \theta_{JN}, t_C\}$

$\bar{\gamma} = \{\gamma_1, \gamma_2, \gamma_3\}$ = free extra parameters of $\mathcal{H}_B$ that are fixed in the narrow model $\mathcal{H}_N$

I. $$\gamma_1 = \frac{\cos\theta_1 - \cos\theta_{UD}(q, \chi_1, \chi_2)}{2}$$

II. $$\gamma_2 = \frac{\cos\theta_2 - \cos\theta_{UD}(q, \chi_1, \chi_2)}{2}$$

III. $$\gamma_3 = \frac{1}{\pi}\arctan\left(\frac{\sin\phi_{12}}{\cos\phi_{12}}\right),$$

Narrow model $\mathcal{H}_N \in \mathcal{H}_B$: 12 parameters $\rightarrow \overrightarrow{\theta_B} = \{\phi, \bar{\gamma} = 0\}$

Originally aligned BBH

3 constraints:
$\bar{\gamma} = \{\gamma_1, \gamma_2, \gamma_3\} = \{0,0,0\}$

Savage-Dickey Ratio for nested models

$\mathcal{H}_{\mathrm{B}}$: $\overrightarrow{\theta_{\mathrm{A}}} = \{\phi, \bar{\gamma}\}$

$\mathcal{H}_{\mathrm{N}}$: $\overrightarrow{\theta_{\mathrm{B}}} = \{\phi, \bar{\gamma} = 0\}$

$\mathcal{H}_{\mathrm{N}} \in \mathcal{H}_{\mathrm{B}}$

$$\boxed{\mathcal{B} = \frac{Z(d|\mathcal{H}_{\mathrm{N}})}{Z(d|\mathcal{H}_{\mathrm{B}})} \sim \frac{p(\bar{\gamma} = 0 \,|d, \mathcal{H}_{\mathrm{B}})}{\pi(\bar{\gamma} = 0 \,|\mathcal{H}_{\mathrm{B}})}}$$

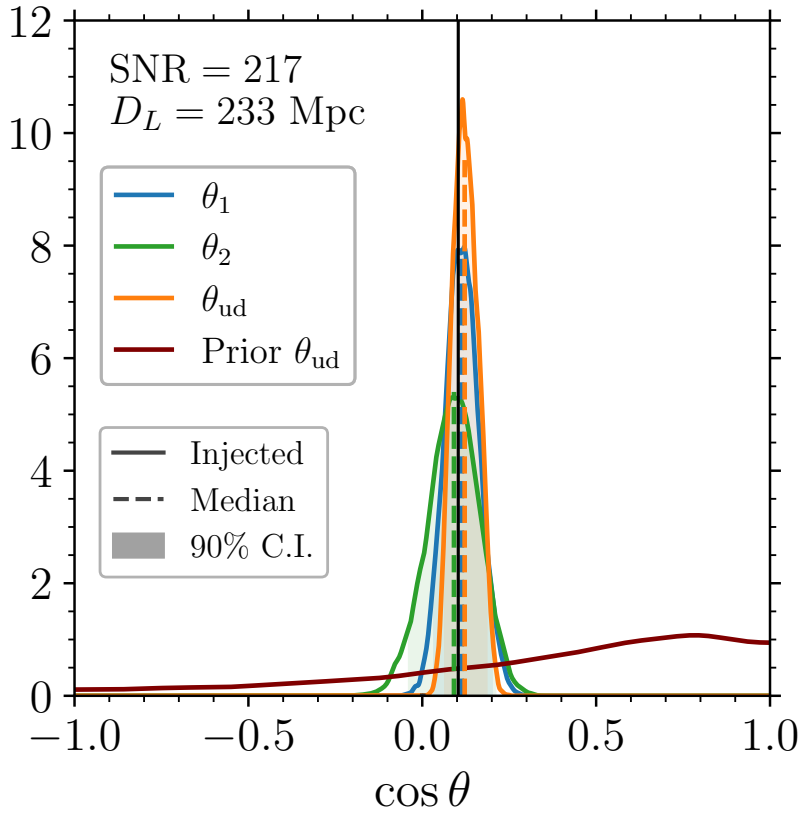
We just need the broad model $\mathcal{H}_{\mathrm{B}}$ to compute the Bayes factor

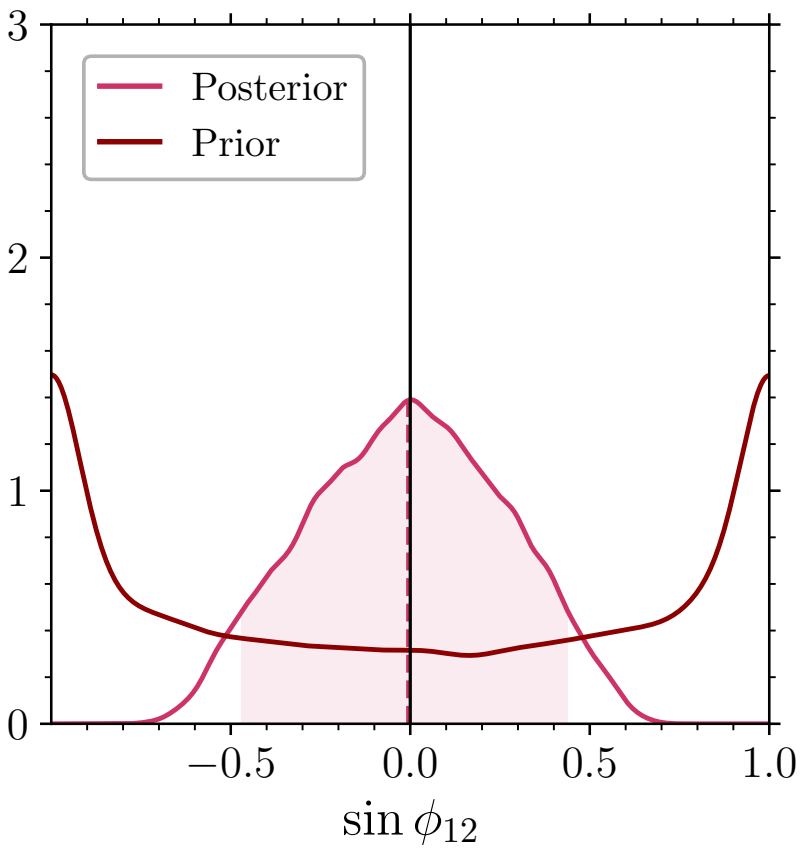
Injection for a single event

INJECTED VALUES

q= 0.79
$\chi_1 = 0.92$
$\chi_1 = 0.94$
$\boldsymbol{\theta_1 = \theta_2 = 1.47 = \theta_{\mathrm{ud}}}$
$\boldsymbol{\phi_{12} = 0.0}$
$d_L = 233.4$ Mpc
$\theta_{JN} = 0.37$
$\phi_{JL} = 5.71$
$dec = 0.24$
$ra = 6.11$
$\psi = 2.28$
$t_{geocent} = -0.069$
$\phi = 5.12$

$\boldsymbol{f_{ref} = 20\ Hz}$
q= 0.79
$r_+ = 266\ M \gg r = 10\ M$





I. + **II.** $\cos\theta_1 = \cos\theta_2 = \boxed{\dfrac{\chi_1 - q\chi_2}{\chi_1 + q\chi_2}}$

III. $\phi_{12} = 0$

$\ln\mathcal{B} = 6.3 \rightarrow$ Strong evidence in favor of the nested model

Backpropagation to 0 Hz

INJECTED VALUES

$M_{1,detector} = 49.5$
$M_{1,detector} = 39.4$
$\boldsymbol{\theta_1 = 1.47}$
$\boldsymbol{\theta_2 = 1.47}$
$\boldsymbol{\phi_{12} = 0.0}$
$\chi_1 = 0.92$
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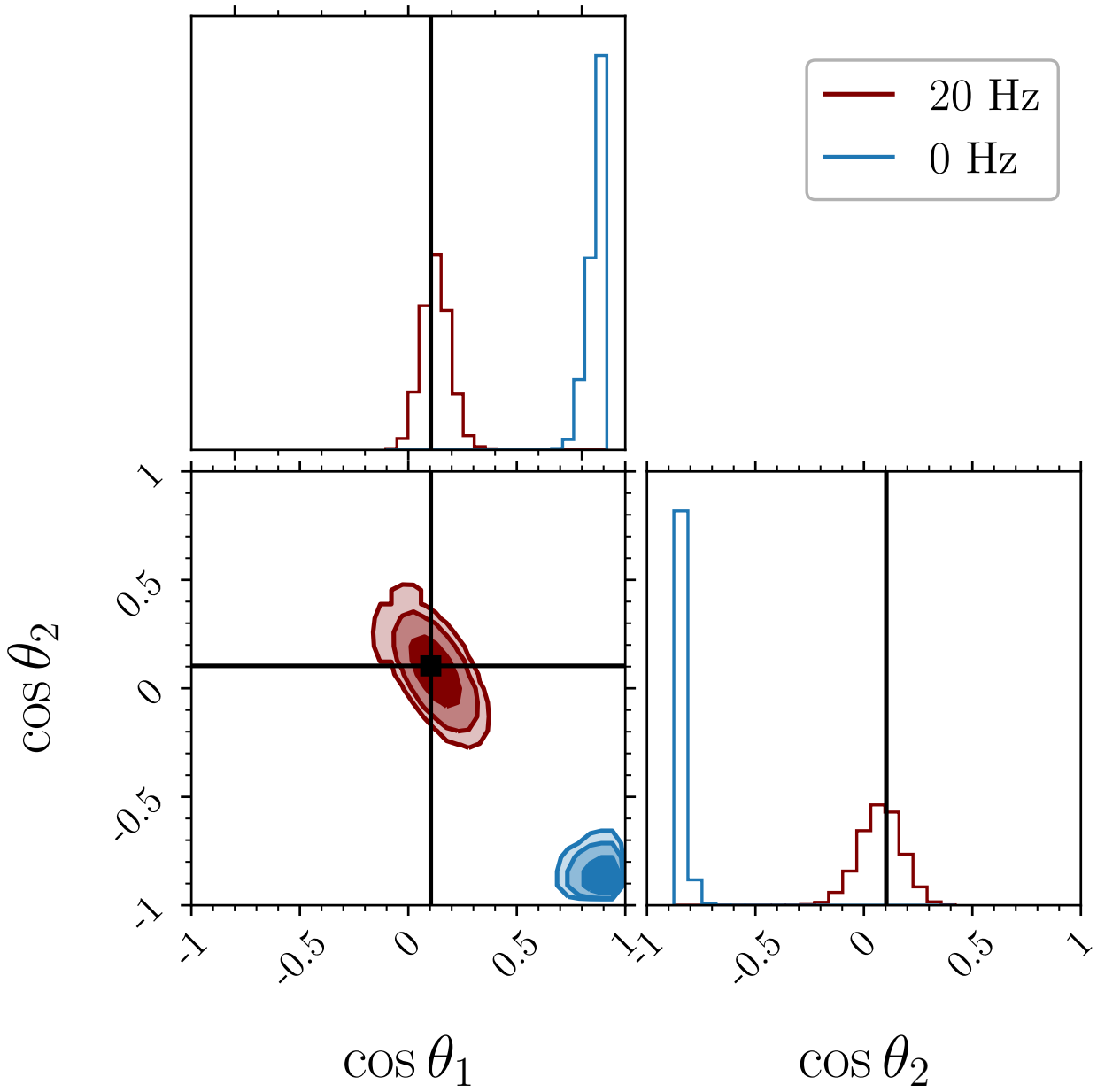
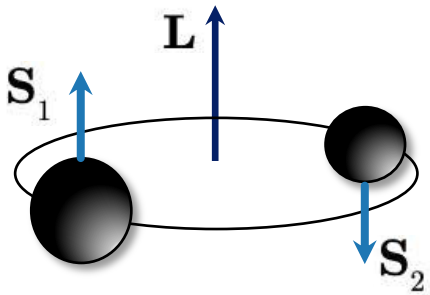


Up-down configuration (0Hz)

$\cos\theta_1 = 1 \rightarrow \theta_1 = 0$
$\cos\theta_2 = -1 \rightarrow \theta_2 = \pi$



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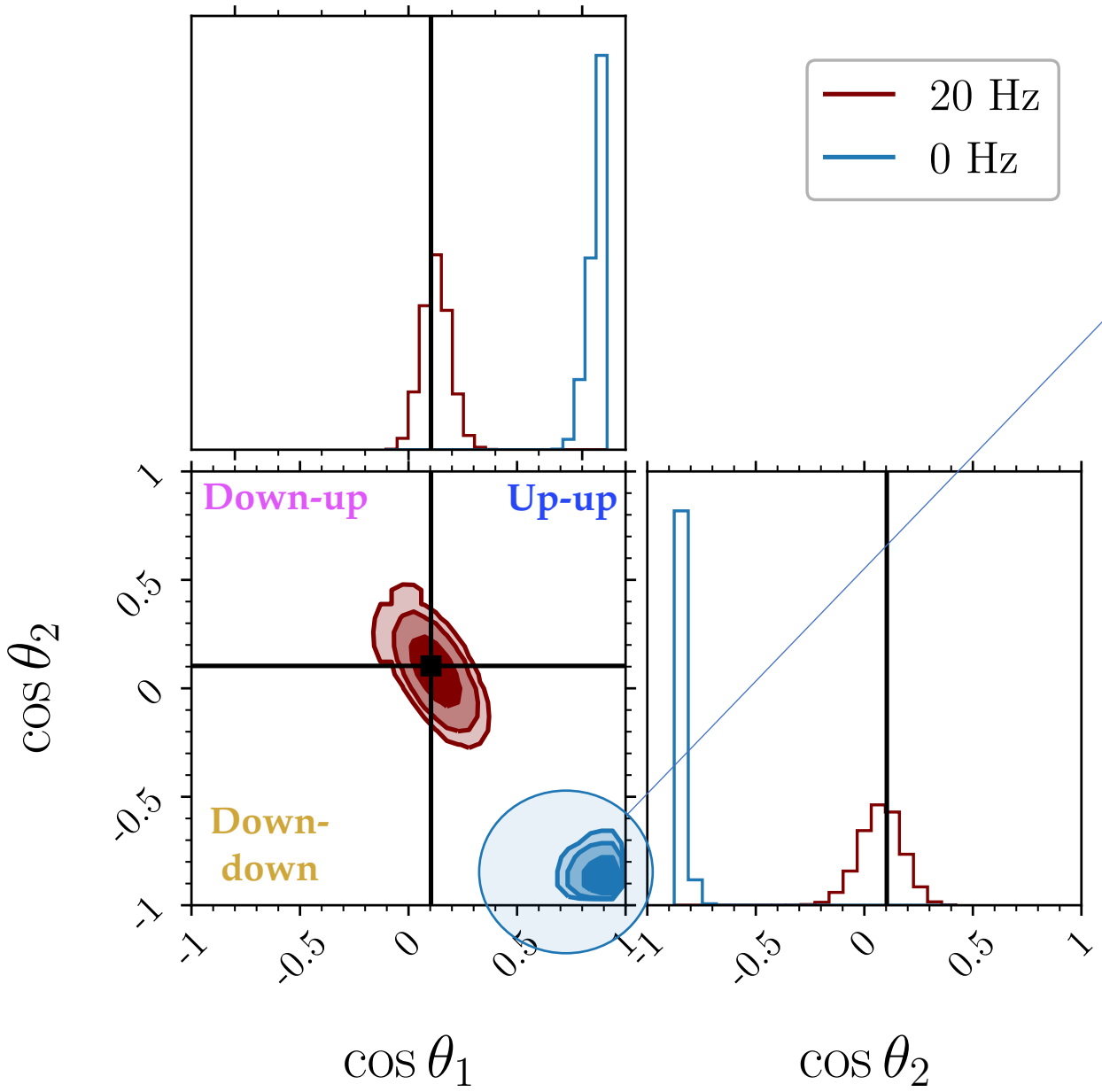

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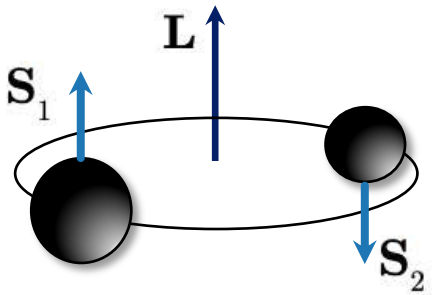
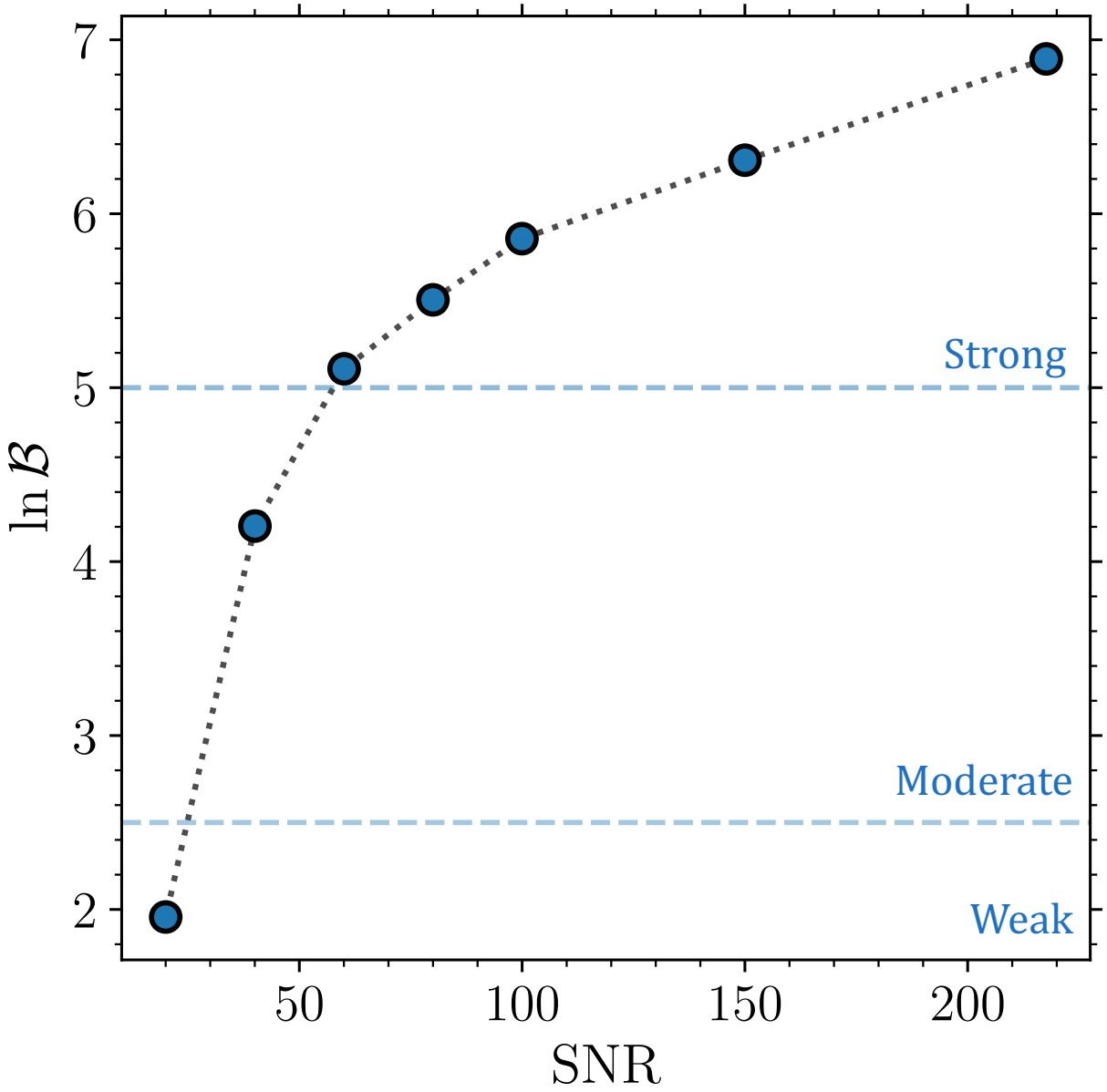


Recovery at different SNR



$D_L = [2538, 1268, 845, 634, 508, 338, 233]$ Mpc
$\rightarrow$ SNR $= [20, 40, 60, 80, 100, 150, 217]$

- As expected, the Bayes factor increases for higher SNR
- For this particular set of parameter, we recover a strong evidence in favor of H_N at SNR > 60

Jeffrey scale

$\ln B_{1,2}$	Strength of evidence
<1.0	inconclusive
1.0 – 2.5	weak
2.5 – 5.0	moderate
> 5.0	strong

Injection campaign

150 synthetic GW signals injected in the endpoint of the up-down instability

I. The **intrinsic parameters** are choosen from the standard priors such that the critical radius r_+ satisfies the condition

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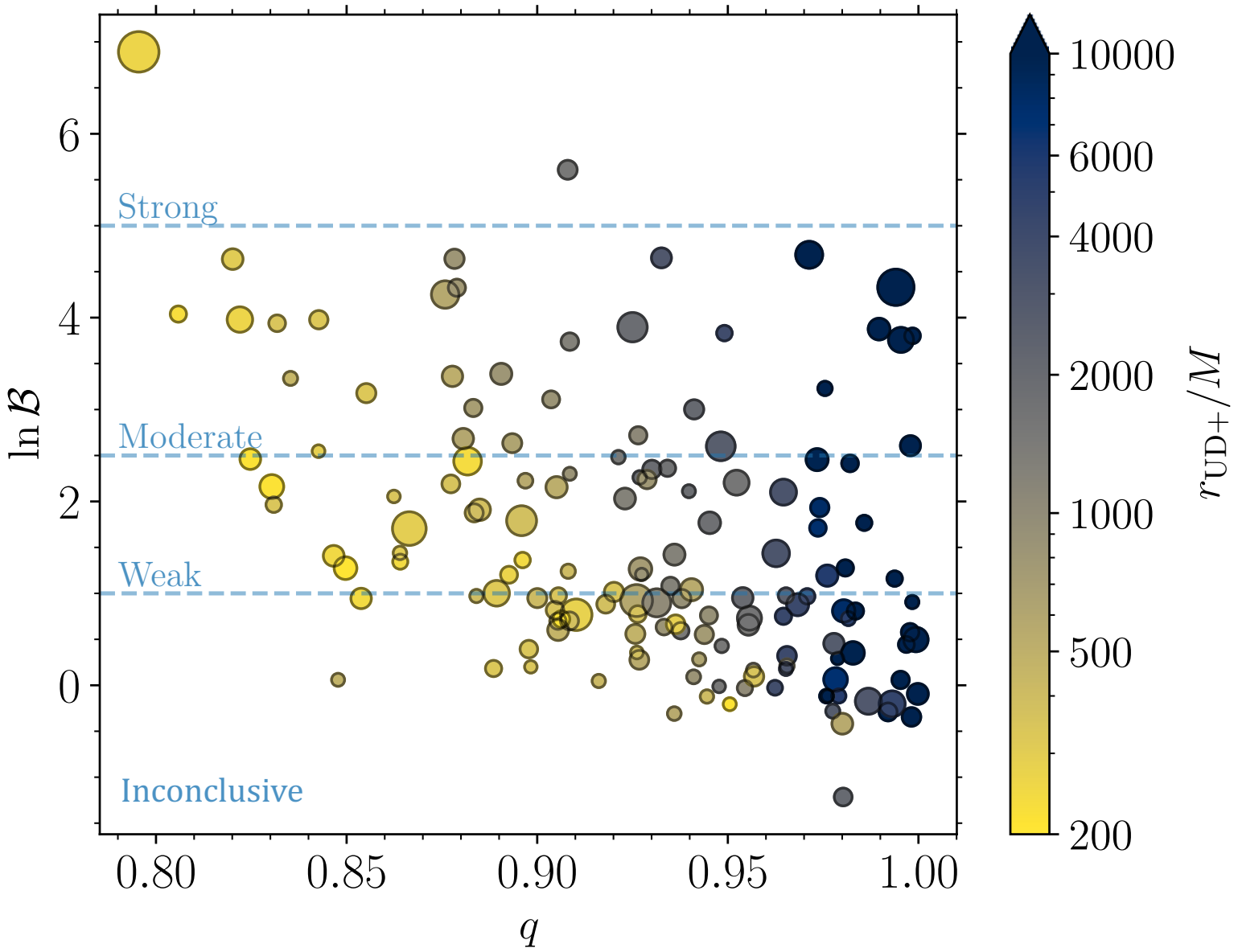
II. The **extrinsic parameters** are selected such that SNR>20



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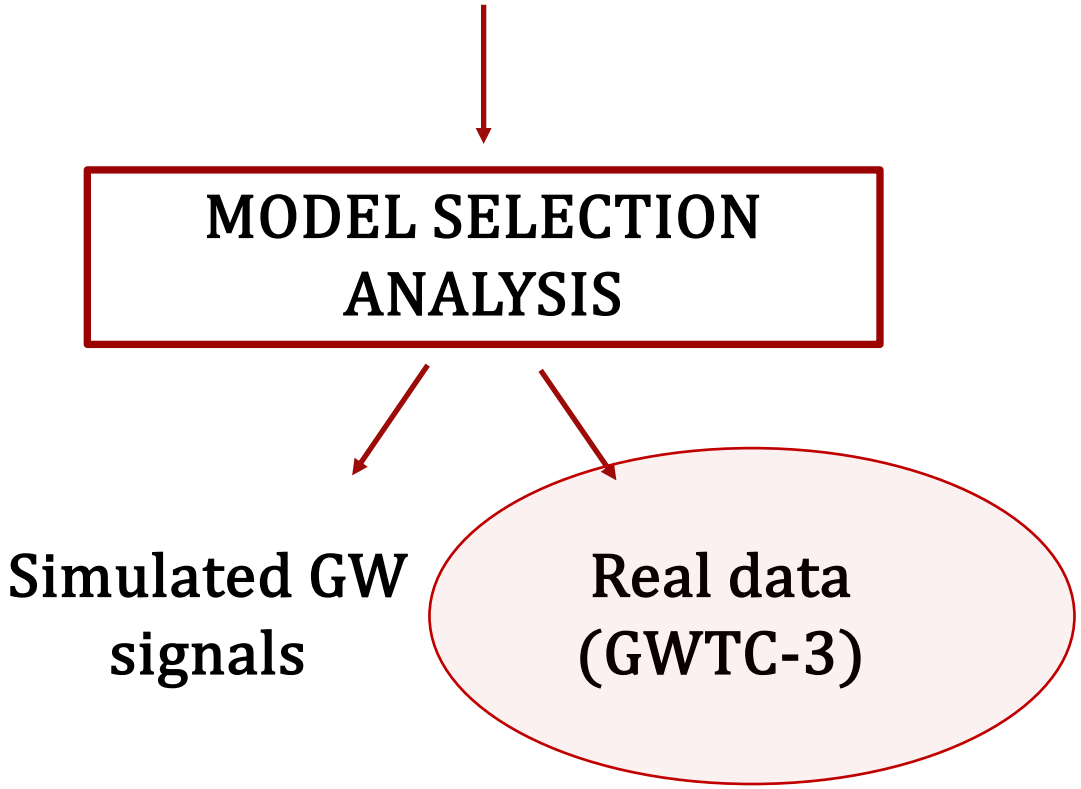
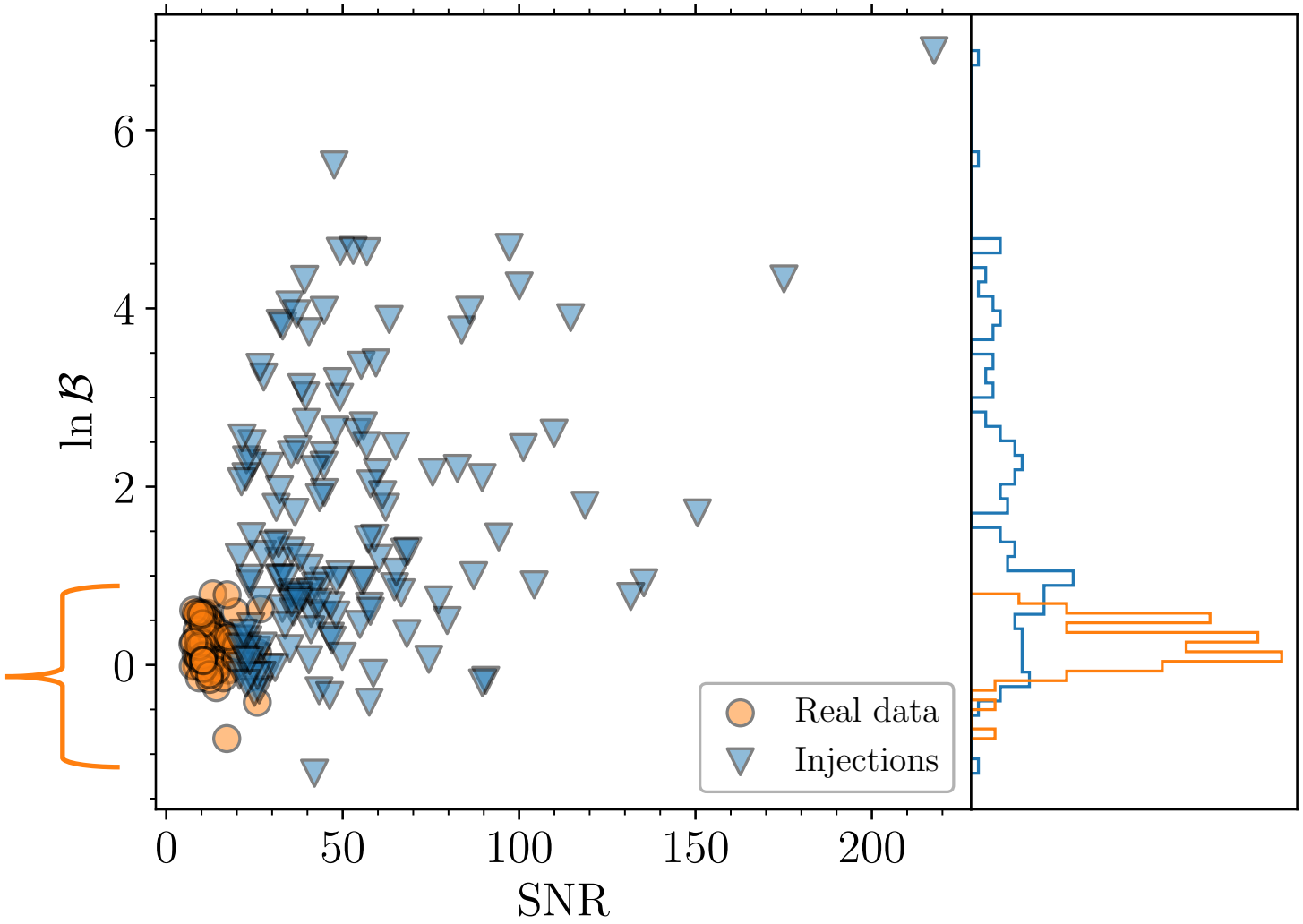


Real data (GWTC-3)

- 69 events of BBH coalescences (BNS excluded)
  - FAR $< 1yr^{-1}$ in at least one search

$|\ln\mathcal{B}| < 1 \rightarrow$
Inconclusive evidence
for GWTC-3



Conclusions

WHERE WE ARE NOW

- Moderate evidence for spin precession in individual events
- Collective evidence for spin precession at the population level (LIGO 2021)
- More accurate spin measurement in O4.

NEXT

TWO PRECESSING SPINS

- We found out that, if the SNR is high enough (>20), LIGO, Virgo and KAGRA will potentially be able to detect two-spin effect in the next observing run (O4).
- No false positive cases.

UP-DOWN BINARIES

- High sensitivity is a necessary but not sufficient condition for the up-down origin to be distinguishable
- Possible mechanism for the formation of precessing binaries in environments where sources are preferentially formed with (anti) aligned spins.

PUBLICY AVAILABLE POSTERIORS:

github.com/ViolaDeRenzis/twoprecessingspins

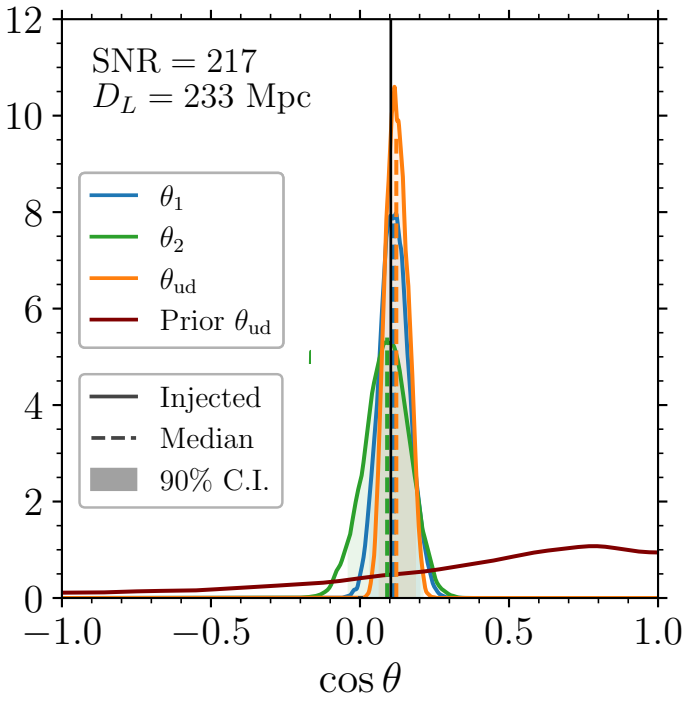
github.com/ViolaDeRenzis/updowninjections

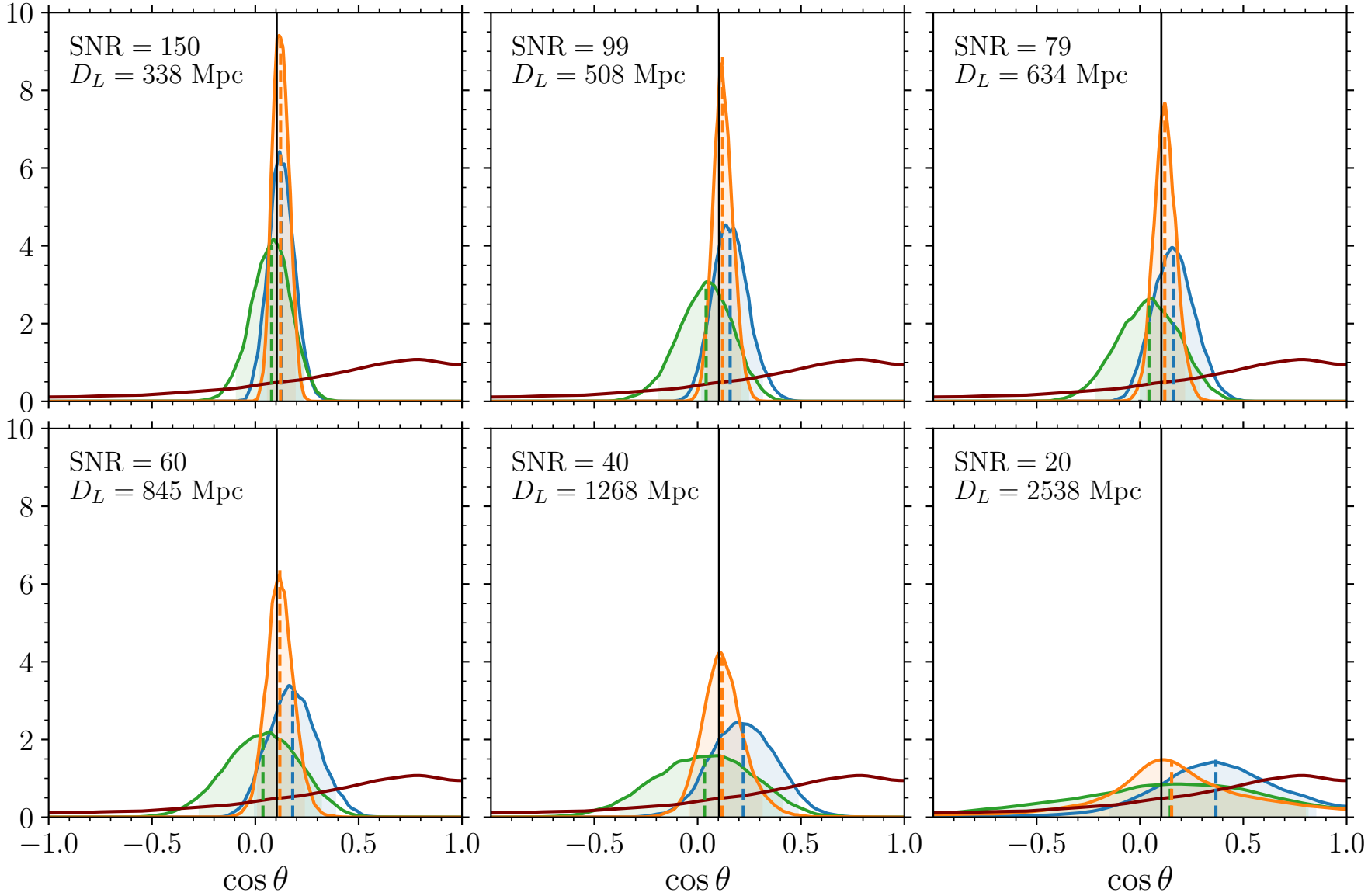
Thanks for the attention!

BACK-UP SLIDES

Up-down

Recovery at different SNR





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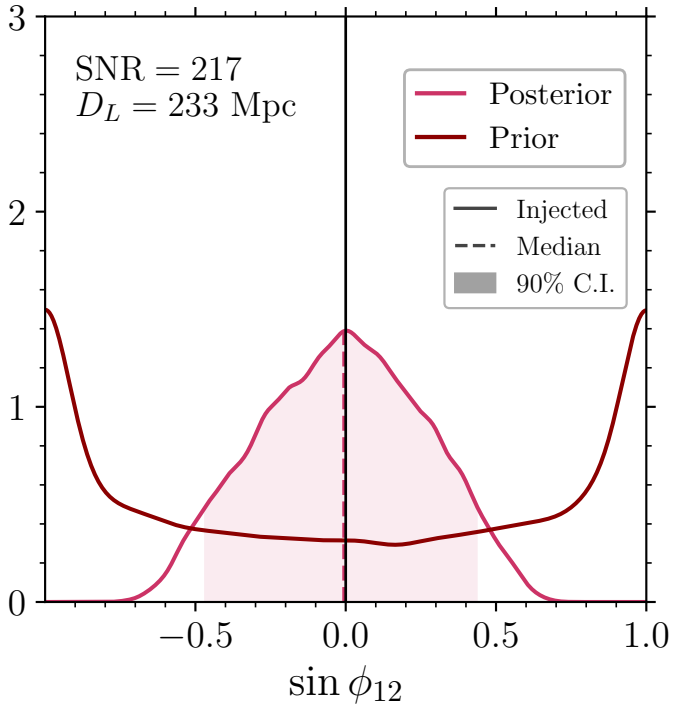

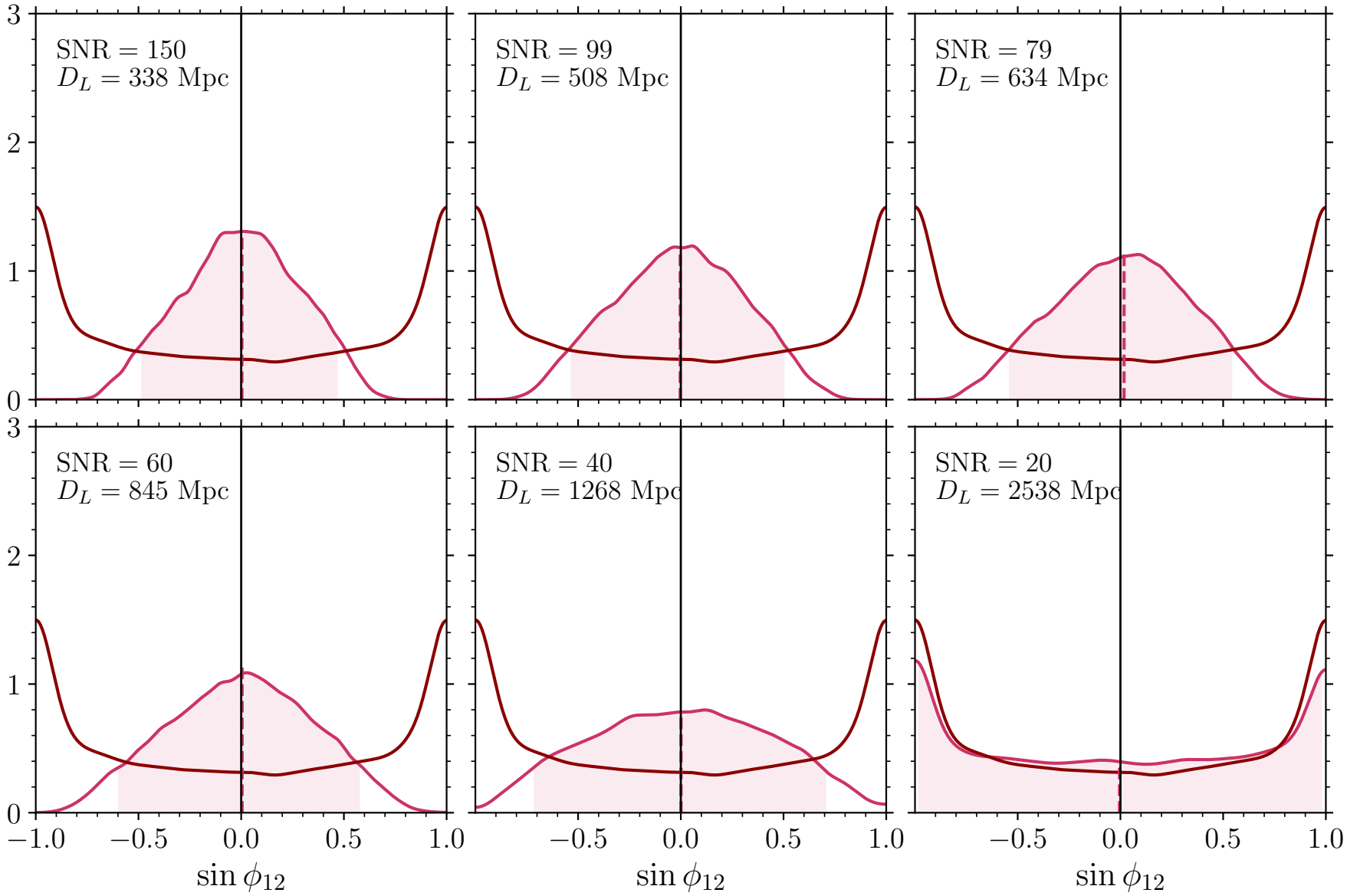

III.

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150 synthetic GW signals injected in the endpoint of the up-down instability

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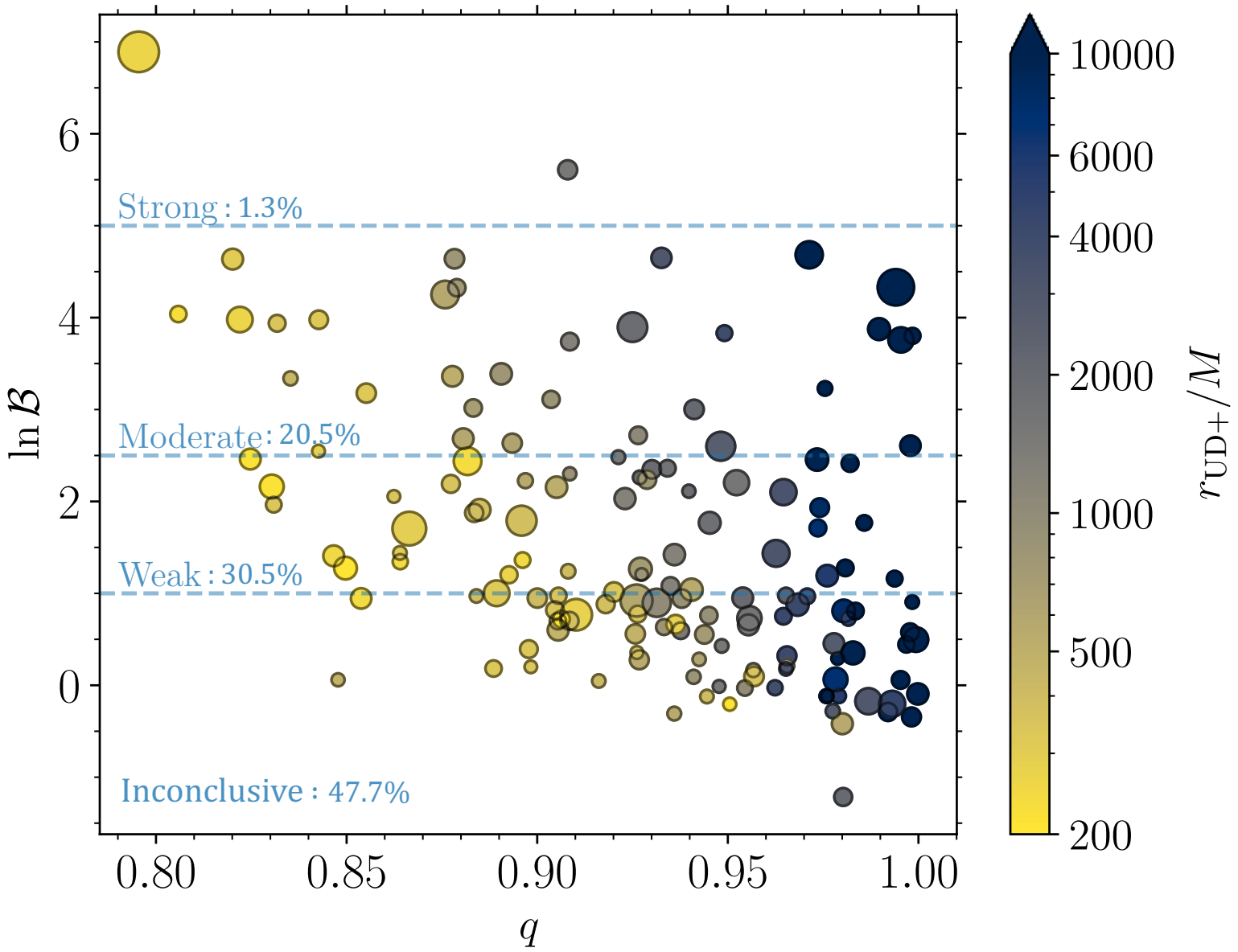

Effect of higher modes

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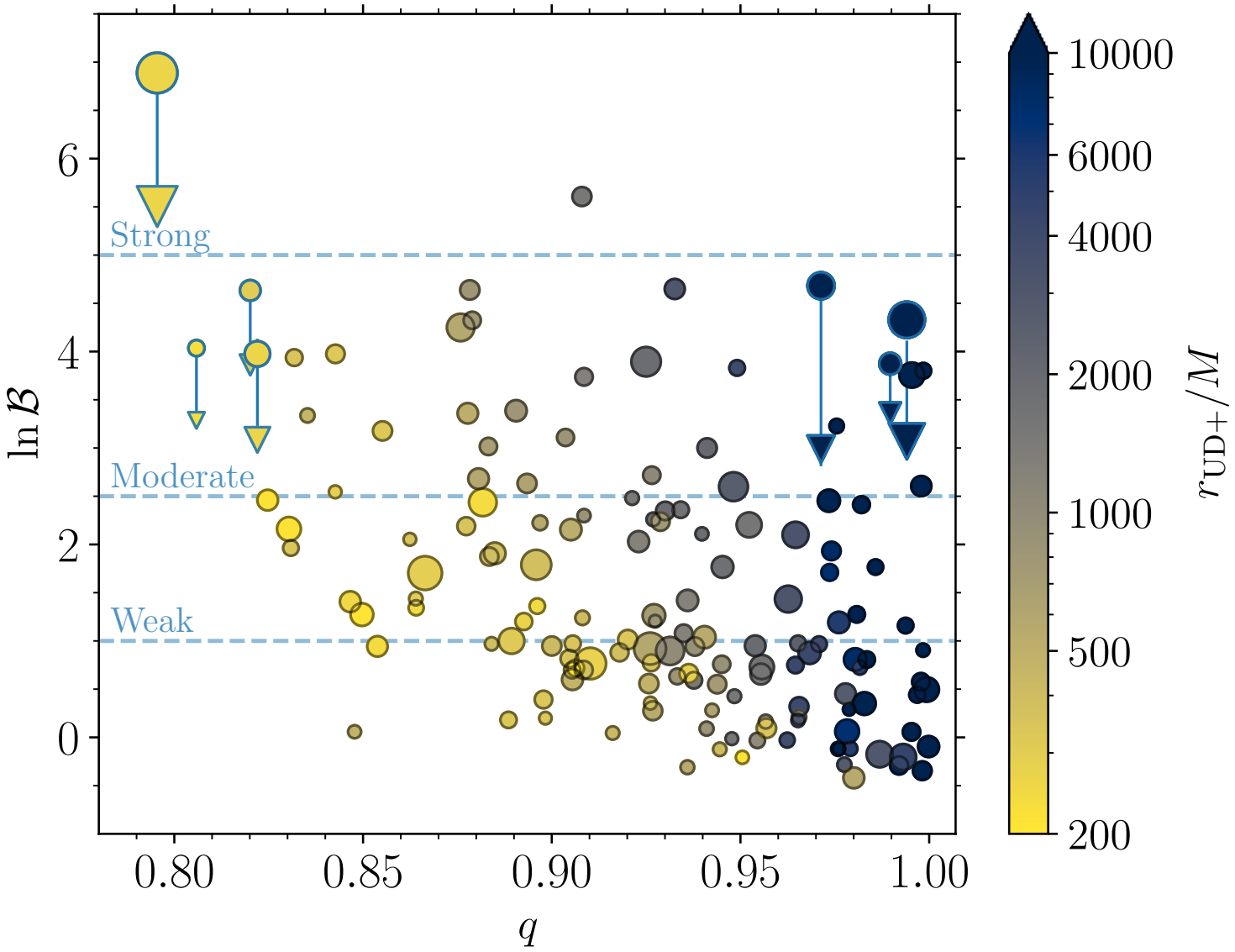

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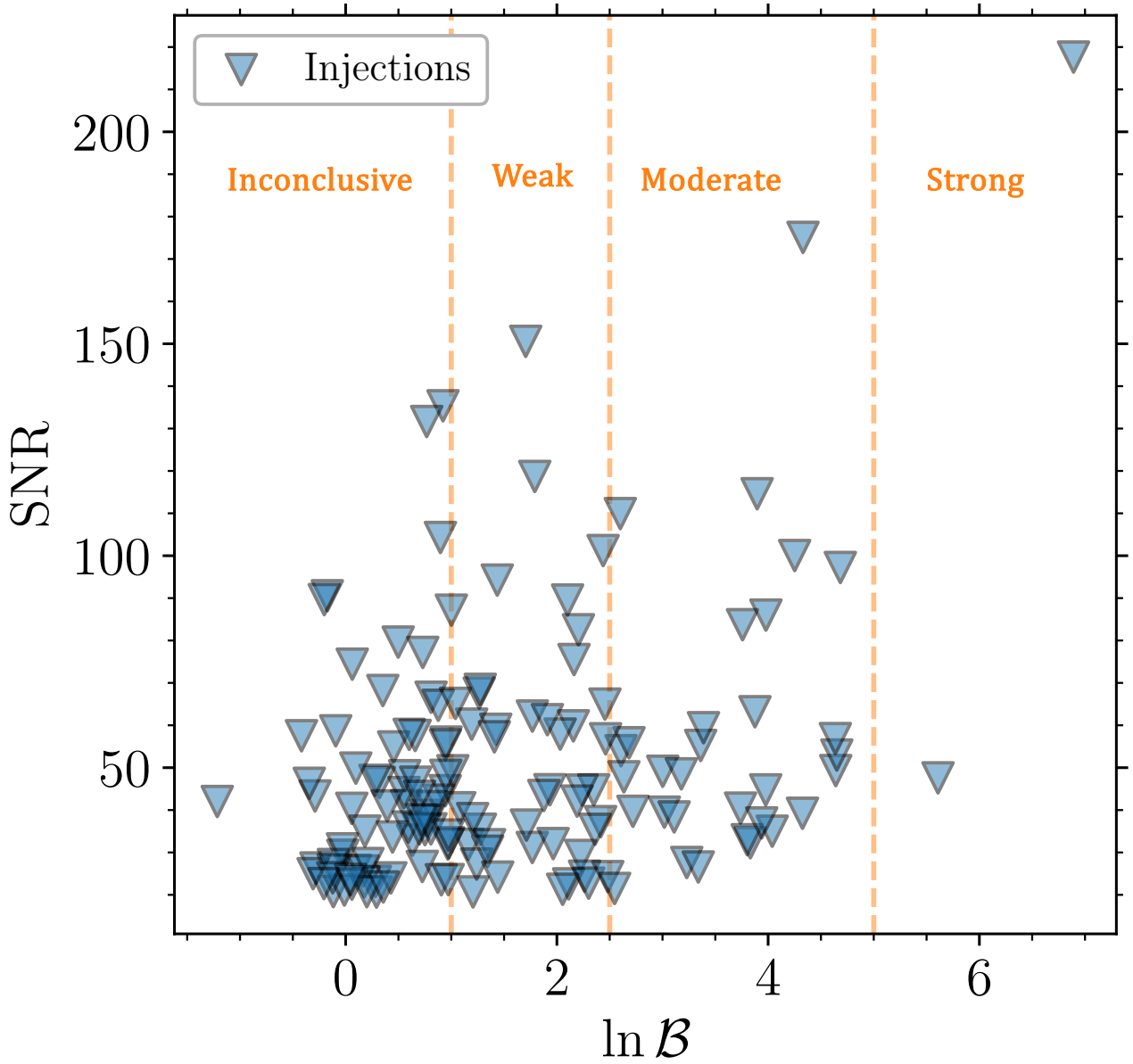

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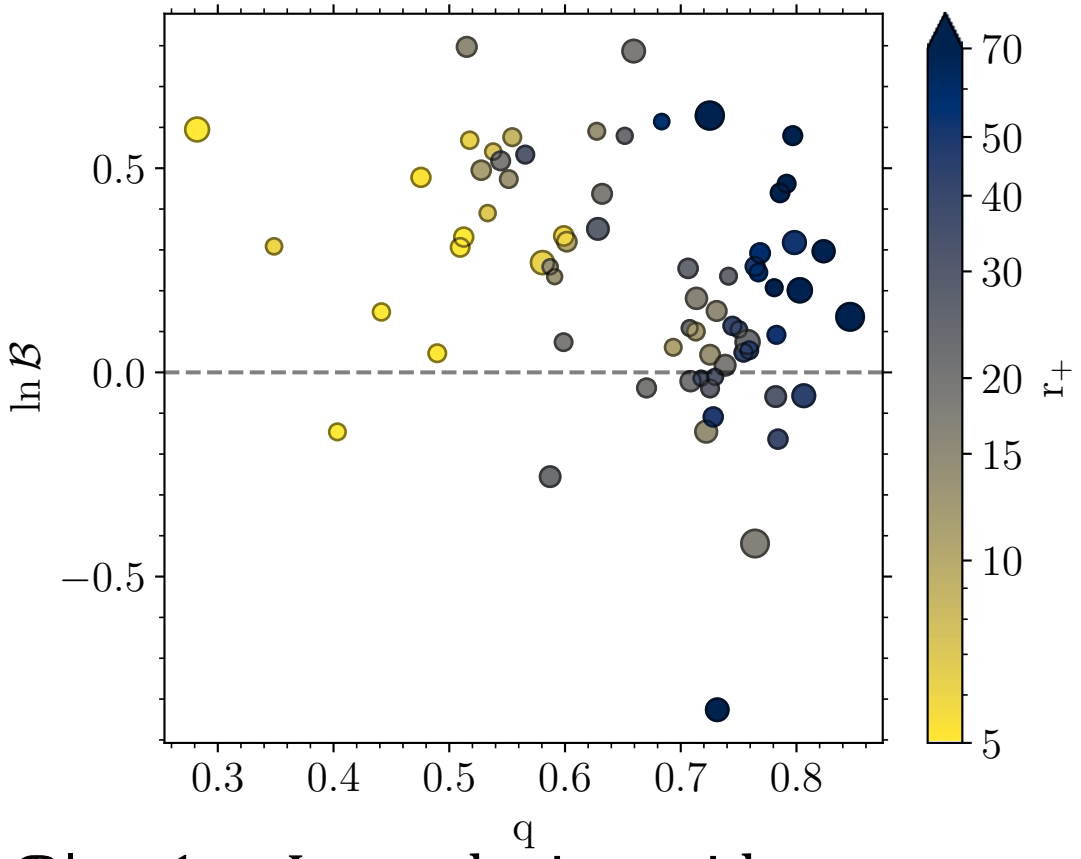
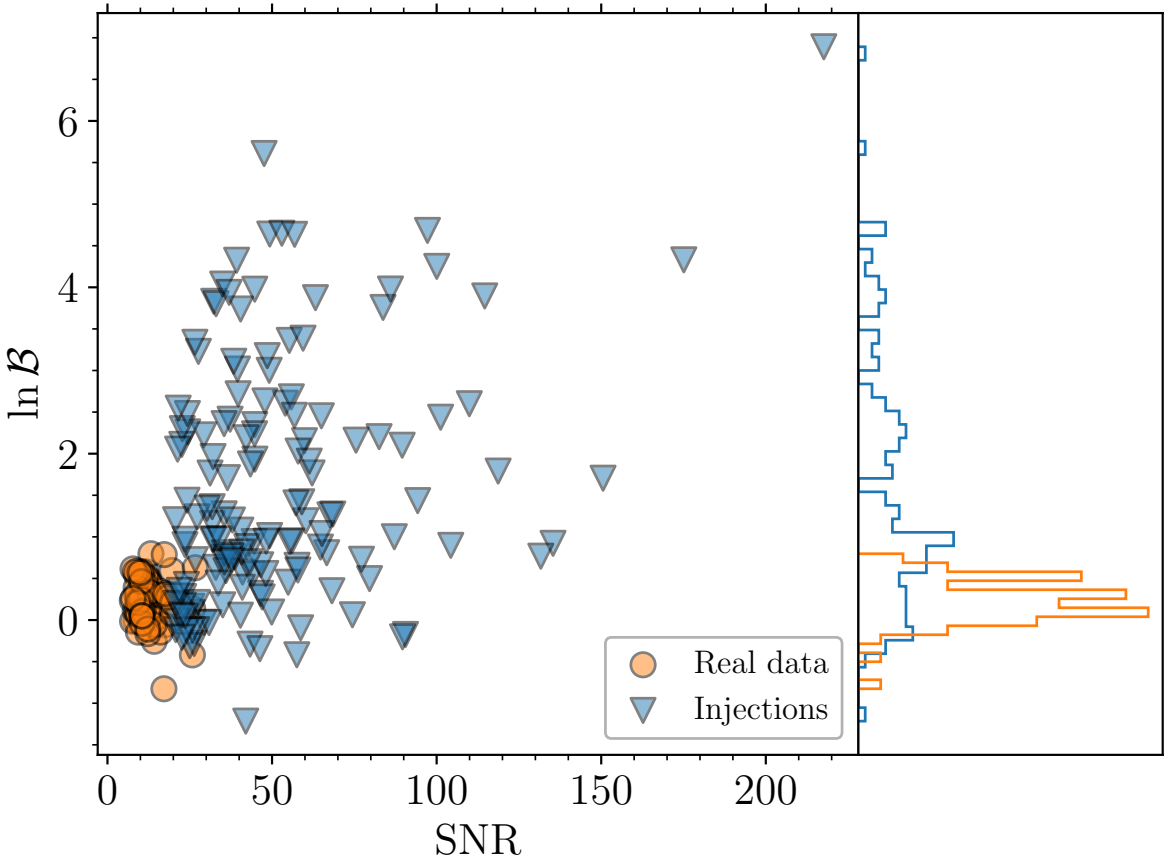
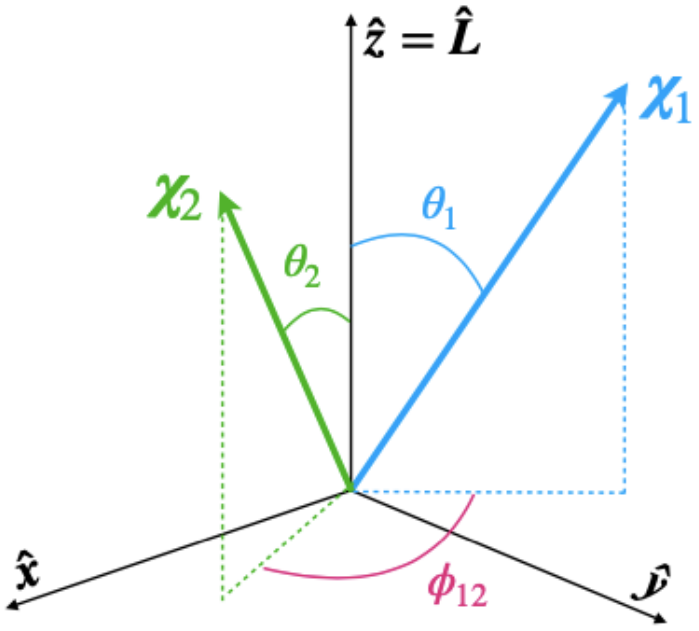
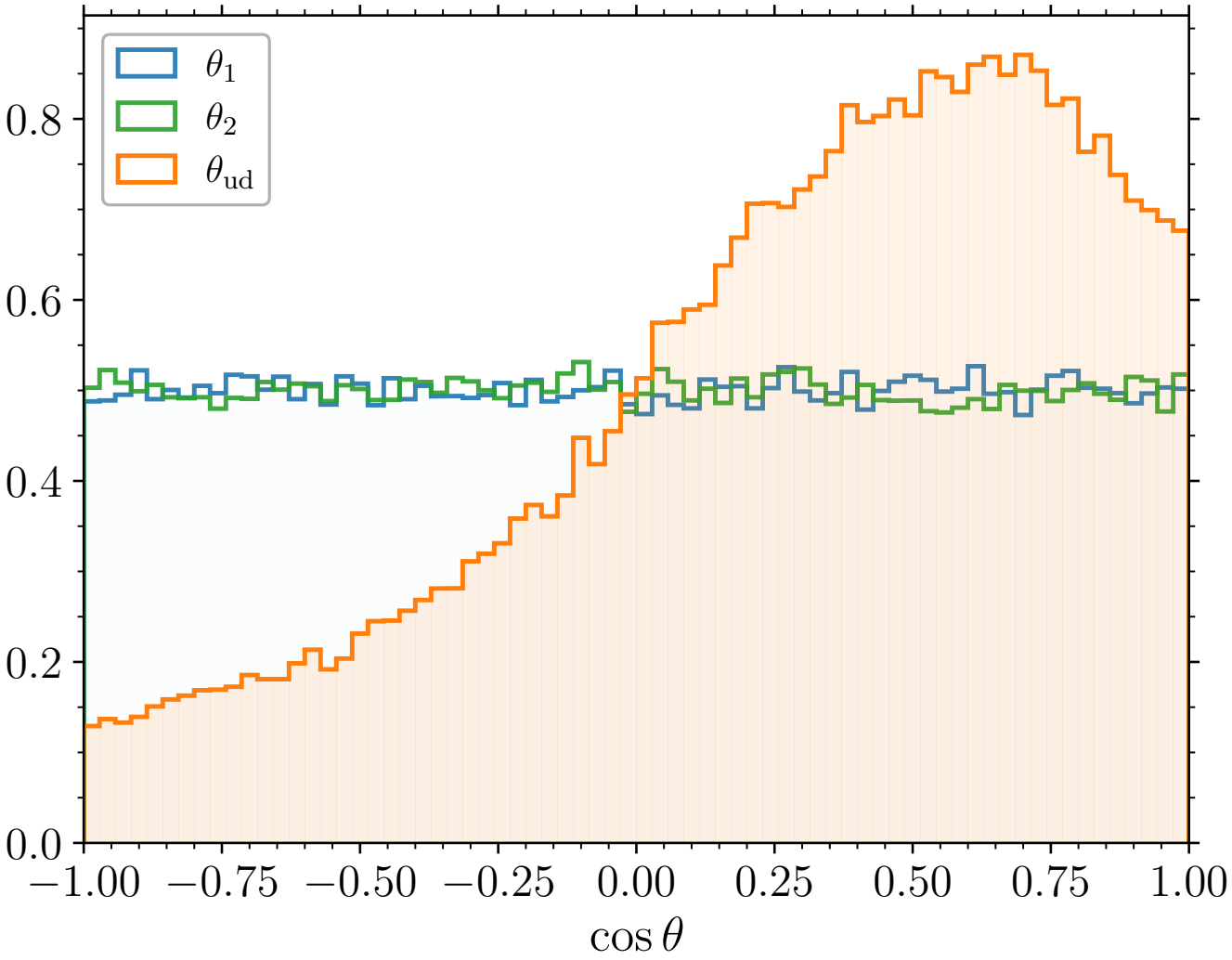




$|\ln\mathcal{B}| < 1 \rightarrow$ Inconclusive evidence

Priors





$$\cos\theta_{ud} = \frac{\chi_1 - q\chi_2}{\chi_1 + q\chi_2}$$

$\cos\theta_1$
$\cos\theta_2$

→ Standard uniform priors

Savage-Dickey Ratio for nested models

- Nested model $\mathcal{H}_\mathrm{N} : \mathcal{H}_\mathrm{B} \wedge \gamma = \gamma_\mathrm{N}(\varphi)$ where φ are the common parameters

- Bayes factor: $\mathcal{B} = \dfrac{\mathcal{Z}(d|\mathcal{H}_\mathrm{N})}{\mathcal{Z}(d|\mathcal{H}_\mathrm{B})}$

- Evidence for the nested model:

$$\mathcal{Z}(d|\mathcal{H}_N) = \int \mathcal{L}(d|\varphi, \mathcal{H}_\mathrm{N})\, \pi(\varphi|\mathcal{H}_\mathrm{N}) \mathrm{d}\varphi$$

$$= \int \underbrace{\mathcal{L}(d|\varphi, \gamma{=}\gamma_\mathrm{N}(\varphi), \mathcal{H}_\mathrm{B})}_{\text{Bayes' theorem}}\, \pi(\varphi|\gamma{=}\gamma_\mathrm{N}(\varphi), \mathcal{H}_\mathrm{B})\, \mathrm{d}\varphi$$

$$\frac{p(\varphi, \gamma{=}\gamma_\mathrm{N}(\varphi)|d, \mathcal{H}_\mathrm{B})\mathcal{Z}(d|\mathcal{H}_\mathrm{B})}{\pi(\varphi, \gamma{=}\gamma_\mathrm{N}(\varphi)|\mathcal{H}_\mathrm{B})}$$

$$\mathcal{B} = \frac{\mathcal{Z}(d|\mathcal{H}_\mathrm{N})}{\mathcal{Z}(d|\mathcal{H}_\mathrm{B})} = \int \mathrm{d}\varphi\, p(\varphi, \gamma{=}\gamma_\mathrm{N}(\varphi)|d, \mathcal{H}_\mathrm{B}) \frac{\pi(\varphi|\gamma{=}\gamma_\mathrm{N}(\varphi), \mathcal{H}_\mathrm{B})}{\pi(\varphi, \gamma{=}\gamma_\mathrm{N}(\varphi)|\mathcal{H}_\mathrm{B})}$$

Savage-Dickey Ratio for nested models

$$\mathcal{B} = \frac{\mathcal{Z}(d|\mathcal{H}_{\mathrm{N}})}{\mathcal{Z}(d|\mathcal{H}_{\mathrm{B}})} = \int \mathrm{d}\varphi\, p(\varphi, \gamma = \gamma_{\mathrm{N}}(\varphi)|d, \mathcal{H}_{\mathrm{B}}) \boxed{\frac{\pi(\varphi|\gamma = \gamma_{\mathrm{N}}(\varphi), \mathcal{H}_{\mathrm{B}})}{\pi(\varphi, \gamma = \gamma_{\mathrm{N}}(\varphi)|\mathcal{H}_{\mathrm{B}})}}$$

Rule of conditional probability

$$\boxed{\pi(\gamma = \gamma_{\mathrm{N}}(\varphi)|\mathcal{H}_{\mathrm{B}}) = \int \pi(\varphi', \gamma = \gamma_{\mathrm{N}}(\varphi)|\mathcal{H}_{\mathrm{B}})\mathrm{d}\varphi'}$$

The general expression for the Bayes factor in the case of nested models is:

$$\mathcal{B} = \int \frac{p(\varphi, \gamma = \gamma_{\mathrm{N}}(\varphi)|d, \mathcal{H}_{\mathrm{B}})}{\int \pi(\varphi', \gamma = \gamma_{\mathrm{N}}(\varphi)|\mathcal{H}_{\mathrm{B}})\mathrm{d}\varphi'} \mathrm{d}\varphi$$

From here, we can recover the classic expression for the Savage Dickey ratio

Savage-Dickey Ratio for nested models

We can perform a suitable change of variables:

$$\{\varphi, \gamma\} \longrightarrow \{\bar{\varphi} = \varphi, \bar{\gamma} = \gamma - \gamma_{\mathrm{N}}(\varphi)\}$$

The determinant of the resulting Jacobian is

$$\det \begin{pmatrix} \partial\bar{\varphi}/\partial\varphi & \partial\bar{\varphi}/\partial\gamma \\ \partial\bar{\gamma}/\partial\varphi & \partial\bar{\gamma}/\partial\gamma \end{pmatrix} = \det \begin{pmatrix} 1 & 0 \\ -\mathrm{d}\gamma_{\mathrm{N}}/\mathrm{d}\varphi & 1 \end{pmatrix} = 1$$

such that, for any probability distribution P, one can write

$$P(\varphi, \gamma = \gamma_{\mathrm{N}}(\varphi)) = P(\varphi, \bar{\gamma} = 0)$$

$$\rightarrow \quad \mathcal{B} = \int \frac{p(\varphi, \gamma = \gamma_{\mathrm{N}}(\varphi) | d, \mathcal{H}_{\mathrm{B}})}{\int \pi(\varphi', \gamma = \gamma_{\mathrm{N}}(\varphi) | \mathcal{H}_{\mathrm{B}}) \mathrm{d}\varphi'} \mathrm{d}\varphi = \frac{\int p(\varphi, \bar{\gamma} = 0 | d, \mathcal{H}_{\mathrm{B}}) \mathrm{d}\varphi}{\int \pi(\varphi', \bar{\gamma} = 0 | \mathcal{H}_{\mathrm{B}}) \mathrm{d}\varphi'} = \frac{p(\bar{\gamma} = 0 | d, \mathcal{H}_{\mathrm{B}})}{\pi(\bar{\gamma} = 0 | \mathcal{H}_{\mathrm{B}})}$$

Case of the updown instability

We perform the following change of variables:

$$\{q, \chi_1, \chi_2, \theta_1, \theta_2, q\} \rightarrow \{q, \chi_1, \chi_2, \gamma_1, \gamma_2, \gamma_3\}$$

where

$$\begin{aligned} \cos\theta_1 &= \frac{\chi_1 - q\chi_2}{\chi_1 + q\chi_2}\,, \\ \cos\theta_2 &= \frac{\chi_1 - q\chi_2}{\chi_1 + q\chi_2}\,, \\ \phi_{12} &= 0\,, \end{aligned} \quad \longrightarrow \quad \begin{aligned} \gamma_1 &= \frac{\cos\theta_1 - \cos\theta_{\rm ud}(q, \chi_1, \chi_2)}{2}\,, \\ \gamma_2 &= \frac{\cos\theta_2 - \cos\theta_{\rm ud}(q, \chi_1, \chi_2)}{2}\,, \\ \gamma_3 &= \frac{1}{\pi}\arctan\left(\frac{\sin\phi_{12}}{\cos\phi_{12}}\right)\,, \end{aligned}$$

The up-down instability is mapped in the point $\gamma = \{0,0,0\}$ and the Bayes factor can be computed as

$$\mathcal{B} = \frac{p(\bar{\gamma} = 0 | d, \mathcal{H}_{\rm B})}{\pi(\bar{\gamma} = 0 | \mathcal{H}_{\rm B})}$$

Multi-timescale analysis

- Orbital timescale: $t_{orb} \sim r^{3/2}$
- Precession timescale: $t_{pre} \sim r^{5/2}$
- Radiation-reaction timescale: $t_{RR} \sim r^{4}$

In the post-Newtonian regime (large separations):

$$r \gg M \longrightarrow t_{orb} \ll t_{pre} \ll t_{RR}$$

Each part of the binary dynamics can be addressed independently

Multi-timescale analysis

$t_{orb} \ll t_{pre}$ → Study precession in BBHs averaging the motion over the orbital period

- 2PN orbit-averaged spin precession equations

$$\frac{d\mathbf{S_i}}{dt} = \mathbf{\Omega_i} \times \mathbf{S_i}$$

$$\frac{d\mathbf{L}}{dt} = (\mathbf{\Omega_L} \times \hat{L})L + \frac{dL}{dt}\hat{\mathbf{L}}$$

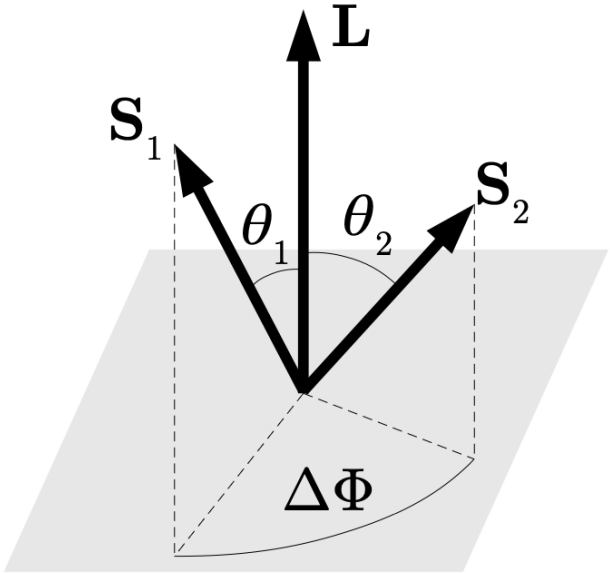
[Damour, 2008]

$$\begin{cases} \mathbf{\Omega}_L = \Omega_1\chi_1\hat{\mathbf{S}}_1 + \Omega_2\chi_2\hat{\mathbf{S}}_2 \\ \Omega_1 = \frac{M^2}{2r^3(1+q)^2}\left[4 + 3q - \frac{3q\chi_{\rm eff}}{(1+q)}\frac{M^2}{L}\right] \\ \Omega_2 = \frac{qM^2}{2r^3(1+q)^2}\left[4q + 3 - \frac{3q\chi_{\rm eff}}{(1+q)}\frac{M^2}{L}\right] \end{cases}$$

- **9D problem:** 2 spin vectors and 1 orbital vector

Dimensionality reduction

- **9D problem:** 2 spin vectors and 1 orbital vector
- **7D:** 2 BH spin magnitudes are conserved
- **4D**: choose a reference frame
- **3D:** χ_{eff} is a conserved quantity a 2PN



$$\boxed{t_{pre} \ll t_{RR}}$$

- **1D problem:** two additional conserved quantities on the short precessional timescale

$$\frac{dL}{dt} = 0 \rightarrow L = |\boldsymbol{L}|$$
$$J = |\boldsymbol{L} + \boldsymbol{S_1} + \boldsymbol{S_2}|$$

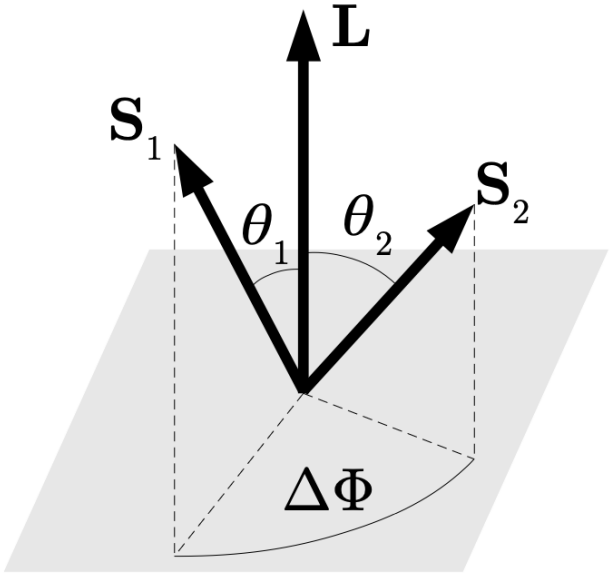
Motion on the precession timescale

$$t_{orb} \ll t_{pre} \ll t_{RR}$$

The entire precessional dynamics can be parametrized with a single variable, the total spin magnitude

$$S = |\boldsymbol{S_1} + \boldsymbol{S_2}|$$

[Kesden, 2015]



Perturbation of the aligned configurations

- Small perturbations to aligned-spin configurations evolve as an **harmonic oscillator**

$$\boxed{\frac{d^2}{dt^2}(S^2 - S_*^2) + \omega^2(S^2 - S_*^2) \simeq 0}$$

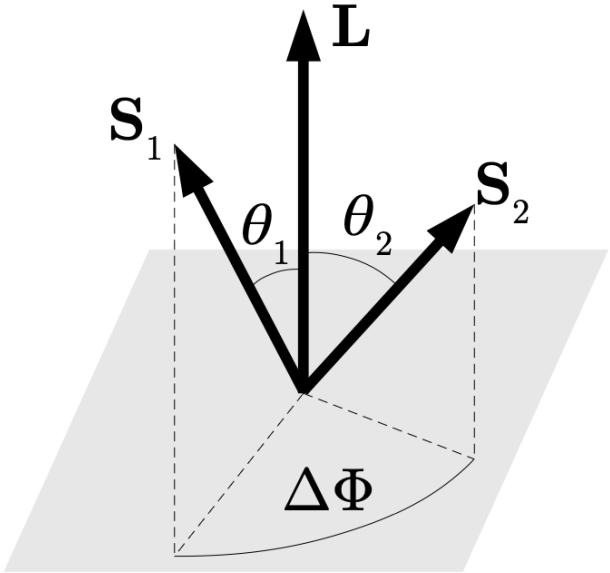
STABILITY?

- Real frequency $\omega^2 > 0$ → small amplitude oscillations (stable configuration)
- If $\omega^2 > 0$ → $S^2 = \text{const}$ → onset of the instability
- Imaginary frequency $\omega^2 < 0$ → **dynamical instability**

Spin-orbit resonances

- Spin-orbit resonances: the three angular momenta remain coplanar

$$\Delta\Phi = 0, \pi$$



Savage-Dickey Ratio for nested models

$$M_B \in M_A$$

Model M_A: 15 parameters

$$\overrightarrow{\theta_A} = \{\underbrace{m_1, m_2, \chi_1, \chi_2, \phi_{JL}, d_L, ra, dec, \psi, \phi, \theta_{JN}, t_C}_{\phi = \text{common parameters with } M_B}, \underbrace{\theta_1, \theta_2, \phi_{12}}_{\delta = \text{extra parameters of } M_A \text{ that are fixed to certain } \delta_0 \text{ in } M_B}\}$$

Model M_B: 12 parameters $\rightarrow \overrightarrow{\theta_B} = \phi$ and $\delta = \{\theta_1, \theta_2, \phi_{12}\} = \delta_0$

$$Z(d \mid M_B) = \int d\phi \, P(d \mid \phi, M_B) \, \pi(\phi \mid M_B) =$$

$$= \int d\phi \, P(d \mid \phi, \delta = \delta_0, M_A) \, \pi(\phi \mid \delta = \delta_0, M_A) = P(d \mid \delta = \delta_0, M_A)$$

Savage-Dickey Ratio for nested models

$$\boldsymbol{M_B \in M_A}$$

$$Z(d \mid M_B) = \int \mathrm{d}\phi \, P(d \mid \phi, M_B) \, \pi(\phi \mid M_B) =$$

$$= \int \mathrm{d}\phi \, P(d \mid \phi, \delta = \delta_0, M_A) \, \pi(\phi \mid \delta = \delta_0, M_A) = P(d \mid \delta = \delta_0, M_A)$$

By Bayes' theorem we can rewrite this last line (which has the shape of a likelihood) as

$$P(d \mid \delta = \delta_0, M_A) = \frac{P(\delta = \delta_0 \mid d, M_A) Z(d \mid M_A)}{\pi(\delta = \delta_0 \mid M_A)}$$

$$\longrightarrow \boxed{Z(d \mid M_B) = \frac{P(\delta = \delta_0 \mid d, M_A) Z(d \mid M_A)}{\pi(\delta = \delta_0 \mid M_A)}}$$

Savage-Dickey Ratio for nested models

$$\boldsymbol{M_B \in M_A}$$

$$Z(d \mid M_B) = \frac{P(\delta = \delta_0 | d, M_A) P(d | M_A)}{P(\delta = \delta_0 | M_A)}$$

Bayes factor:

$$\mathrm{BF} = \frac{Z(\mathrm{d} \mid \mathrm{M_B})}{Z(\mathrm{d} \mid \mathrm{M_A})} =$$

$$= Z(\mathrm{d} \mid \mathrm{M_B}) \cdot \frac{1}{Z(\mathrm{d} \mid \mathrm{M_A})} =$$

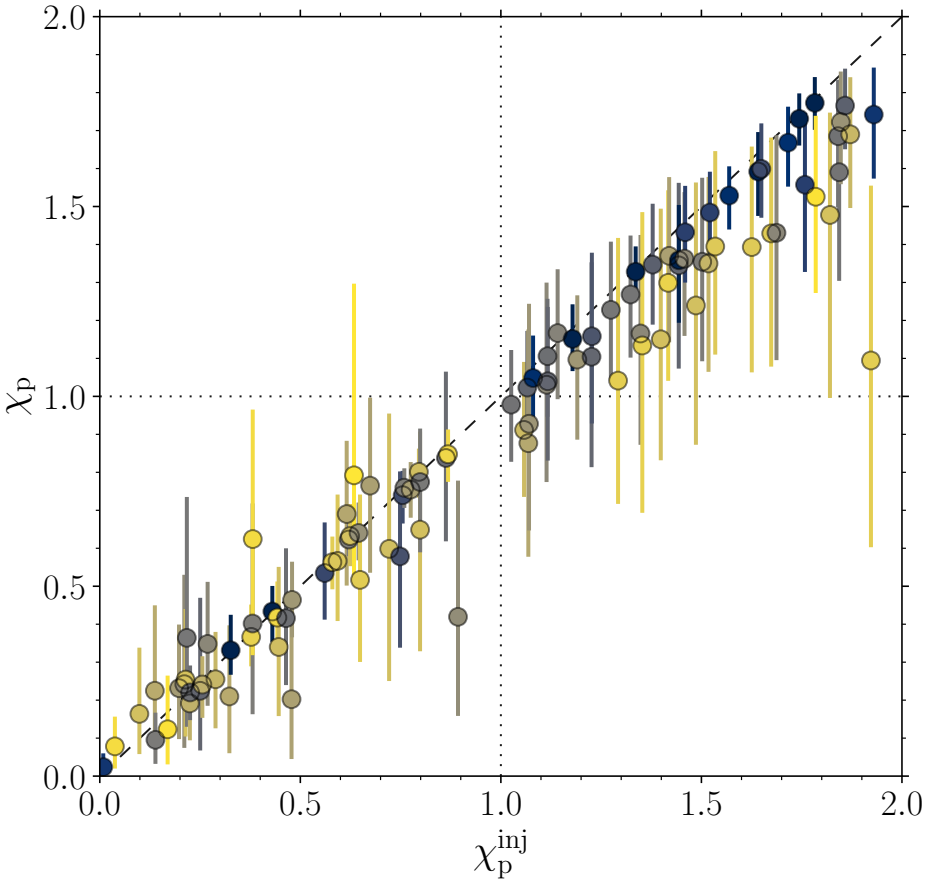
$$= Z(\mathrm{d} \mid \mathrm{M_B}) \cdot \frac{1}{Z(\mathrm{d} \mid \mathrm{M_A})} =$$

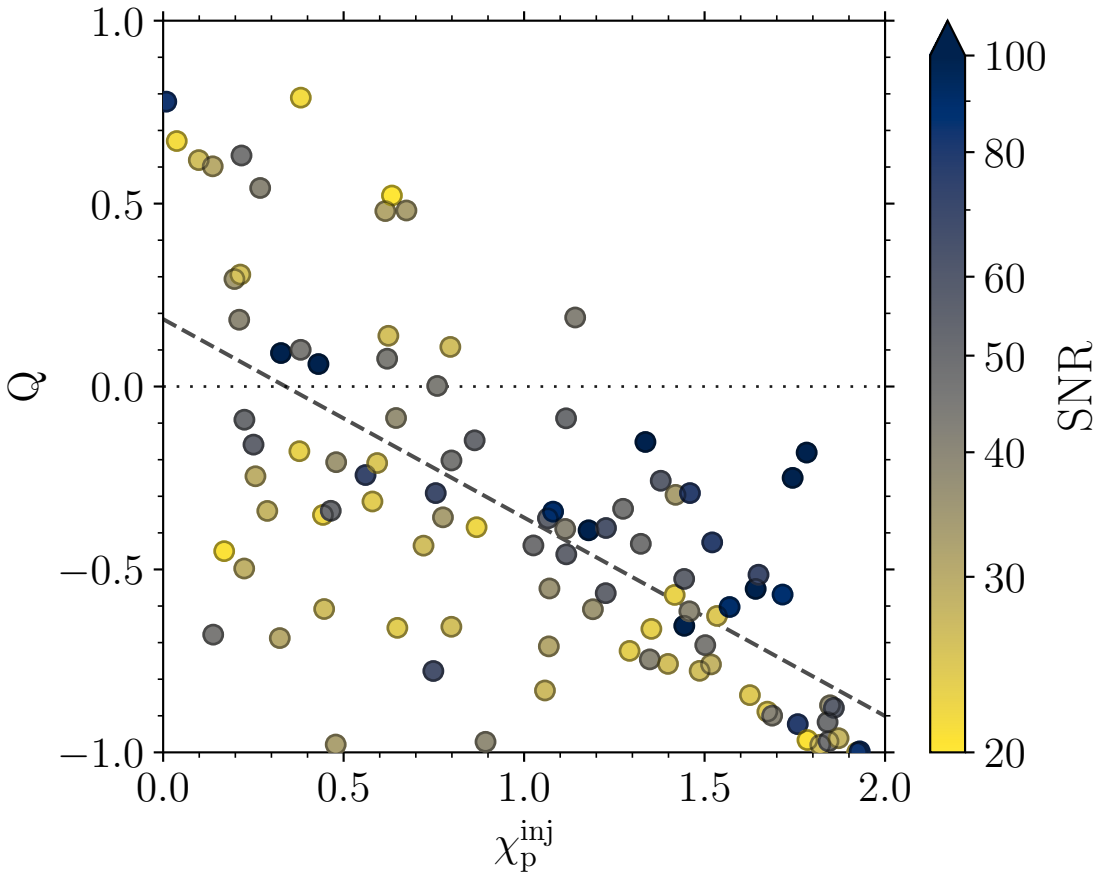
$$= \frac{P(\delta = \delta_0 | d, M_A) \cancel{Z(d | M_A)}}{P(\delta = \delta_0 | M_A)} \cdot \frac{1}{\cancel{Z(\mathrm{d} \mid \mathrm{M_A})}} =$$

$$= \frac{P(\delta = \delta_0 | d, M_A)}{P(\delta = \delta_0 | M_A)}$$

Two precessing spins

Statistical bias at $\chi_p > 1$





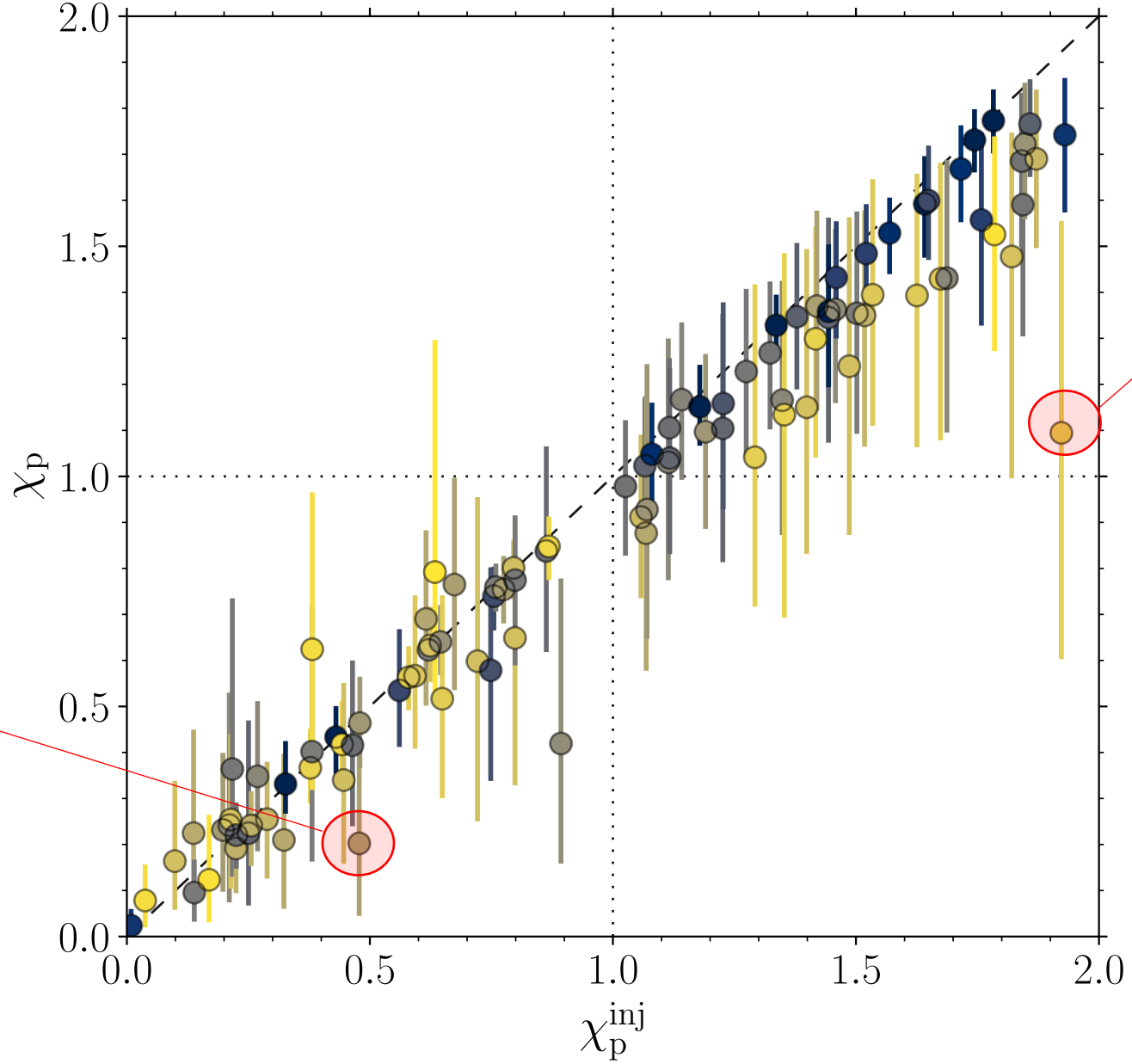
$Q > 0$:
overestimation

$Q < 0$:
underestimation

Statistical errors cause an underestimation of χ_p whenever $\chi_p > 1$

$$Q \equiv 2 \int_{-\infty}^{\chi_p^{inj}} P(\chi_p)\, d\chi_p - 1 \in [-1, 1]$$

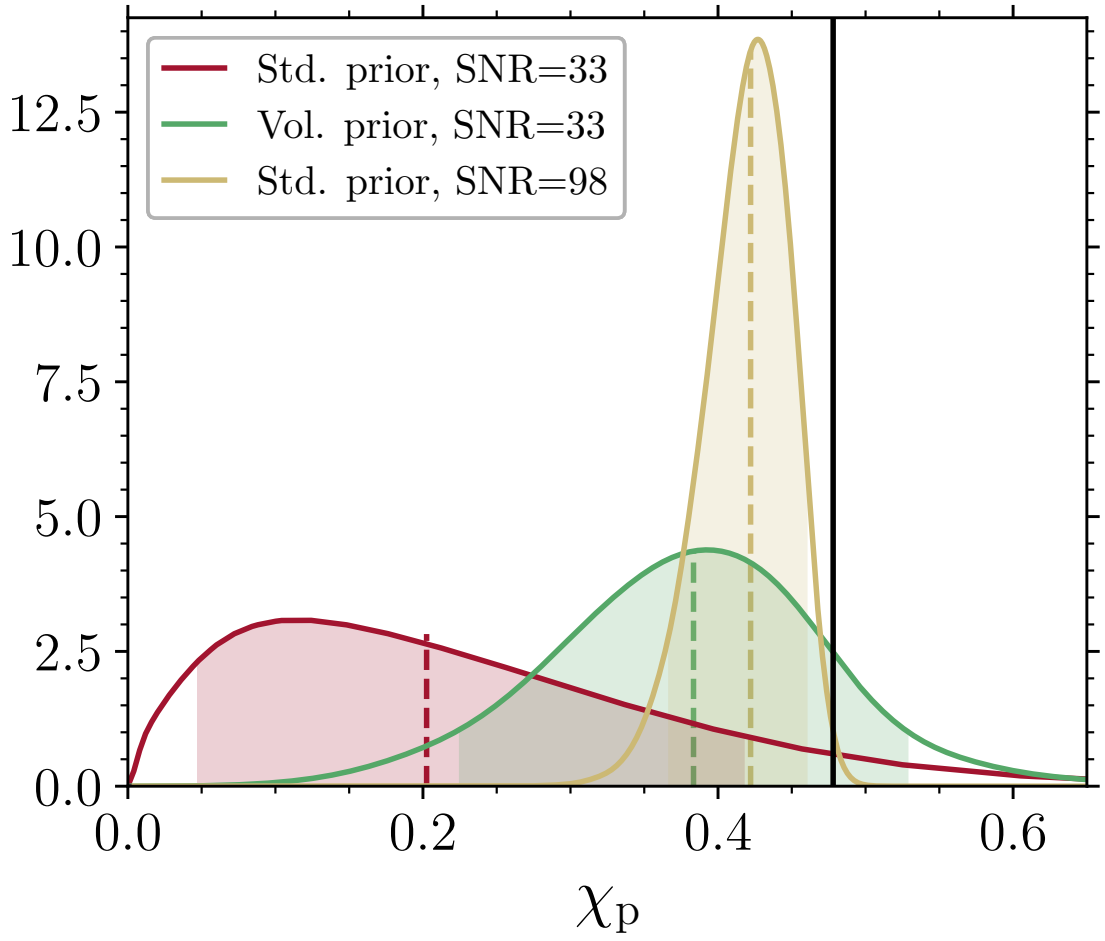
Prior effects: standard BBH priors

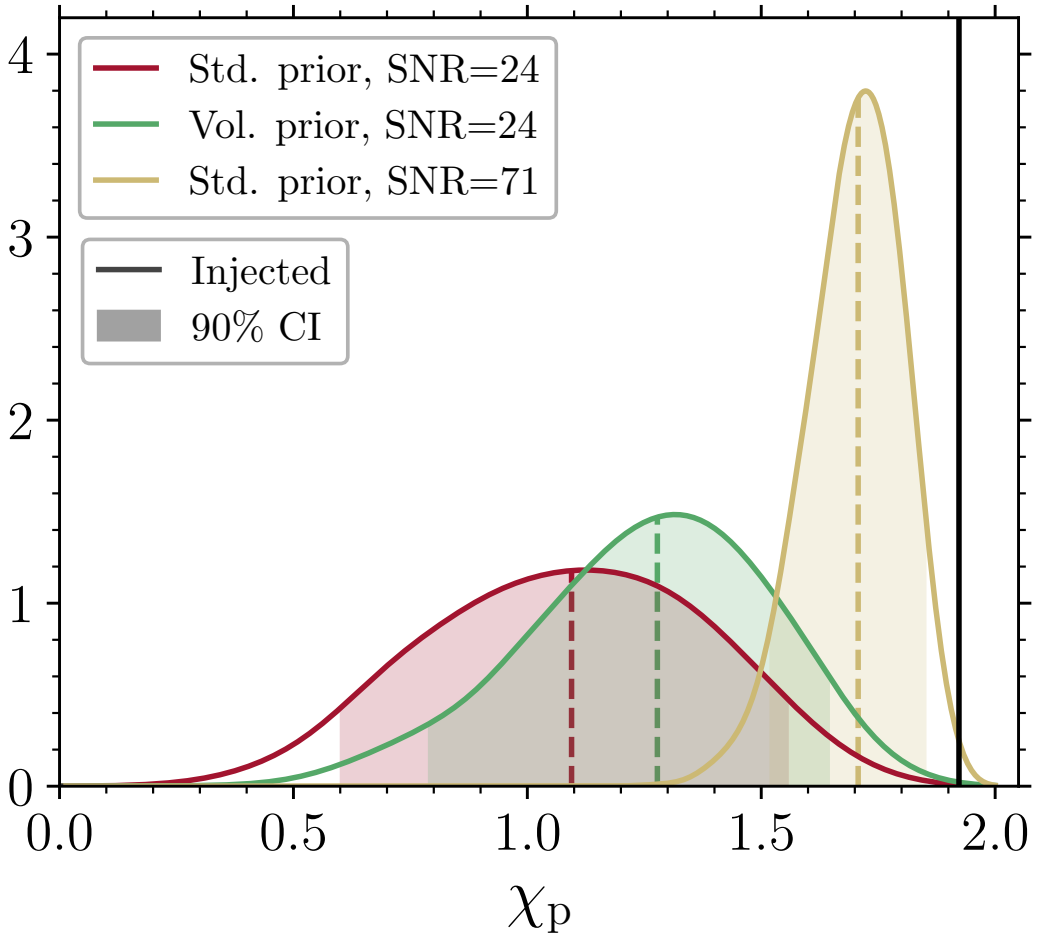


$\chi_p^{inj} = 1.92$
$\chi_p = 1.10^{+0.46}_{-0.49}$
$Q = -1$
SNR=24

$\chi_p^{inj} = 0.48$
$\chi_p = 0.20^{+0.22}_{-0.16}$
$Q = -0.98$
SNR=33

Prior effects





$\chi_p^{inj} = 0.48$

$\chi_p = \{0.20^{+0.22}_{-0.16}, 0.38^{+0.14}_{-0.16}, 0.42^{+0.04}_{-0.05}\}$

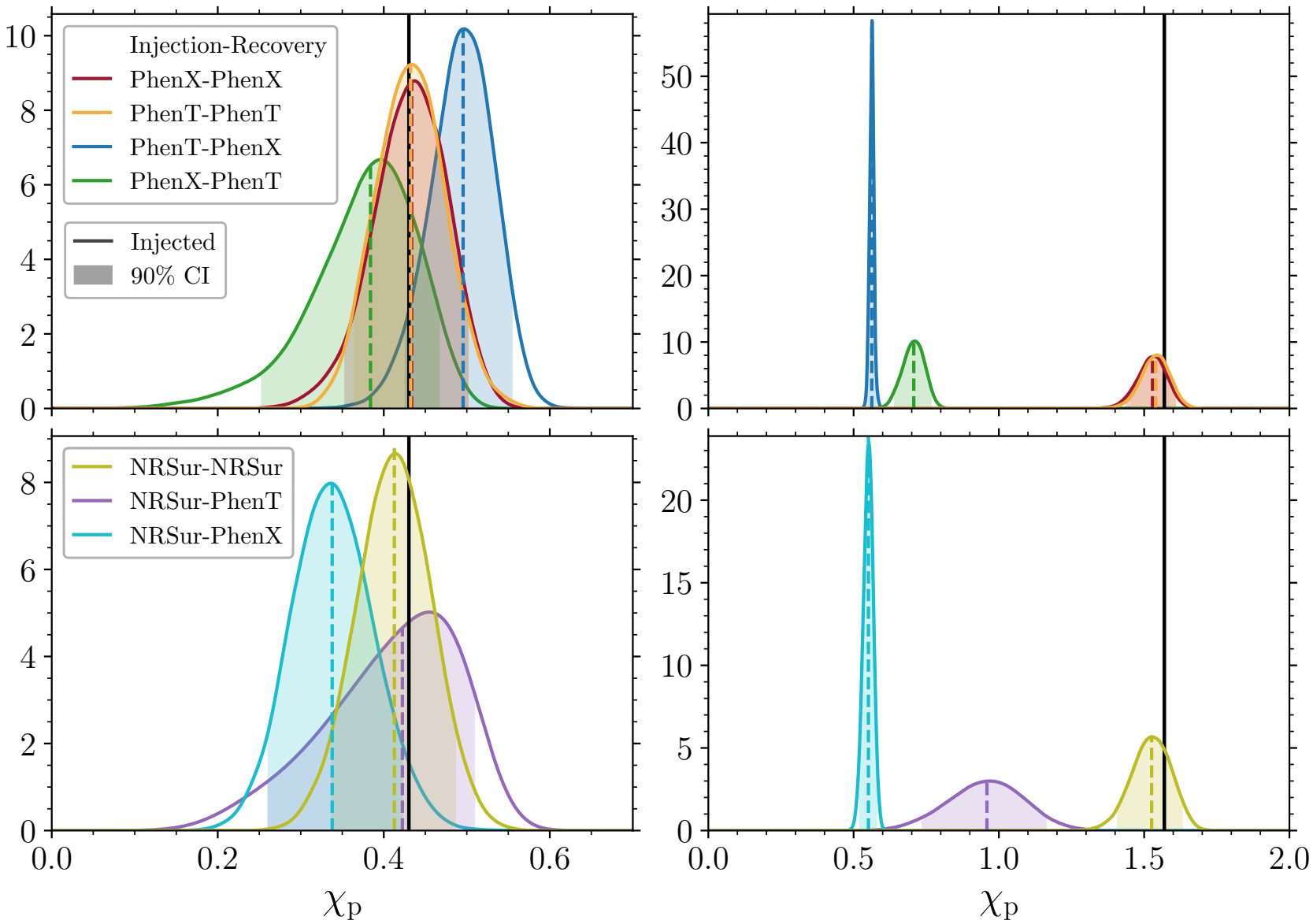
$Q = \{-0.98, -0.75, -1\}$

$\chi_p^{inj} = 1.92$

$\chi_p = \{1.10^{+0.46}_{-0.49}, 1.28^{+0.37}_{-0.49}, 1.71^{+0.14}_{-0.19}\}$

$Q = \{-1, -1, -1\}$

Waveform systematics

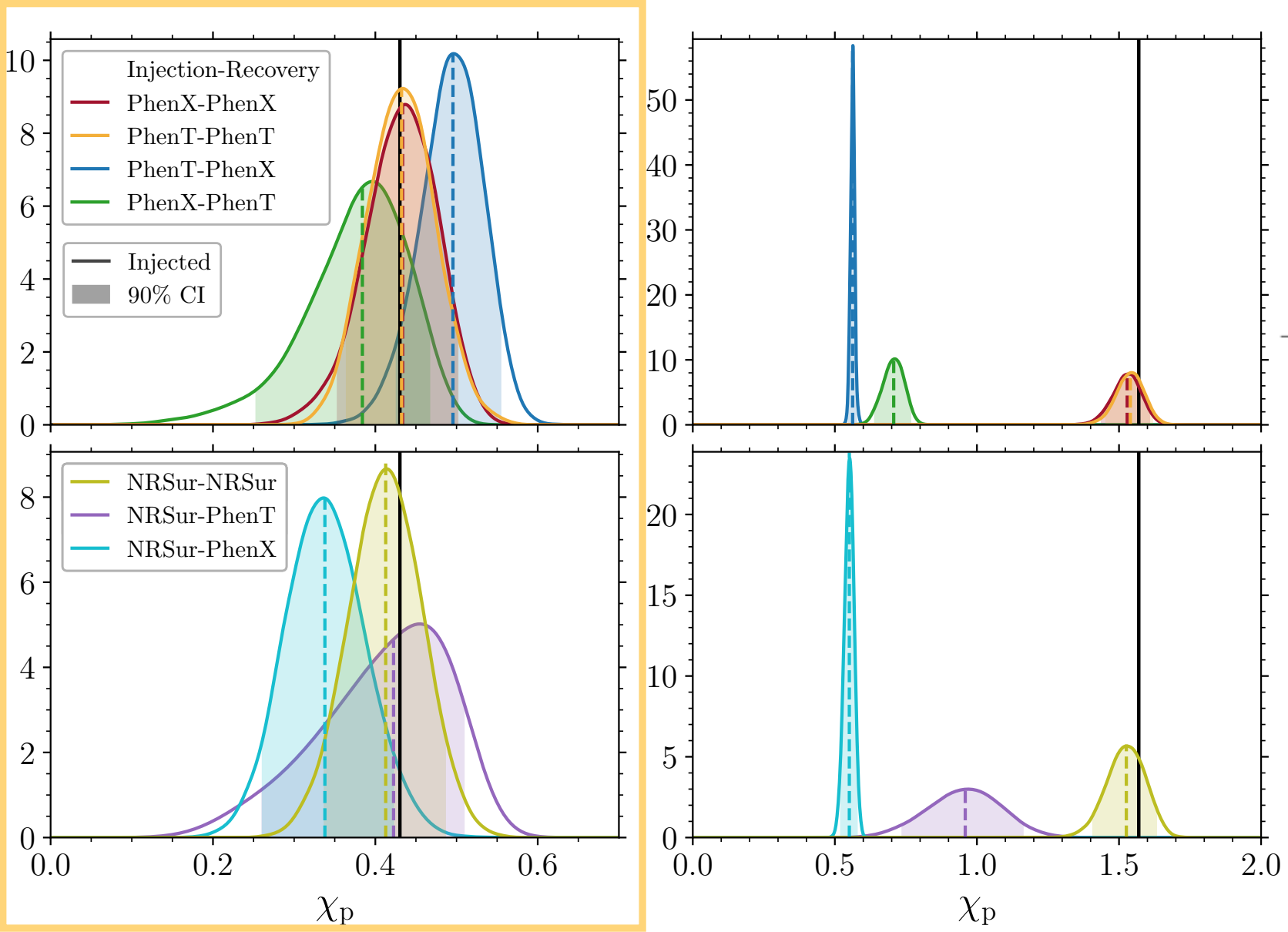


PhenX=IMRPhenomXPHM
PhenT=IMRPhenomTPHM
NRSur=NRSur7dq4

	$\chi_p^{inj} = 0.43$	$\chi_p^{inj} = 1.57$
M	$131.1 M_\odot$	$130.8 M_\odot$
SNR (PhenX)	107.3	90.3
SNR (PhenT)	93.6	81.7
SNR (NRSur*)	100.2	75.6

- High total mass → short signal → weaker precession signature
- $SNR\left(\chi_p^{inj} = 0.43\right) > SNR\left(\chi_p^{inj} = 1.57\right)$

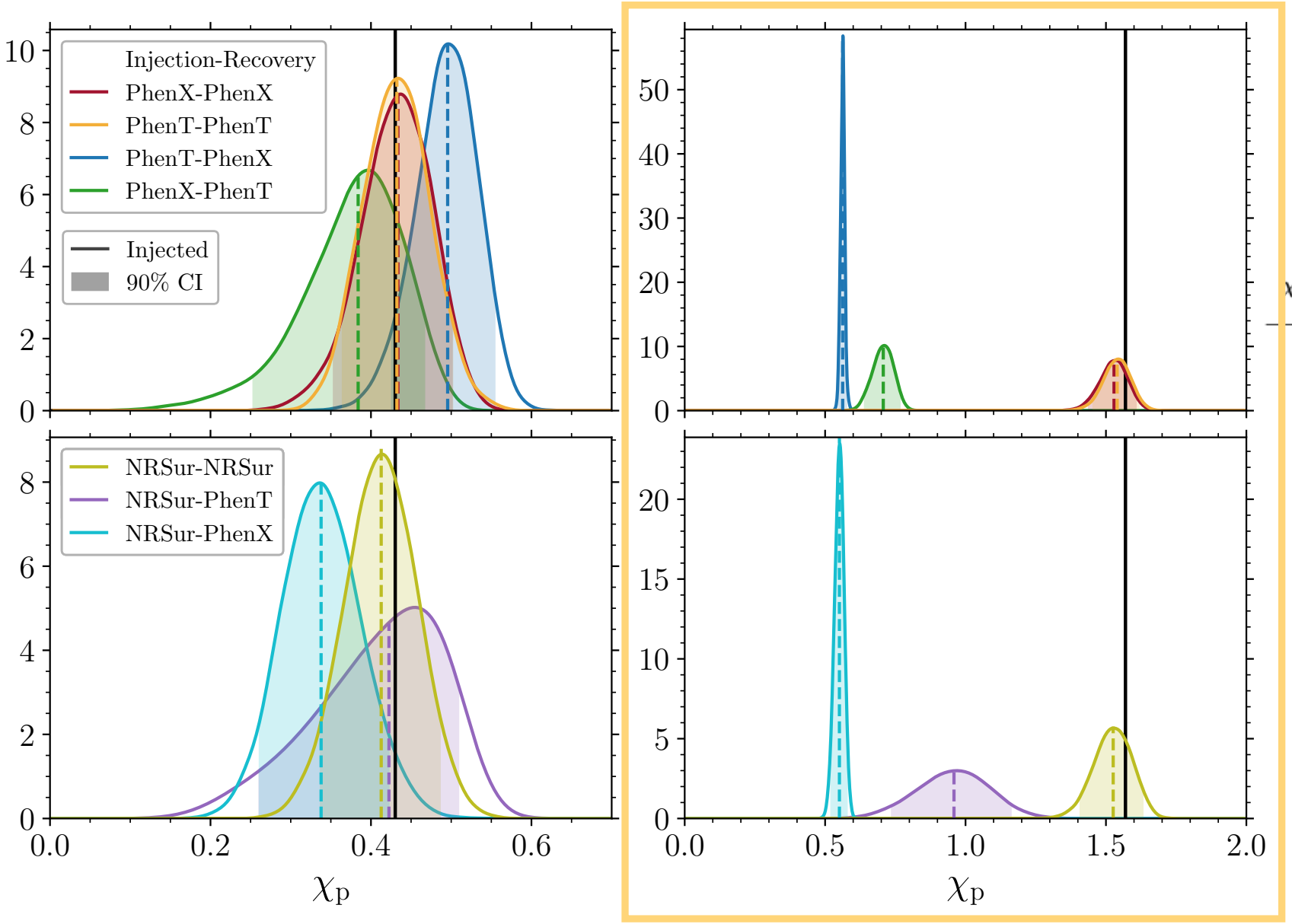
Waveform systematics



Low χ_p injection

$\chi_p^{inj} = 0.43$	PhenX		PhenT		NRSur	
	Q	χ_P	Q	χ_P	Q	χ_P
PhenX	0.06	$0.43^{+0.07}_{-0.08}$	-0.6	$0.38^{+0.08}_{-0.13}$	–	–
PhenT	0.88	$0.5^{+0.06}_{-0.07}$	0.04	$0.43^{+0.07}_{-0.07}$	–	–
NRSur	-0.93	$0.34^{+0.08}_{-0.08}$	-0.08	$0.42^{+0.09}_{-0.16}$	-0.3	$0.41^{+0.07}_{-0.08}$

Waveform systematics

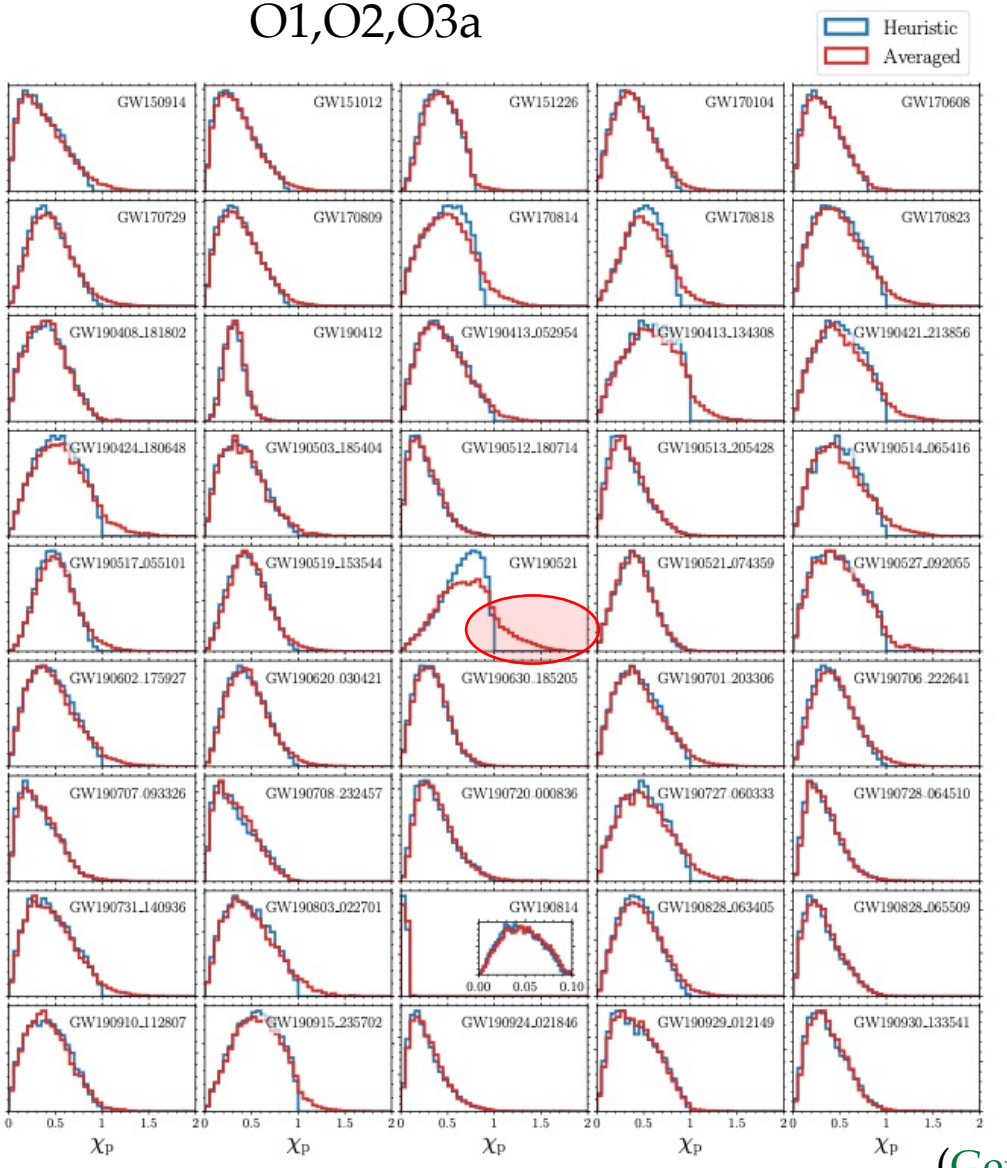


High χ_p injection

$\chi_p^{inj} = 1.57$	PhenX		PhenT		NRSur	
	Q	χ_P	Q	χ_P	Q	χ_P
PhenX	-0.6	$1.53^{+0.08}_{-0.09}$	-1	$0.71^{+0.06}_{-0.07}$	–	–
PhenT	-1	$0.56^{+0.01}_{-0.01}$	-0.47	$1.54^{+0.08}_{-0.08}$	–	–
NRSur	-1	$0.55^{+0.03}_{-0.03}$	-1	$0.96^{+0.2}_{-0.22}$	-0.48	$1.53^{+0.1}_{-0.12}$

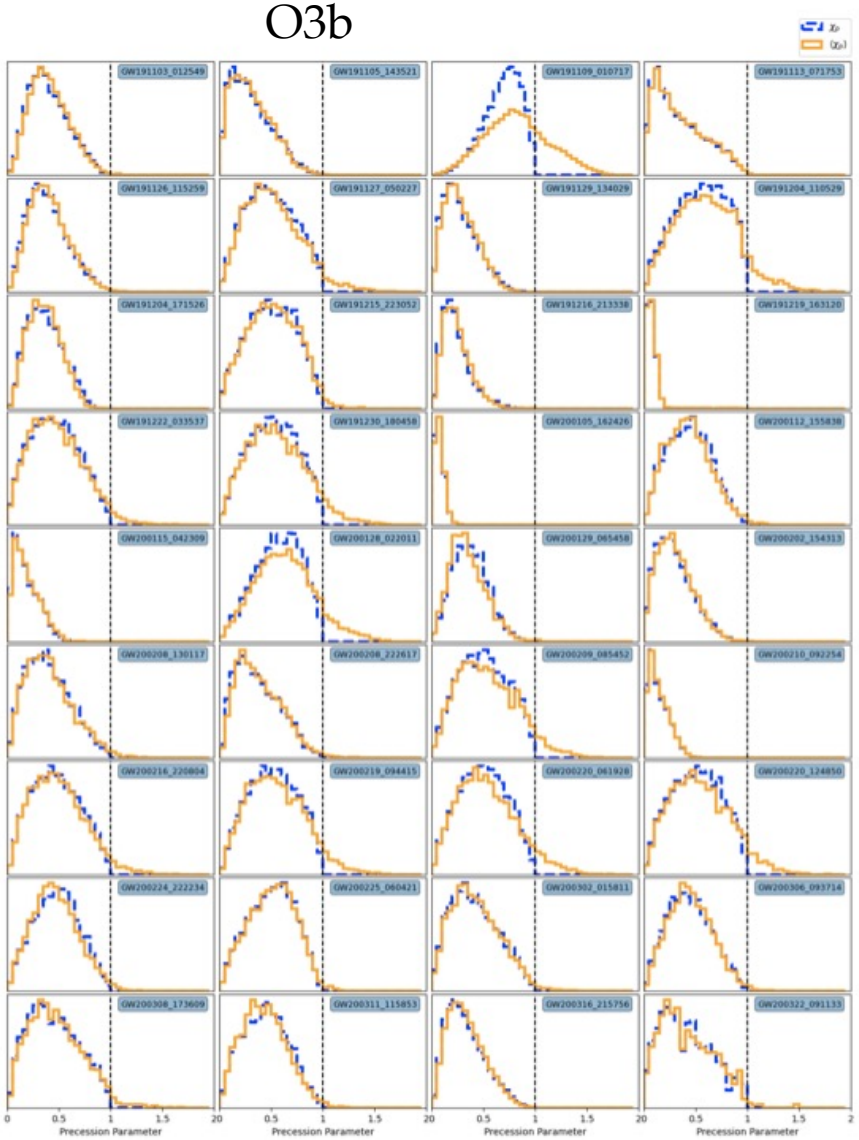
Two spin-effects in real data

O1,O2,O3a



(Gerosa+ 2020)

O3b



(Henshaw+ 2022)

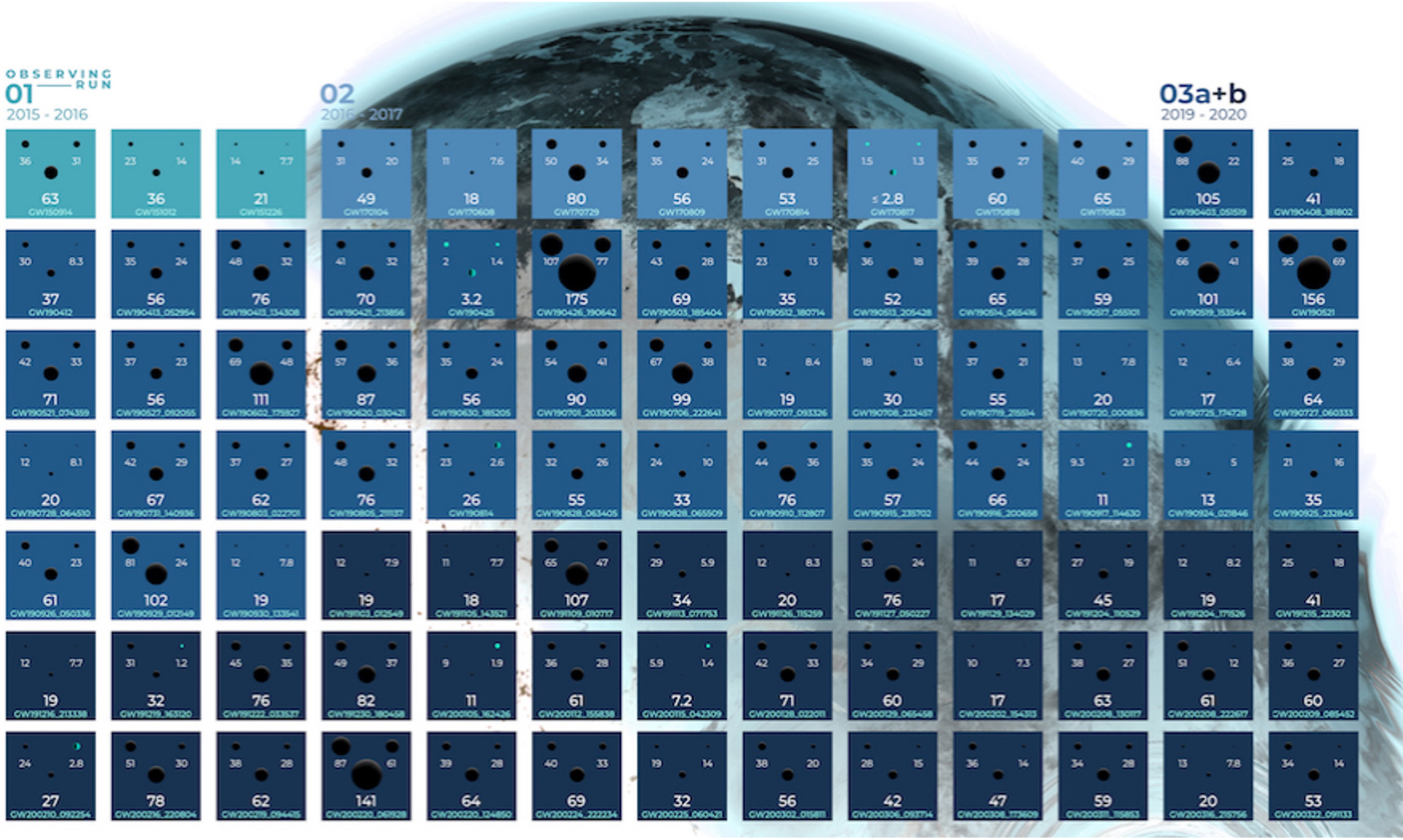
GW detections (GWTC-3)

2015: first GW detection

Observing runs
O1: 2015-2016
O2: 2016-2017
O3: 2019-2020
O4: March 2023

Total number of events is now **90**

- **Binary black holes** (BBH)
  Binary neutron stars (BNS)
- **Neutron star–black hole binaries** (NSBH)



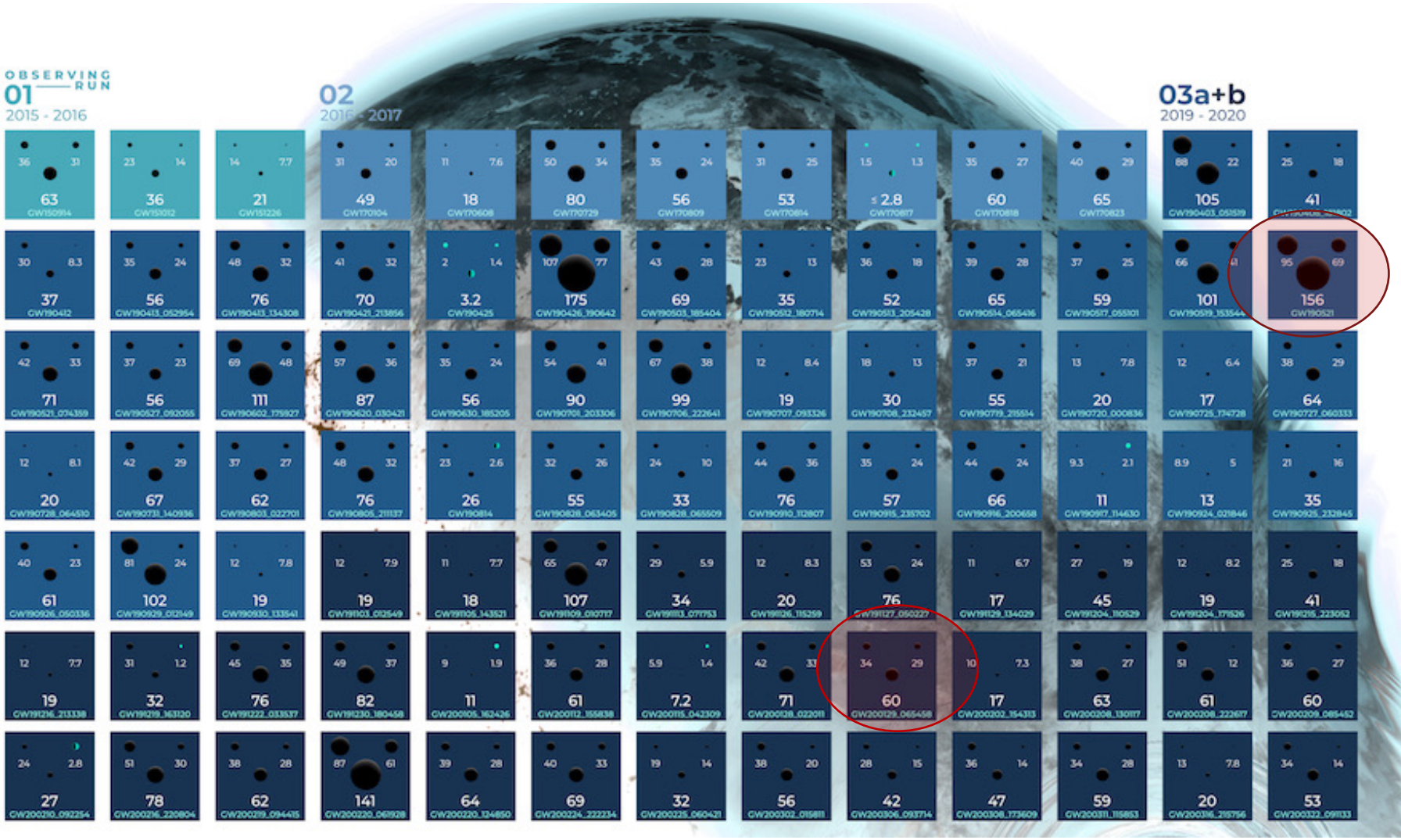
(LVK Collaboration)

Spin precession in individual events

Moderate evidence for highly precessing spins in GWTC-3.

Promising candidates for spin precession:

- **GW190521** (potential degeneracies with the eccentricity [Romero-Shaw et al. 2020])
- **GW200129** [Hannam et al. 2021] (possible issues in the glitch mitigation analysis [Payne et al. 2021])



(LVK Collaboration)

Standard spin priors

```
prior = {
'chirp_mass' : bilby.gw.prior.UniformInComponentsChirpMass(name='chirp_mass', minimum=10, maximum=60),
'mass_ratio' : bilby.gw.prior.UniformInComponentsMassRatio(name='mass_ratio', minimum=0.125, maximum=1),
'mass_1'     : bilby.gw.prior.Constraint(name='mass_1', minimum=5, maximum=100),
'mass_2'     : bilby.gw.prior.Constraint(name='mass_2', minimum=5, maximum=100),
'a_1'        : bilby.prior.analytical.Uniform(name='a_1', minimum=0, maximum=0.99),
'a_2'        : bilby.prior.analytical.Uniform(name='a_2', minimum=0, maximum=0.99),
'tilt_1'     : bilby.prior.analytical.Sine(name='tilt_1'),
'tilt_2'     : bilby.prior.analytical.Sine(name='tilt_2'),
'phi_12'     : bilby.prior.analytical.Uniform(name='phi_12', minimum=0, maximum=2 * np.pi, boundary='periodic'),
'phi_jl'     : bilby.prior.analytical.Uniform(name='phi_jl', minimum=0, maximum=2 * np.pi, boundary='periodic'),
'luminosity_distance' : bilby.gw.prior.UniformSourceFrame(name='luminosity_distance', minimum=1e2, maximum=5e3,
unit='Mpc'),
'dec'      : bilby.prior.analytical.Cosine(name='dec'),
'ra'       : bilby.prior.analytical.Uniform(name='ra', minimum=0, maximum=2 * np.pi, boundary='periodic'),
'theta_jn' : bilby.prior.analytical.Sine(name='theta_jn'),
'psi'      : bilby.prior.analytical.Uniform(name='psi', minimum=0, maximum=np.pi, boundary='periodic'),
'phase'    : bilby.prior.analytical.Uniform(name='phase', minimum=0, maximum=2 * np.pi, boundary='periodic')
}
```

$p(\chi)\mathrm{d}\chi \propto \mathrm{d}\chi$ → Spins uniform in magnitudes and isotropic in directions

Volumetric spin priors

```
prior = {
'mass_1'     : bilby.gw.prior.Constraint(name='mass_1', minimum=5, maximum=100),
'mass_2'     : bilby.gw.prior.Constraint(name='mass_2', minimum=5, maximum=100),
'chirp_mass' : bilby.gw.prior.UniformInComponentsChirpMass(name='chirp_mass', minimum=10, maximum=60),
'mass_ratio' : bilby.gw.prior.UniformInComponentsMassRatio(name='mass_ratio', minimum=0.125, maximum=1),
'a_1'        : PowerLaw(alpha=2, name='a_1', minimum=0, maximum=0.99),
'a_2'        : PowerLaw(alpha=2, name='a_1', minimum=0, maximum=0.99),
'tilt_1'     : bilby.prior.analytical.Sine(name='tilt_1'),
'tilt_2'     : bilby.prior.analytical.Sine(name='tilt_2'),
'phi_12'     : bilby.prior.analytical.Uniform(name='phi_12', minimum=0, maximum=2 * np.pi, boundary='periodic'),
'phi_jl'     : bilby.prior.analytical.Uniform(name='phi_jl', minimum=0, maximum=2 * np.pi, boundary='periodic'),
'luminosity_distance' : bilby.gw.prior.UniformSourceFrame(name='luminosity_distance', minimum=1e2, maximum=1500,
unit='Mpc'),
'dec'      : bilby.prior.analytical.Cosine(name='dec'),
'ra'       : bilby.prior.analytical.Uniform(name='ra', minimum=0, maximum=2 * np.pi, boundary='periodic'),
'theta_jn' : bilby.prior.analytical.Sine(name='theta_jn'),
'psi'      : bilby.prior.analytical.Uniform(name='psi', minimum=0, maximum=np.pi, boundary='periodic'),
'phase'    : bilby.prior.analytical.Uniform(name='phase', minimum=0, maximum=2 * np.pi, boundary='periodic'),
'geocent_time' : bilby.core.prior.Uniform(minimum=-0.1, maximum=0.1, name='geocent_time', latex_label='$t_c$',
unit='$s$')
}
```

$p(\chi)\mathrm{d}\chi \propto \chi^2\mathrm{d}\chi$ → Spin vectors drawn uniformly in volume

Uninformative BBH priors

Intrinsic parameters

```
prior = {
'chirp_mass' : bilby.gw.prior.UniformInComponentsChirpMass(name='chirp_mass', minimum=10, maximum=60),
'mass_ratio' : bilby.gw.prior.UniformInComponentsMassRatio(name='mass_ratio', minimum=0.125, maximum=1),
'mass_1'     : bilby.gw.prior.Constraint(name='mass_1', minimum=5, maximum=100),
'mass_2'     : bilby.gw.prior.Constraint(name='mass_2', minimum=5, maximum=100),
'a_1'        : bilby.prior.analytical.Uniform(name='a_1', minimum=0, maximum=0.99),
'a_2'        : bilby.prior.analytical.Uniform(name='a_2', minimum=0, maximum=0.99),
'tilt_1'     : bilby.prior.analytical.Sine(name='tilt_1'),
'tilt_2'     : bilby.prior.analytical.Sine(name='tilt_2'),
'phi_12'     : bilby.prior.analytical.Uniform(name='phi_12', minimum=0, maximum=2 * np.pi, boundary='periodic'),
'phi_jl'     : bilby.prior.analytical.Uniform(name='phi_jl', minimum=0, maximum=2 * np.pi, boundary='periodic'),
```

Uniform mass prior $m_{1,2} \in [5,100] M_{\odot}$

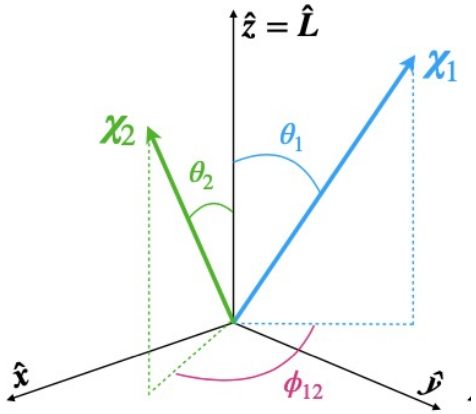
Uninformative BBH priors

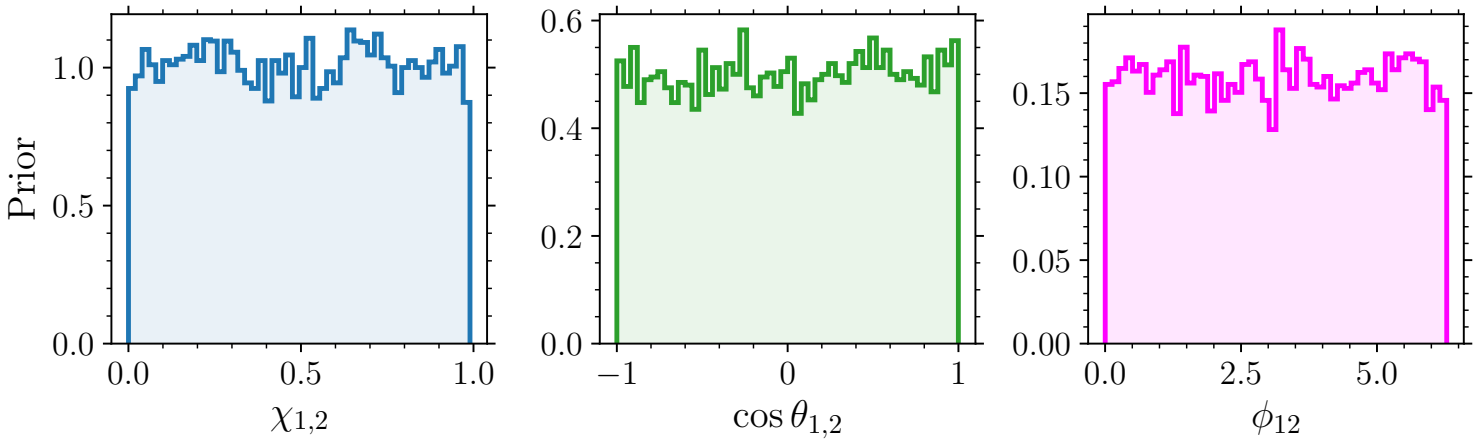
Intrinsic parameters

```
prior = {
'chirp_mass' : bilby.gw.prior.UniformInComponentsChirpMass(name='chirp_mass', minimum=10, maximum=60),
'mass_ratio' : bilby.gw.prior.UniformInComponentsMassRatio(name='mass_ratio', minimum=0.125, maximum=1),
'mass_1'     : bilby.gw.prior.Constraint(name='mass_1', minimum=5, maximum=100),
'mass_2'     : bilby.gw.prior.Constraint(name='mass_2', minimum=5, maximum=100),
'a_1'        : bilby.prior.analytical.Uniform(name='a_1', minimum=0, maximum=0.99),
'a_2'        : bilby.prior.analytical.Uniform(name='a_2', minimum=0, maximum=0.99),
'tilt_1'     : bilby.prior.analytical.Sine(name='tilt_1'),
'tilt_2'     : bilby.prior.analytical.Sine(name='tilt_2'),
'phi_12'     : bilby.prior.analytical.Uniform(name='phi_12', minimum=0, maximum=2 * np.pi, boundary='periodic'),
'phi_jl'     : bilby.prior.analytical.Uniform(name='phi_jl', minimum=0, maximum=2 * np.pi, boundary='periodic'),
```

$p(\chi)\mathrm{d}\chi \propto \mathrm{d}\chi$

Spins uniform in magnitudes and isotropic in directions





Uniform in $\chi_{1,2} \in [0, 0.99]$

Uniform in $\cos\theta_{1,2} \in [-1, 1]$

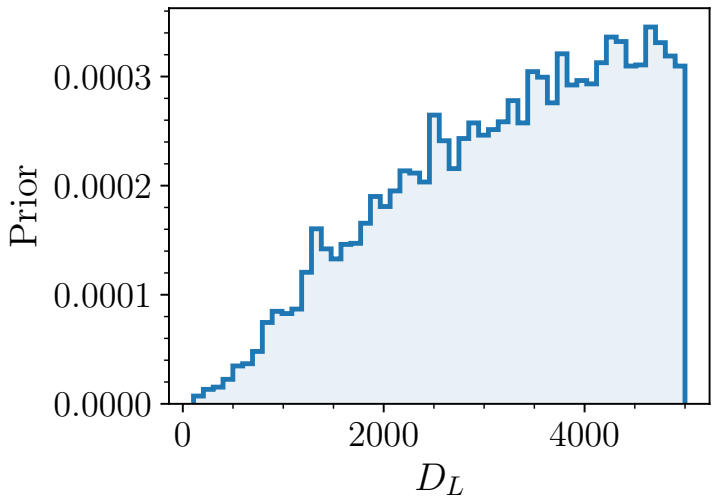
Uniform in $\phi_{12} \in [0, 2\pi]$

Uninformative BBH priors

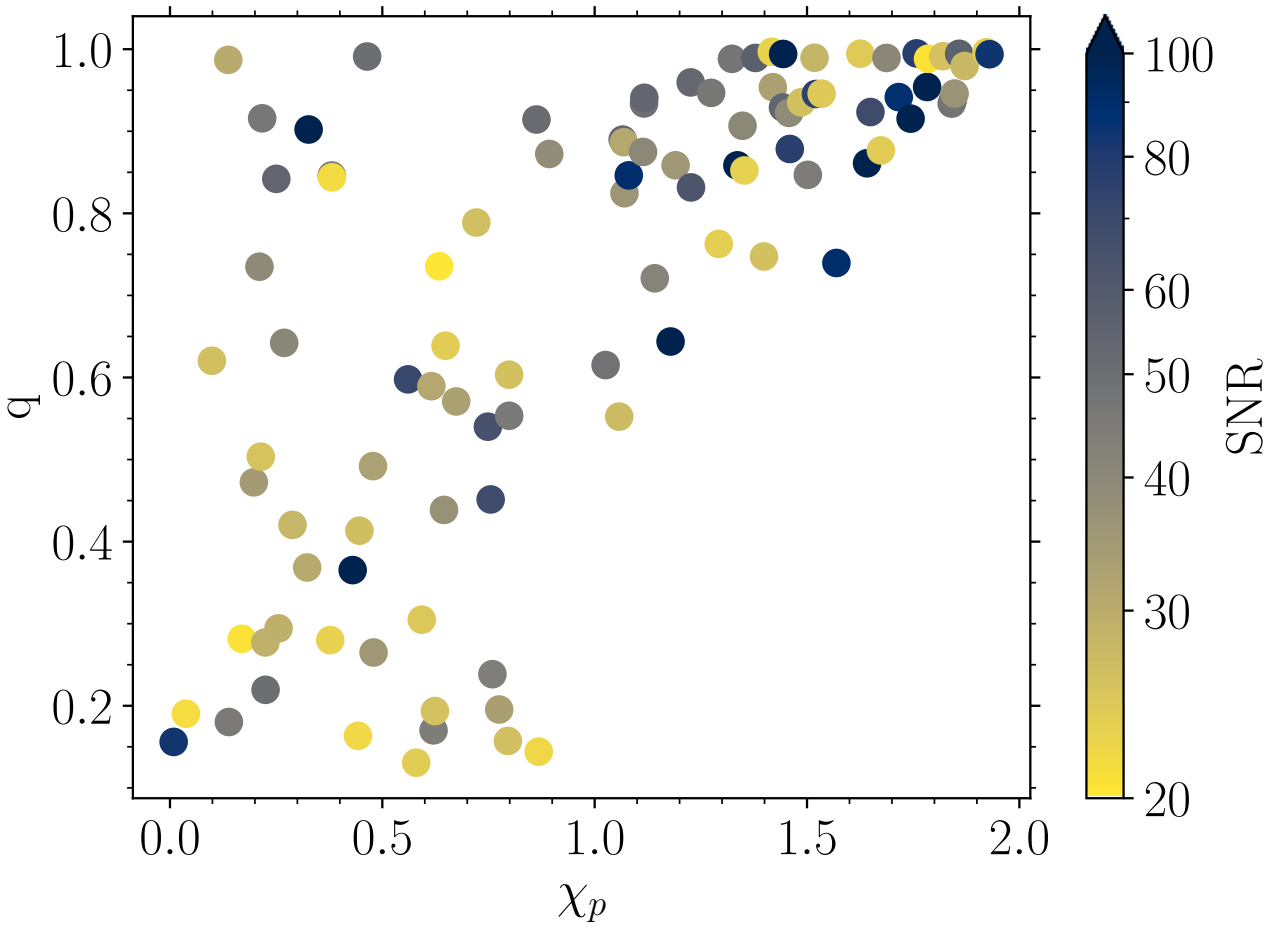
Extrinsic parameters

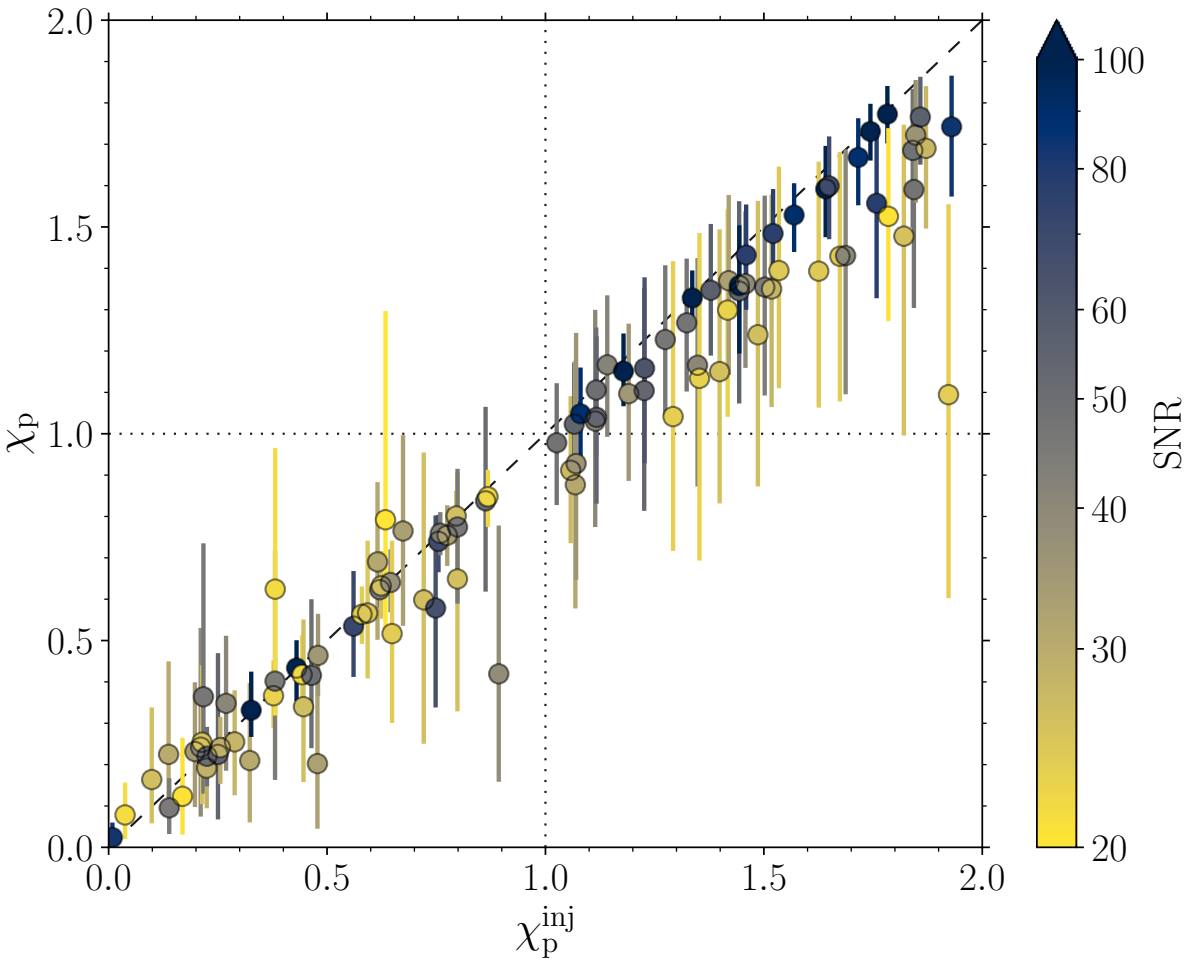
```
'luminosity_distance' : bilby.gw.prior.UniformSourceFrame(name='luminosity_distance', minimum=1e2, maximum=5e3,
unit='Mpc'),
'dec'       : bilby.prior.analytical.Cosine(name='dec'),
'ra'        : bilby.prior.analytical.Uniform(name='ra', minimum=0, maximum=2 * np.pi, boundary='periodic'),
'theta_jn'  : bilby.prior.analytical.Sine(name='theta_jn'),
'psi'       : bilby.prior.analytical.Uniform(name='psi', minimum=0, maximum=np.pi, boundary='periodic'),
'phase'     : bilby.prior.analytical.Uniform(name='phase', minimum=0, maximum=2 * np.pi, boundary='periodic')
}
```

Luminosity distance uniform in comoving volume
$D_L \in [100,5000]$ Mpc



Dataset of 100 injections: mass ratio q





High χ_p → mass ratio ~ 1 → High SNR

χ_p calculation

$t_{orb} \ll t_{pre} \ll t_{RR}$

$$\boxed{\frac{d\mathbf{L}}{dt} = \frac{d\hat{\mathbf{L}}}{dt}L + \frac{dL}{dt}\hat{\mathbf{L}} = (\mathbf{\Omega}_L \times \hat{\mathbf{L}})L + \frac{dL}{dt}\hat{\mathbf{L}}}$$

Precession Radiation reaction

$$\begin{cases} \mathbf{\Omega}_L = \Omega_1\chi_1\hat{\mathbf{S}}_1 + \Omega_2\chi_2\hat{\mathbf{S}}_2 \\ \Omega_1 = \dfrac{M^2}{2r^3(1+q)^2}\left[4+3q-\dfrac{3q\chi_{\rm eff}}{(1+q)}\dfrac{M^2}{L}\right] \\ \Omega_2 = \dfrac{qM^2}{2r^3(1+q)^2}\left[4q+3-\dfrac{3q\chi_{\rm eff}}{(1+q)}\dfrac{M^2}{L}\right] \end{cases}$$

Amount of relativistic precession:

$$\left|\frac{d\hat{\mathbf{L}}}{dt}\right|^2 = (\Omega_1\chi_1\sin\theta_1)^2 + (\Omega_2\chi_2\sin\theta_2)^2 + 2\Omega_1\Omega_2\chi_1\chi_2\sin\theta_1\sin\theta_2\cos\Delta\Phi$$

Heuristic χ_p:

$$\chi_p^{\rm heu} = \frac{1}{2\Omega_1}\left(\left|\frac{d\hat{\mathbf{L}}}{dt}\right|_+ + \left|\frac{d\hat{\mathbf{L}}}{dt}\right|_-\right) = \max\left(\chi_1\sin\theta_1, q\frac{4q+3}{(4+3q)}\chi_2\sin\theta_2\right)$$

Aritmetic heuristic mean between two configurations: $\boldsymbol{\cos\Delta\Phi = \pm 1}$
normalized with Ω_1

Two problems:

I. The configurations are not always geometrically possible
II. The angles $\theta_1, \theta_2, \Delta\Phi$ all vary on the precession timescale.

How to fix the problems with heuristic χ_p?

I. Generalized χ_p → Retain all the variation occurring on the precession timescale

$$\chi_p^{\mathrm{gen}} = \frac{1}{\Omega_1}\left(\left|\frac{\mathrm{d}\hat{\mathbf{L}}}{dt}\right|\right) = \left[\underbrace{(\chi_1 \sin\theta_1)^2 + \left(q\,\frac{4q+3}{(4+3q)}\chi_2 \sin\theta_2\right)^2}_{\text{Heuristic } \chi_p} + 2q\,\frac{4q+3}{(4+3q)}\chi_1\chi_2 \sin\theta_1 \sin\theta_2 \cos\Delta\phi\right]^{1/2}$$

Retain the dependence on $\Delta\Phi$

II. Averaged χ_p → Average all the variation occurring on the precession timescale

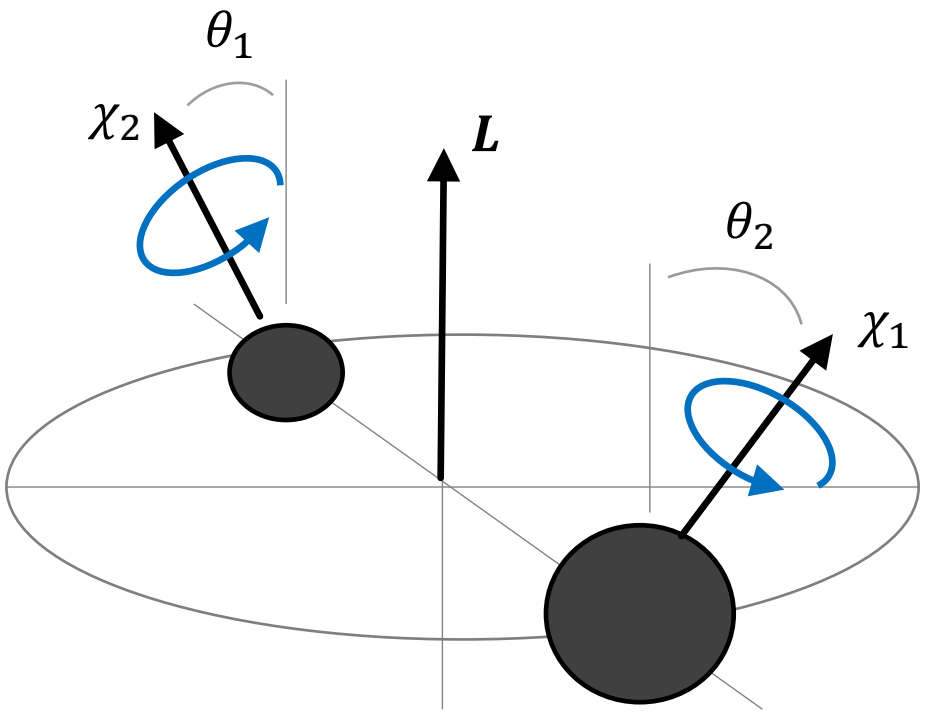
$$\langle \chi_p^{av} \rangle = \frac{\int \chi_p(\psi)\left(\frac{d\psi}{dt}\right)^{-1} d\psi}{\int \left(\frac{d\psi}{dt}\right)^{-1} d\psi}$$

ψ = quantity that parametrize the precession cycle

(Gerosa+ 2015)

Mass and spin measurements

No hair theorem:
Kerr BHs are uniquely described by their **mass M** and their **spin S**



- We can measure with great accuracy the **chirp mass**:

$$M_c = \frac{(m_1 m_2)^{3\backslash 5}}{(m_1 + m_2)^{1\backslash 5}}$$

- **Spins** provide an highly subdominant contribution to the emitted radiation

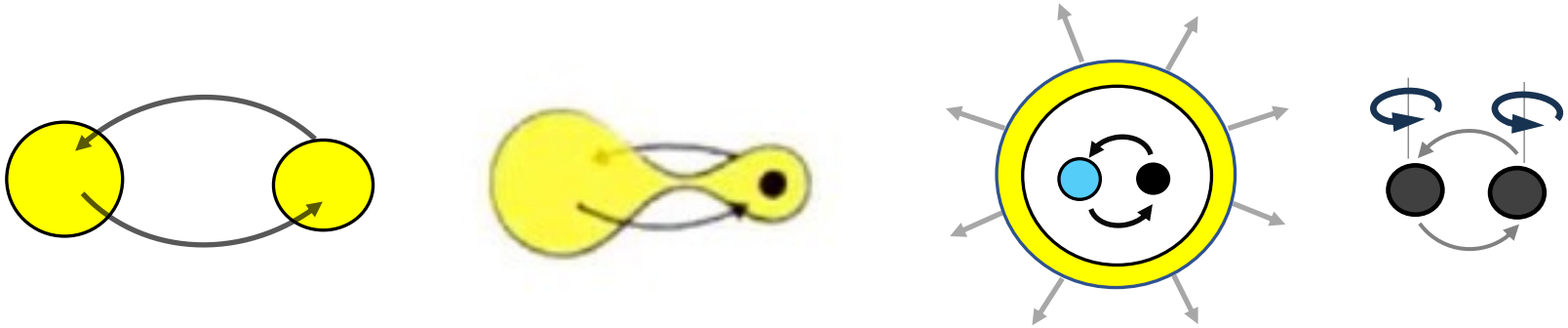
Measurements of component **spin magnitudes** and **tilt angles**

Measurements of **effective spin parameters** χ_{eff} and χ_{p}

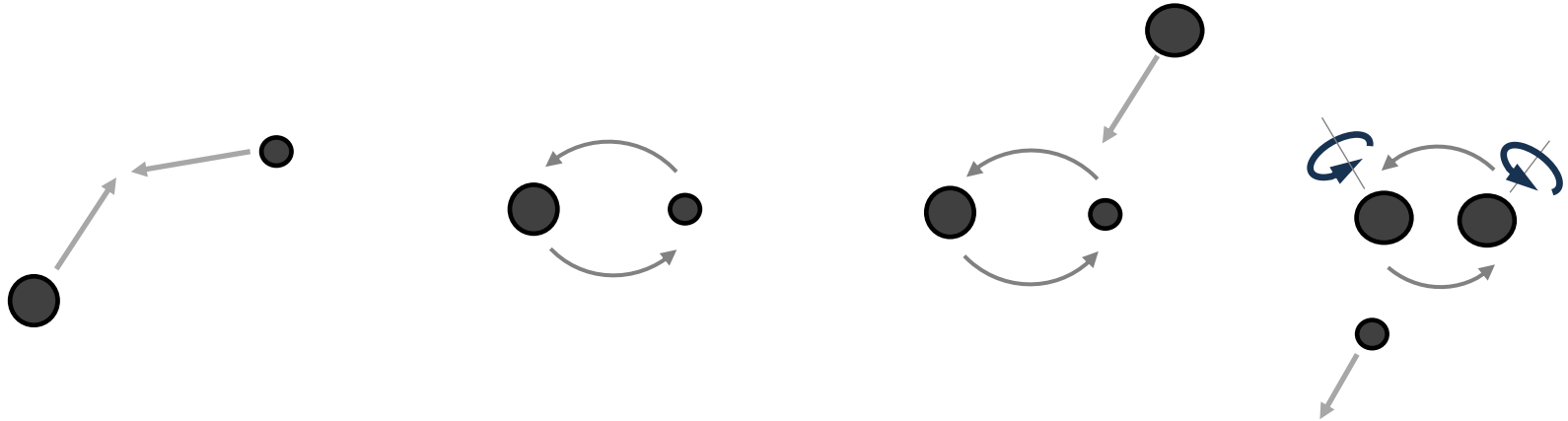
Spin distribution

The directions of the BH spins are believed to be clean tracers of the astrophysical formation pathway

Isolated formation channel:
preferentially aligned spins
(non precessing binaries)



Dynamical formation channel:
isotropically oriented spins
(misalignment → spin precession)



[Mandel and Farmer, 2022]