Anisotropic Neutron Stars: Exploring Quasinormal Modes in Full General Relativity

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Introduction

- Recent discovery of gravitational waves opened a new window to look at the heavenly bodies.
- Nonradial oscillation of neutron stars can generate gravitational waves.
- In general relativistic theory, we know that background metric and fluid get coupled to each other.
- Perturbation on the system(neutron star), can oscillate both metric as well as fluid.
- Outside the star, the fluid perturbation becomes zero, but metric perturbation can propagate as gravitational waves, where the imprint of properties of the compact stars gets incorporated.

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Anisotropic Compact Stars in Equilibrium

- Anisotropy in the neutron stars can arise due to various reasons, presence of magnetic field [1], kion condensation [2], presence of superfluidity [3] etc.

- Due to the presence of anisotropy, the Tolman–Oppenheimer–Volkoff equation also gets modified.

- For a spherically symmetric, nonrotating object, the metric is given by

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (1)$$

- The energy-momentum tensor of anisotropic fluid is given by $T^{\nu}_{\mu} = diag(-\rho, p_r, p_t, p_t)$.

Anisotropic Compact Stars in Equilibrium

- From the Einstein field equations, we can get the hydrostatic equilibrium equation, which can be written as,

$$\frac{dp_r}{dr} = -\frac{(\rho + p_r)(m(r) + 4\pi p_r r^3)}{r(r - 2m(r))} + \frac{2\chi}{r}. \quad (2)$$

  where $\chi = p_t - p_r$, is the anisotropic parameter and $m(r) = 4\pi \int_0^r \rho r'^2 dr'$. This is the modified TOV equation for anisotropic self gravitating compact object.

- To solve this, we have considered BSk21 EoS.

- In our work we shall consider a simple phenomenological ansatz, where $\chi = \tau p_r \mu$, this is called quasi local anisotropy parameter. Where τ describes anisotropic strength and μ is the quasilocal variable. Here we consider the quasi local variable is identical to local compactness, which means $\mu = 2m(r)/r$.

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Perturbation Scheme

- Due to spherical symmetry of background and we are interested in linearized theory, to perturb the system we have considered Regge-Wheeler gauge, where the perturbation decomposes in two independent part (polar and axial) according to their parity.
- Here we will consider the polar part (even parity) of the perturbation which is given by,

$$h_{\alpha\beta} = -r^l \begin{pmatrix} e^{\nu} H_0(r) & i\omega r H_1(r) & 0 & 0 \\ i\omega r H_1(r) & e^{\lambda} H_2(r) & 0 & 0 \\ 0 & 0 & r^2 K(r) & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta K(r) \end{pmatrix} Y_l^m e^{i\omega t}, \tag{3}$$

Perturbation Scheme

- The displacement of the fluid is given by,

$$\xi^\mu = \begin{pmatrix} 0 \\ r^{l-1} e^{-\lambda/2} W(r) \\ -r^{l-2} V(r) \partial_\theta \\ -\frac{r^{l-2}}{\sin^2\theta} V(r) \partial_\phi \end{pmatrix} Y_l^m e^{i\omega t}, \tag{4}$$

- The perturbation of the velocity is given by,

$$\delta u^\sigma = \begin{pmatrix} -\frac{H}{2} \\ i\omega r^{-1} e^{-\lambda/2} W \\ -i\omega r^{-2} V \partial_\theta \\ -i\omega (r\sin\theta)^{-2} V \partial_\phi \end{pmatrix} r^l e^{-\nu/2} Y_l^m e^{i\omega t}. \tag{5}$$

Perturbation Scheme

- Linearized Einstein equation

$$\delta G_\alpha^\beta = 8\pi \delta T_\alpha^\beta. \tag{6}$$

- Linearized conservation of energy-momentum tensor equation

$$\delta(\nabla_\beta T^{\alpha\beta}) = 0. \tag{7}$$

- Define

$$\Delta p_r = -r^l e^{-\nu/2} X Y_l^m e^{i\omega t}. \tag{8}$$

Perturbation Scheme

- With these expressions in our hand, we can derive the governing equations of oscillation. The equations are given by as follows.
- From $\delta G_0^2 = 8\pi\delta T_0^2$ we can write,

$$H_1' = \frac{e^\lambda}{r}H + \frac{e^\lambda}{r}K - \frac{l+1+2mr^{-1}e^\lambda - 4\pi r^2 e^\lambda(p_r-\rho)}{r}H_1 - \frac{16\pi e^\lambda(\rho+p_r+\chi)}{r}V \quad (9)$$

- In the similar way, from $\delta G_0^1 = 8\pi\delta T_0^1$

$$K' = \frac{1}{r}H - \left[(l+1)r^{-1} - \frac{1}{2}\nu'\right]K + \frac{l(l+1)}{2r}H_1 - \frac{8\pi e^{\lambda/2}(p_r+\rho)}{r}W \quad (10)$$

Perturbation Scheme

- From the definition of X in Eq. (8)

$$W' = \frac{1}{2}e^{\lambda/2}rH + e^{\lambda/2}rK - \frac{e^{\lambda/2}l(l+1)}{r}V - \frac{(l+1)}{r}W + \frac{re^{\frac{\lambda-\nu}{2}}}{c_s^2\left(p_r+\rho+\frac{2\chi}{3}\right)}X \tag{11}$$

- From $\delta(\nabla_\beta T^{2\beta}) = 0$

$$\omega^2(\rho+p_r+\chi)V = e^{\nu/2}X - \frac{1}{2}e^{\nu}(p_r+\rho)H + \frac{e^{\nu-\lambda/2}p_r'}{r}W - e^{\nu}\delta\chi, \tag{12}$$

Perturbation Scheme

From $\delta G^2_1 = 8\pi\delta T^2_1$ and $\delta G^1_1 = 8\pi\delta T^1_1$

$$\left[3m + \frac{1}{2}(l-1)(l+2) + 4\pi r^3 p_r\right] H = 8\pi e^{-\nu/2} r^3 X$$
$$- \left[-e^{-\nu-\lambda} r^3 \omega^2 + \frac{1}{2} l(l+1)(m + 4\pi r^3 p_r)\right] H_1 + \chi 16 e^{-\lambda/2} \pi r W$$
$$- \left[-\frac{1}{2}(l-1)(l+2)r + e^{-\nu} r^3 \omega^2 + \frac{e^\lambda}{r}(m + 4\pi r^3 p_r)(3m - r + 4\pi r^3 p_r)\right] K, \tag{13}$$

Perturbation Scheme

$$
\begin{aligned}
X' = {} & -2\frac{e^{\nu/2}}{r}\delta\chi - \left[\frac{l}{r} + \frac{\chi(6+r\nu')}{rc_s^2(3(p_r+\rho)+2\chi)}\right] X + (p_r+\rho)e^{\nu/2}\left[\frac{1}{2r} - \frac{\nu'}{2}\right] \\
& H - \left[\frac{e^{\nu/2}l(l+1)(p_r+\rho)\nu'}{2r^2} - \chi\frac{2l(l+1)e^{\nu/2}}{r^3}\right] V \\
& + \left[\frac{e^{\nu/2}(p_r+\rho)}{2}\left(\frac{l(l+1)}{2r} + r\omega^2 e^{-\nu}\right) + \frac{\chi e^{\nu/2}l(l+1)}{2r}\right] H_1 \\
& + \left[\frac{1}{2}e^{\nu/2}(p_r+\rho)\left(\frac{3}{2}\nu' - \frac{1}{r}\right) + \chi\frac{e^{\nu/2}(-6+r\nu')}{2r}\right] K \\
& + \left[-\frac{e^{\nu/2}(p_r+\rho)}{r}\left(e^{\lambda/2-\nu}\omega^2 + 4\pi e^{\lambda/2}(p_r+\rho) - \frac{1}{2}r^2(r^{-2}e^{-\lambda/2}\nu')'\right)\right. \\
& \left. -\chi'\frac{2e^{(\nu-\lambda)/2}}{r^2} + \frac{e^{(\nu+\lambda)/2}}{r^3}\chi\left(6 - 14mr^{-1} - 8\pi r^2 p_r\right)\right] W \qquad (14)
\end{aligned}
$$

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Outline of solution

- Inside the star we have to solve the set of 4 first order differential equation for oscillation (H_1, K, W, X) and three equations for background(m, p_r, ν) with two algebraic relation (V, H).

- Initial conditions come from regularity condition at the center of the star. One can get the expressions for initial condition by expanding the equations (both oscillation and background) in Taylor series up to zeroth order.

- The boundary condition is the Lagrangian perturbation at surface should be zero $(\Delta p_r = 0)$.

- Outside the star fluid perturbation is zero. But metric perturbation is finite. The metric perturbation functions (H_1, K) combines to a single second order differential equation, which is called Zerilli equation.

Outline of solution

- The Zerilli equation is given by

$$\frac{d^2 Z}{dr^{*2}} + \left(\omega^2 - V_z(r^*)\right) Z = 0 \ , \tag{15}$$

where,

$$V_z(r^*) = \frac{(1 - 2M/r)}{r^3(n_l r + 3M)^2}\left(2n_l^2(n_l + 1)r^3 + 6n_l^2 M r^2 \right.$$
$$\left. + 18 n_l M^2 r + 18M^3\right) \ . \tag{16}$$

and $n_l = (l(l+1))/2) - 1$, and $r^* = r + 2Mln((r/2M) - 1)$.

- In the asymptotic limit, we can think the solution of this equation as a combination of ingoing and outgoing wave.

Outline of the solution

- As, we are interested in the solutions where the solution of this equation is totally outgoing, so we have integrated these equations for several frequencies, and calculated the component of the ingoing part. And by finding the frequency at which the outgoing part becomes zero, we get the quasinormal mode of the particular model of the star.
- For that we have integrated the Zerilli equation up to infinity (near about $(50\omega^{-1})$).
- Then match the solution with

$$Z(r^*) = B_{in} Z_{in}(r^*) + B_{out} Z_{out}(r^*) \ , \tag{17}$$

Outline of the solution

- Where,

$$Z_{out}(r^*) = e^{(-i\omega r^*)} \sum_{j=0}^{\infty} \beta_j r^{-j} \ , \tag{18}$$

$$Z_{in}(r^*) = e^{(i\omega r^*)} \sum_{j=0}^{\infty} \bar{\beta}_j r^{-j} \ , \tag{19}$$

- From these expressions we get a recursion relation of β_j s.
- Now matching the numerical solution with the analytical asymptotic form, we can get B_{in}.
- Now we can calculate B_{in} for a several values of ω and fit those values to get a functional form of $B_{in}(\omega)$.
- Now we find the root of that function to get the particular frequency, for which the the ingoing part of the wave is zero, and the wave is purely outgoing.
- Quasi-normal mode frequency is $Re(\omega)$ and damping time is $1/Im(\omega)$.

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Results

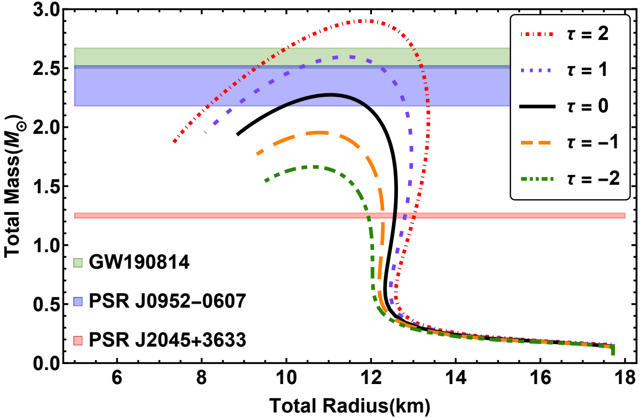

Figure 1: Mass-Radius profiles for anisotropic neutron stars with $-2 \leq \tau \leq 2$ and BSk21 EoS.

Results

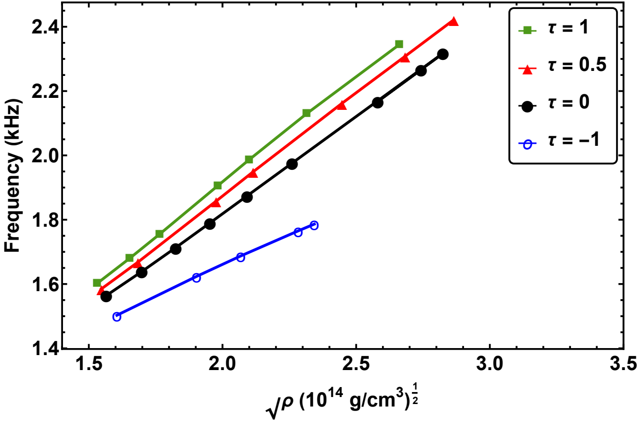

Figure 2: Plot of f mode frequencies with respect to square root of average density for various anisotropic strength.

Results

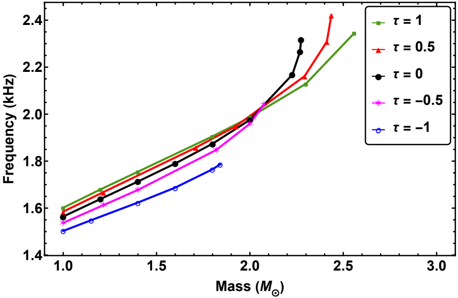

Figure 3: Plot for quasi-normal modes with respect to the total mass of the stars for various anisotropic strength within the stars.

Results

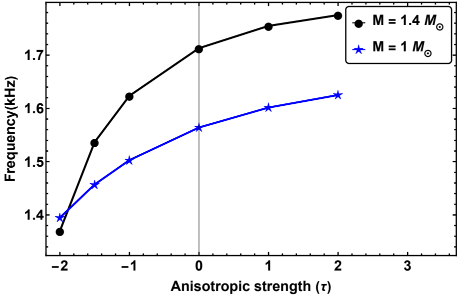

Figure 4: Plot for quasi-normal modes with respect to the anisotropic strength for $1M_\odot$ & $1.4M_\odot$ mass stars.

Results

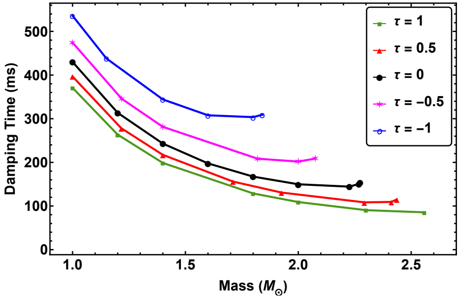

Figure 5: Plot for damping time with respect to mass for various anisotropic strength

Results

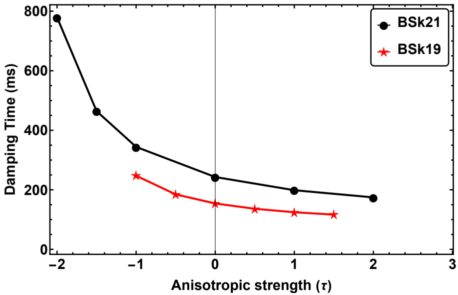

Figure 6: Plot for damping time with respect to anisotropic strength for $1.4M_{\odot}$ neutron stars

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[1] D. Deb, B. Mukhopadhyay, and F. Weber, Effects of Anisotropy on Strongly Magnetized Neutron and Strange Quark Stars in General Relativity, Astrophys. J. 922, 149(2021), arXiv:2108.12436 [astro-ph.HE].
[2] R. F. Sawyer, Condensed pi- phase in neutron star matter, Phys. Rev. Lett. 29, 382 (1972).
[3] R. Kippenhahn, A. Weigert, and A. Weiss, Stellar structure and evolution, Vol. 192 (Springer, 1990).

Thank you!