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GR & DM

How dragging and General Relativity could explain the missing mass problem

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INTRODUCTION
Strategies for the Missing Mass Problem

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Most natural idea:
Existing invisible mass

NB: ***all the DM evidences have gravitational nature***

Is the Missing Mass a clue of misunderstanding in gravity?

Dark matter:
- MaCHOs?
- Hot DM (sterile neutrinos)?
- Cold DM (WIMPs)?

- Galaxy rotation curves
- Virial of clusters
- Gravitational lensing
- Temperature of hot gases
- Bullet clusters
- CMB anisotropies
- SNIa redshift measures
- Etc…

All gravitational attractions or space-time distortions, i.e. gravitational wells

Attempts to modify the Newtonian Gravity (MOND)

Milgrom 1983,
Bekenstein&Milgrom 1984,
Bekenstein 2004

Already have a modified gravity: GR!

INTRODUCTION
GR is more than post-Newtonian corrections

Intuition: GR = Newton + post-Newtonian corrections

Galactic dynamics in low energy régime:
- Sub-relativistic speeds
- Weak forces

PN terms have magnitude $\sim \frac{v^2}{c^2}$: Negligible corrections

Ciotti 2022,
Lasenby+ 2023,
Costa+ 2023,
Glampedakis&Jones 2023

Claim: ***GR includes totally non-Newtonian phenomena!***

Astesiano+3 2022

A galaxy is an extended source: Needs of global metric

Not all metrics are *globally* Newtonian

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0i} \\ g_{0i} & g_{ij} \end{pmatrix},$$

where g_{0i} dragging term

Low energy limit ≠ Newtonian limit

Re-weight DM amount in disc galaxies

Astesiano+5 2022

DM phenomena = fake DM from GR + true DM

Know amount and features of true DM: improve detection experiment!

THEORETICAL FRAMEWORK
Dragging metrics

Stationarity and axisymmetry: $ds^2 = -c^2 e^{2\nu} dt^2 + g_{\varphi\varphi}(d\varphi - \chi dt)^2 + e^{\mu}(dr^2 + dz^2)$

Perfect fluid: $T_{\mu\nu} = \left(\rho + \frac{p}{c^2}\right) U_\mu U_\nu + p g_{\mu\nu}$

No velocity dispersion: $U^\mu = \frac{1}{\sqrt{-H}}(\partial_t + \Omega \partial_\varphi)$

8 fields vs 7 Einstein Equations + $U_\mu U^\mu \equiv -1$

E.g.: $p = 0 \Rightarrow g_{\varphi\varphi} = r^2$

$v \cong r\Omega$ “observed speed”, $w \cong r\chi$ “dragging speed”,
$v_Z \cong v - w$ “ZAMO (Zero Angular Momentum Observer) speed”

Redshift $z \cong {}^{v}/_{c}$

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THEORETICAL FRAMEWORK
Beyond gravitomagnetism

Stationary, linearized EE with

$$g_{00} = -1 + \frac{2\Phi}{c^2}, g_{0j} = -\frac{a_j}{c^2}, g_{ij} = \left(1 + \frac{2\Phi}{c^2}\right)\delta_{ij}$$

Φ gravitational potential,
$\vec{a}$ gravitomagnetic potential

Harmonic gauge $\cancel{2\partial_0\Phi} + \vec{\nabla}\cdot\vec{a} = 0$

$\vec{g} := -\vec{\nabla}\Phi - \cancel{2\partial_0}\vec{a}$ gravitational field,
$\vec{b} := \vec{\nabla}\times\vec{a}$ gravitomagnetic field

$$\begin{cases} \vec{\nabla}\cdot\vec{g} = -4\pi G\rho, \vec{\nabla}\times\vec{g} = -\cancel{{}^{2}/_{c}\,\partial_0\vec{b}} \\ \vec{\nabla}\cdot\vec{b} = 0, \vec{\nabla}\times\vec{b} = 8\pi G\rho\,{}^{\vec{v}}/_{c} + \cancel{{}^{2}/_{c}\,\partial_0\vec{g}} \end{cases}$$

GR effective force $\vec{F} = m\left(\vec{g} + 2\,{}^{\vec{v}}/_{c}\times\vec{b}\right)$

Would return ${}^{w}/_{c} \sim {}^{v^2}/_{c^2} \sim 10^{-7}$

Ciotti 2022, Lasenby+ 2023, Costa+ 2023, Glampedakis&Jones 2023

We explore the case ${}^{w}/_{c} \sim 10^{-4}$:
strong gravitomagnetism.
Non-negligible effects on rotation curves!

Ruggiero+2 2022
Astesiano&Ruggiero 2022

Exploit non-linearity of Einstein Equations

We are looking for solitonic solutions on the dragging term

THEORETICAL FRAMEWORK
BG model and its weaknesses

Balasin&Grumiller 2008

Assumptions:
- stationary,
- axisymmetric,
- <u>“co-rotation”</u>,
- <u>pressure-less dust</u>,
- without velocity dispersion

I.e. $\Omega \equiv 0 \equiv v$
System supported by pure dragging!

Choose $w(r,0) :=$

Claim: Required DM reduced of ${}^{\rho}/_{\rho_{Newt}} \cong {}^{3}/_{4}$!

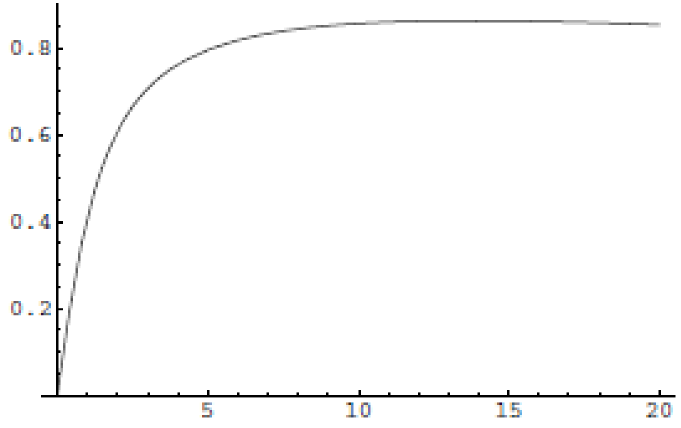

Fig. 3. $V(r,0)/V_0$ plotted up to 20 kpc

NB: Unphysical! w is not the rotation curve. $v \equiv 0$ flat rotation curve!
Dragging $w \cong r\chi$ too big in external region: huge gravitational lensing!

Galoppo+ 2022

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I.e. $\Omega \equiv 0 \equiv v$
System supported by pure dragging!

BG applied to GAIA DR2 catalogue

Crosta+ 2020

Claim: Do not require DM!

<u>BG fits with observations! With less parameters…</u>

Confirmed by GAIA DR3

Crosta+ 2023

Introduction
Theoretical Framework
Empirical Measures
Conclusions

THEORETICAL FRAMEWORK
(η, H) model: the equations

Astesiano+3 2022

Assumptions:
- stationary,
- axisymmetric,
- ~~"co-rotation"~~,
- <u>pressure-less dust</u>,
- without velocity dispersion

Mathematically: choose two functions $H(\eta)$ and $\eta(r,z) = r v_Z(r,z)$

Harmonicity-like $F_{rr} - \frac{1}{r}F_r + F_{zz} = 0$,
s.t. $F(r,z) := 2\eta + r^2 \int \frac{H'}{H}\frac{d\eta}{\eta} - \int \frac{H'}{H}\eta\, d\eta$

$H(\eta)$ and $\eta(r,0)$ free:
2 DoF (1-var functions)

$$\Omega(\eta) = {}^1\!/_2 \int H'(\eta)\, {}^{d\eta}\!/_{\eta}$$

$$8\pi G\rho = \frac{v^2(2-\eta l)^2 - r^2 l^2}{4e^{\mu}} \frac{\eta_r^2+\eta_z^2}{\eta^2} \text{ s.t. } l(\eta) := {}^{H'}\!/_{H}$$

$$g_{tt} = H - 2vr\Omega + \frac{r^2\Omega^2}{-H\gamma^2},$$
$$g_{t\varphi} = rv + \frac{r^2}{\gamma^2 H}\Omega,$$
$$g_{\varphi\varphi} = \frac{r^2}{-H\gamma^2}, \text{ s.t. } \gamma = \gamma(v_Z)$$

THEORETICAL FRAMEWORK
(η, H) model vs BG

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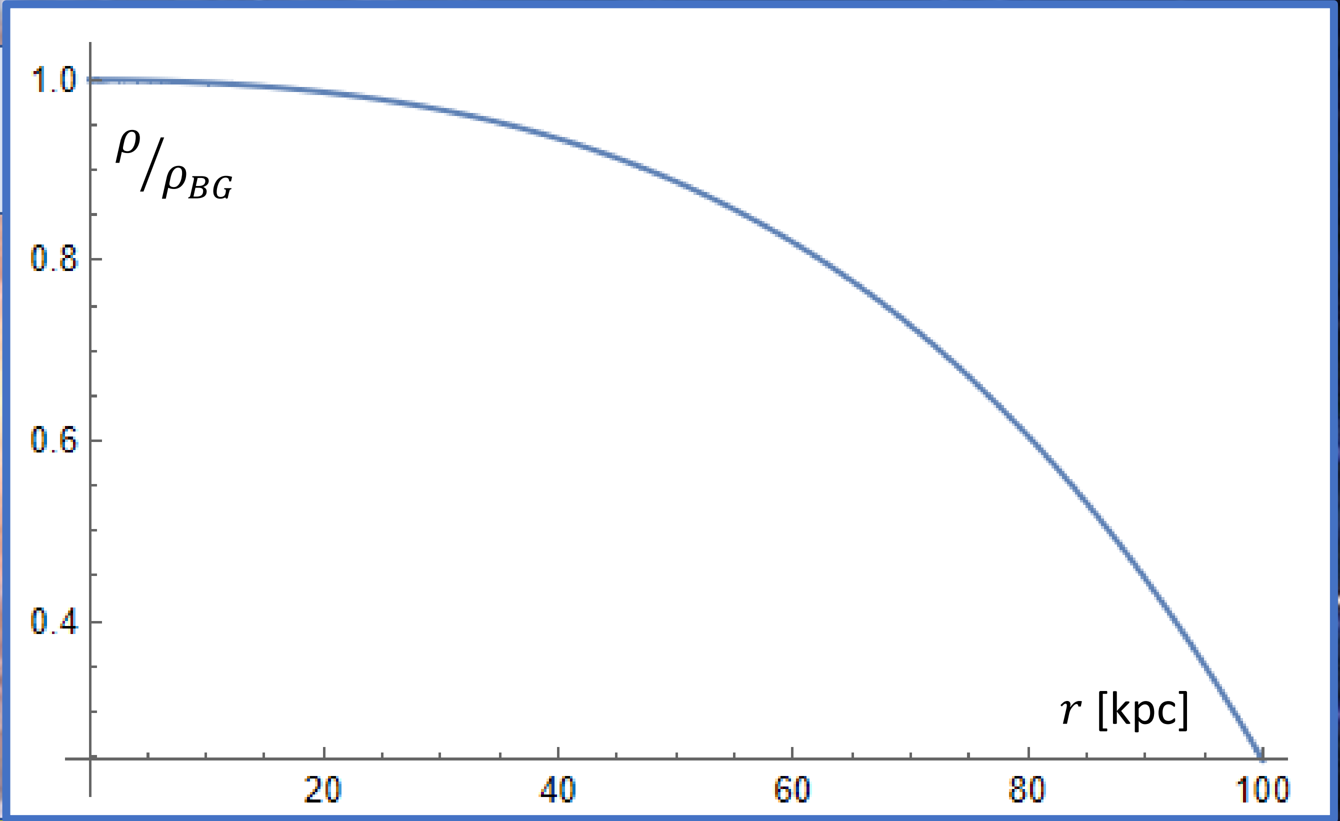
$$8\pi G\rho = \frac{v^2(2-\eta l)^2 - r^2 l^2}{4e^{\mu}} \frac{\eta_r^2+\eta_z^2}{\eta^2} \text{ s.t.}$$

Example: constant $l \equiv {}^{v_c}/_{R_G}$
(almost rigid rotation)



For the same $v(r,z)$ profile: $\rho/\rho_{BG} = 1 - \frac{1}{4}\left(\frac{r}{R_G}\right)^2 \left(\frac{v_c}{v}\right)^2 < 1$!

DM reduction in correspondence of the halo $r \approx R_G$!

THEORETICAL FRAMEWORK
(η, H) model: the physics

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NB: **zero pressure is unphysical!**
$$\boldsymbol{F_{rr} - \frac{1}{r} F_r + F_{zz} = 0}$$ **gives nonsense far from galactic plane**

Newt analog: infinite cylinder

(η, H) model is affordable only near the galactic plane: $|z| \cong 0$

2 DoF (1-var functions), Choose physical parameters

$v(r,0)$: Observed

$w(r,0)$: Not observed

ρ_N for infinite Newtonian cylinder

$$8\pi G\rho \cong \frac{\eta_r^2}{r^2} - r^2\Omega_r^2 = 4\frac{v v_r}{r} + \left(\frac{w}{r} + w_r\right)\left(\frac{w}{r} + w_r - \frac{2v}{r} - 2v_r\right)$$

EMPIRICAL MEASURES
Three ideas

Strong dragging metrics are allowed by Equations.
Have the real disc galaxies such metrics?

How much dragging do we expect?
And how can we measure it?

Three different measures:

1) Transverse redshift vs longitudinal redshift
2) Quadrupole anomaly of the observed CMB
3) Motion of the counter-rotating matter component

Estimation with Newtonian ad-hoc term

Re, coming soon

$$8\pi G(\rho_B + \alpha\rho_{DM}) := 8\pi G\rho := \frac{\eta_r^2}{r^2} - r^2\Omega_r^2 + 2\frac{v^2}{r^2}$$

Fraction $1-\alpha$ of DM explained by dragging $w = v - {}^{\eta}/_{r}$

$$4\pi G(\rho_B + \rho_{DM}) = 2\frac{vv_r}{r} + \frac{v^2}{r^2}$$

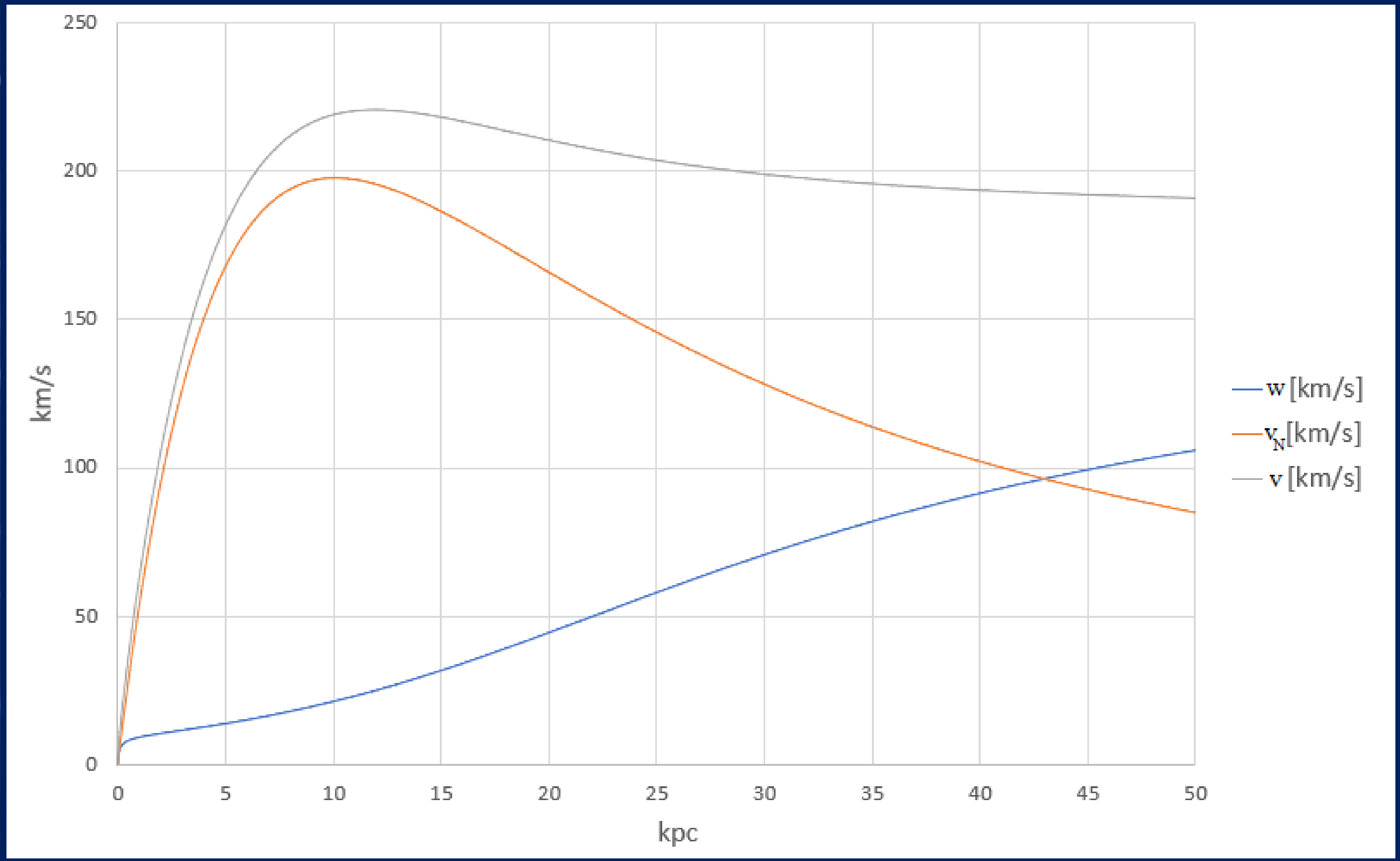
$\alpha = 1 \iff w \equiv 0$: spherically symmetric Newtonian model with 100% of DM

Evaluate for MW: $\rho_B := \rho_{B0} e^{-r/r_B}$ exponential, $\rho_{DM} := \rho_{DM0}\frac{r_{DM}}{r}\left(1+\frac{r}{r_{DM}}\right)^{-2}$ NFW,
$r_B \cong 3$ kpc, $r_{DM} \cong 50$ kpc, $v_{max} \cong 220$ km/s, ${}^{\rho_{B0}}/_{\rho_{DM0}} \cong 230$

At $r_\odot \cong 10$ kpc: $w(r_\odot, 0) \cong (1-\alpha)\cdot 44{,}4$ km/s

Example: $\alpha = {}^{1}/_{2} \Rightarrow w(r_\odot, 0) \approx 22{,}2$ km/s in our neighborhood

EMPIRICAL MEASURES
Estimation with Newtonian ad-hoc term



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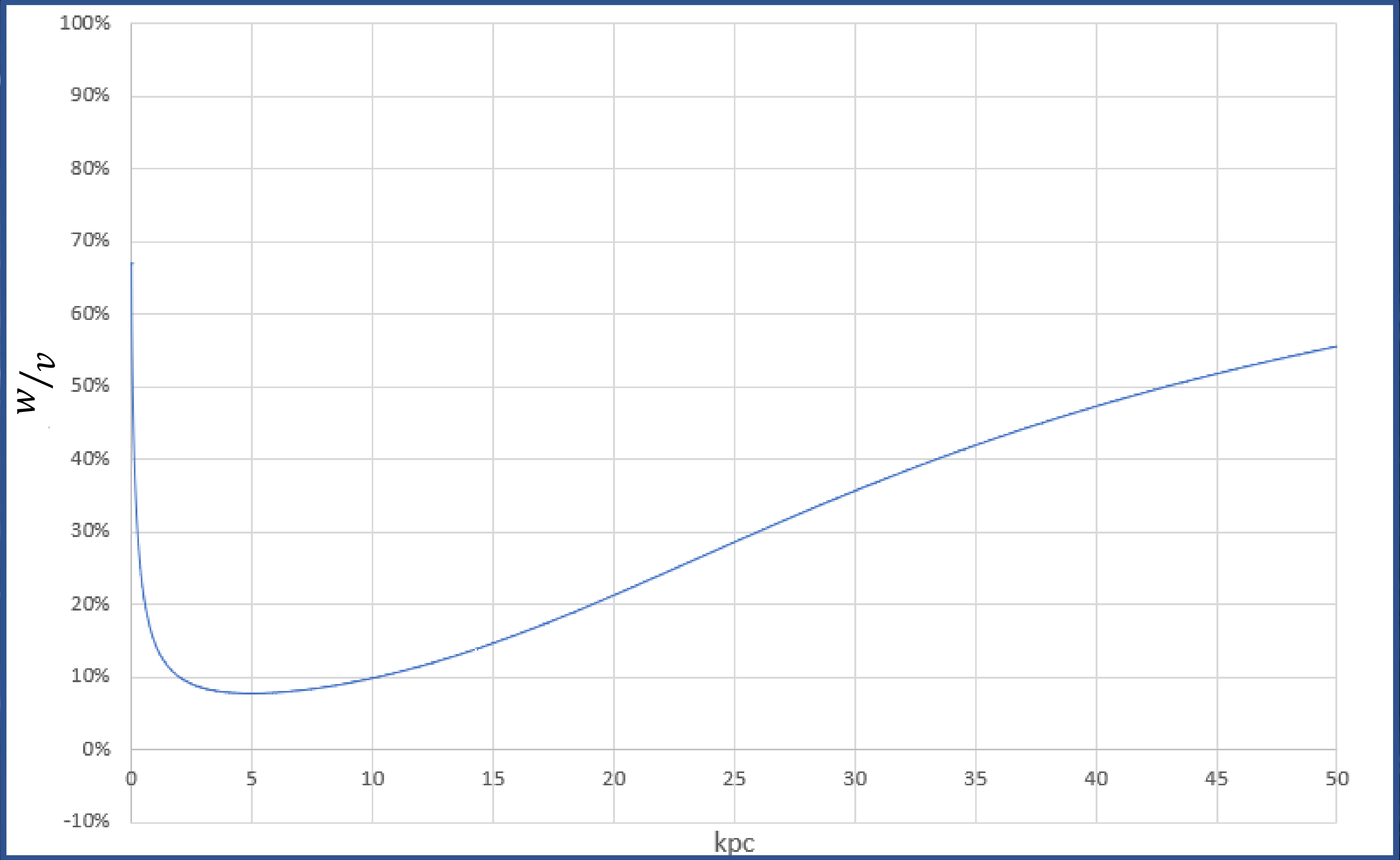
EMPIRICAL MEASURES
Estimation with Newtonian ad-hoc term



Longitudinal and transversal redshift

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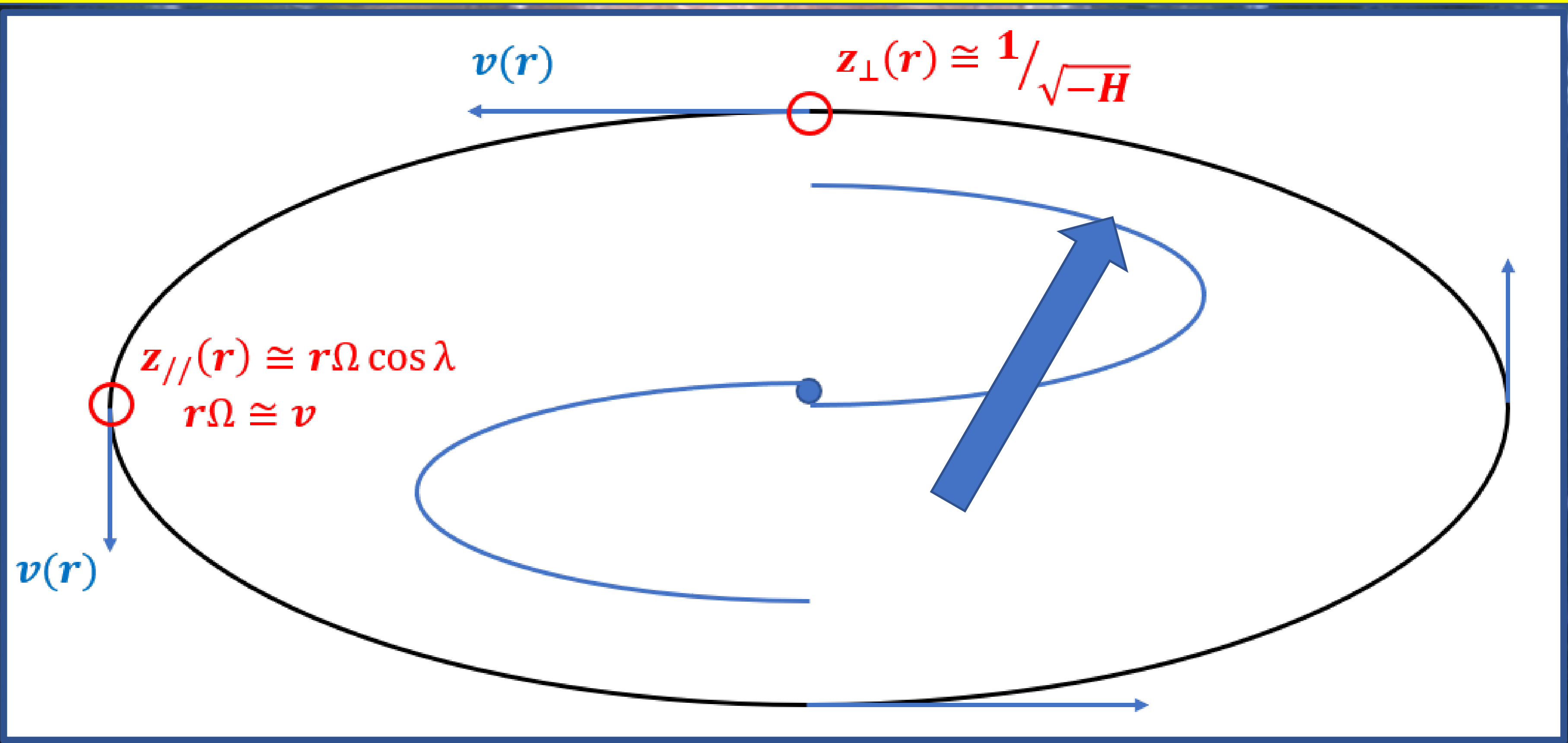
Key idea: (η, H) model has 2 DoF.

<u>Simultaneous measure of both redshifts determines all the model!</u>



Astesiano+5 2022

Key idea: (η, H) model has 2 DoF.

Simultaneous measure of both redshifts determines all the model!

In SR (i.e. $w \equiv 0$):

$$1 + z = \frac{1 + v\cos\theta}{\sqrt{1 - v^2}}$$

1 DoF, redshifts *mutually dependent*

$$2z^{\perp} \cong \left(z^{//}/_{\cos\lambda}\right)^2$$

For GR dragging metric

$$1 + z = \frac{1}{\sqrt{-H}}\left(1 - \frac{\Omega b}{c}\right)\ s.t.\ \frac{b}{c} = \frac{\chi g_{\varphi\varphi}\sin\theta - \sqrt{(\chi g_{\varphi\varphi}\sin\theta)^2 - g_{tt}g_{\varphi\varphi}}}{g_{tt}}\sin\theta$$

$$1 + z^{\perp} = 1/\sqrt{-H}$$

$$1 + z^{//} \cong 1 + v/_c \cos\lambda$$

If broken the
GR is not negligible !

λ galaxy tilting angle

θ line of sight angle

EMPIRICAL MEASURES
Quadrupole of the CMB

Dotti&Re, coming soon

Key idea: ***<u>Same redshift formula for CMB photons!</u>***

In SR: $\sum_{l=0}^{\infty}\sum_{m=-l}^{l} C_l^m Y_l^m(\theta,\Phi) = = T(\theta,\Phi) = \frac{\bar{T}}{1+z} = \frac{\bar{T}}{\gamma(1+v\cos\theta)}$

$C_2^0 \cong {}^4\!/_3\sqrt{{}^{\pi}\!/_5}\,\bar{T}v^2 \;\Rightarrow$ II-ord rel: $\sqrt{5}C_0^0 C_2^0 \cong 2(C_1^0)^2$

$$C_0^0 \cong 2\sqrt{\pi}\bar{T},$$
$$C_1^0 \cong -2\sqrt{{}^{\pi}\!/_3}\,\bar{T}v,$$

In GR: $\sum_{l=0}^{\infty}\sum_{m=-l}^{l} C_l^m Y_l^m(\theta,\Phi) = = T(\theta,\Phi) = \frac{\bar{T}}{1+z_{GR}} = \bar{T}\,\frac{\sqrt{-H}}{1-{}^{\Omega b}\!/_c}$

$C_2^0 \cong {}^4\!/_3\sqrt{{}^{\pi}\!/_5}\,\bar{T}vv_Z$ because 2 DoF

EMPIRICAL MEASURES
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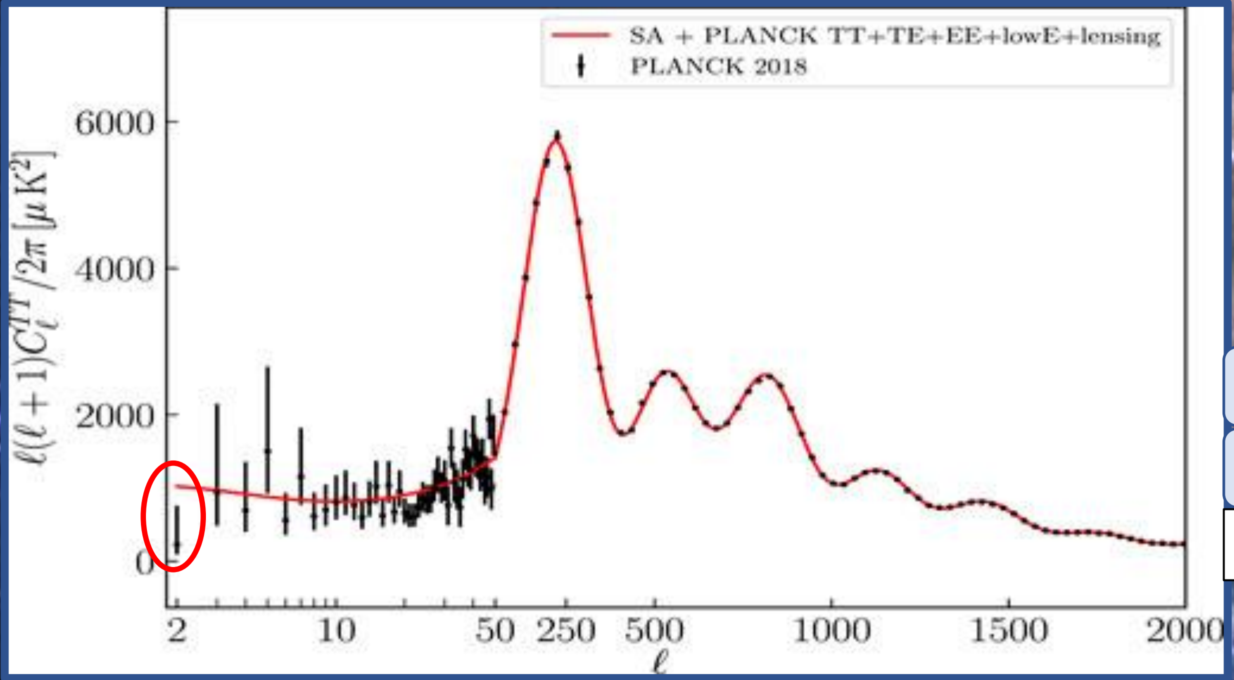
Dotti&Re, coming soon

Key idea: ***Same redshift formula for CMB photons!***



Tension in CMB quadrupole

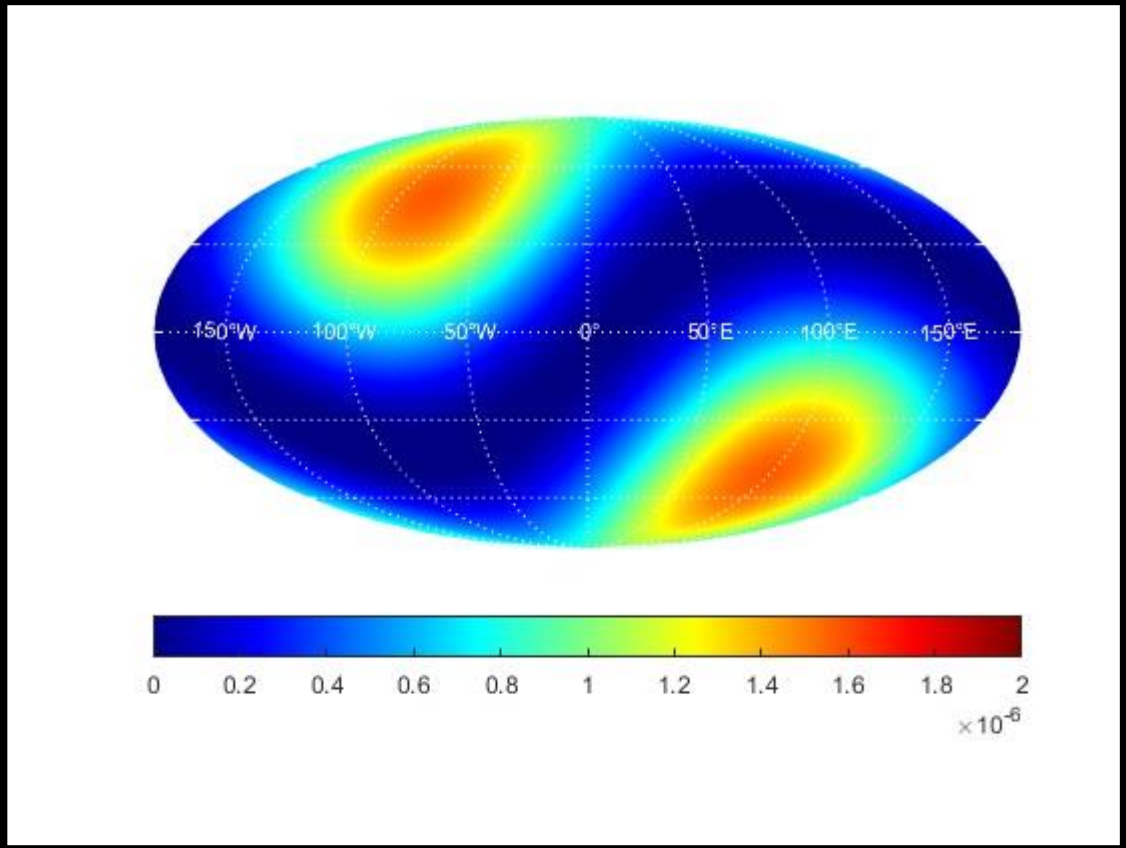
Related to systematic kinetic quadrupole

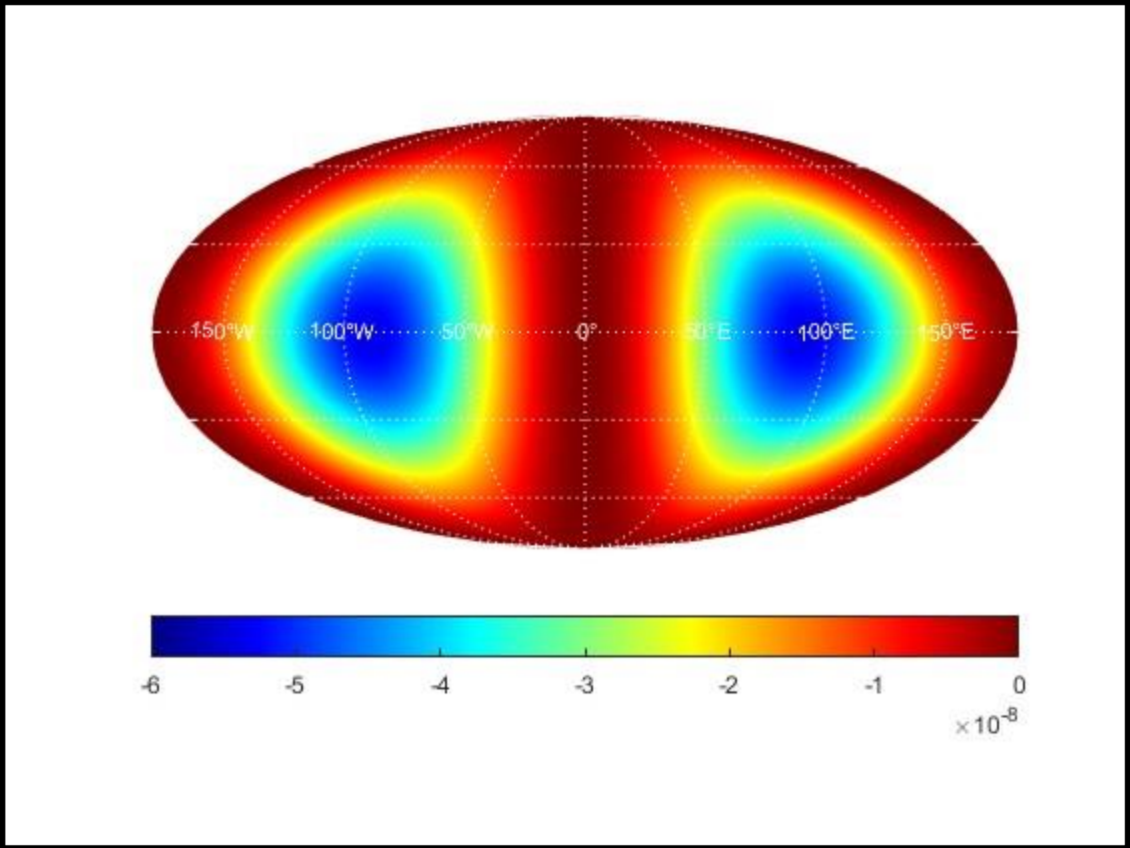
See e.g. Notari&Quartin 2015

EMPIRICAL MEASURES
Quadrupole of the CMB

Dotti&Re, coming soon

Key idea: *Same redshift formula for CMB photons!*





EMPIRICAL MEASURES
Counter-rotating matter

Some disc galaxies have counter-rotating stars or gas

Kuijken+ 1996, Corsini 2014

Key idea: ***The counter-rotating component is also dragged!*** $v_+ + v_- \propto w$

Gorini&Re, coming soon

Consider geodesics for a test particle with tangent motion $\frac{\partial \varphi}{\partial \tau} \cong \tilde{\Omega} \cong \tilde{v}/r$

Without dragging ($w \equiv 0$): depends only on the potential Φ s.t. $g_{tt} = -e^{2\Phi/c^2}$, $\frac{\partial \Phi}{\partial r} = \frac{v^2}{r}$

Geodesic $\ddot{r} \cong \frac{\tilde{v}^2 - v^2}{r}$ $\Rightarrow$ symmetrically $v_\pm \cong \pm v$

With dragging w: $\frac{\partial \Phi}{\partial r} \cong v\left(\frac{v_Z}{r} - \frac{\partial w}{\partial r}\right)$

Geodesic $\ddot{r} \cong \frac{\tilde{v}^2 - v^2}{r} + \frac{v - \tilde{v}}{r} \frac{\partial (rw)}{\partial r}$

asymmetric $v_+ \cong v$, $v_- = -v + \frac{\partial (rw)}{\partial r}$

First-order deviation from Newton!

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CONCLUSIONS
Feasibility of the measures

$z^{//}$ vs $z^{\perp}$ needs to measure II order quantities $\sim 10^{-7}$.
Requires future spectrographs: e.g. HIRES, ANDES.
Galaxy peculiar motions mask $z^{\perp}$: rippling, wobbling, warping, bulge, and bar buckling.

Not affected by peculiar motions! We already have a lot of data!
Looking for a $\sim 5 \cdot 10^{-8}$ anisotropy (II order, again), while typically ${}^{\Delta T}/_{T} \sim 10^{-5}$.
Get C_2^0 integrating on all the sky. We are studying the feasibility.

Looking for a I order quantity!
Galaxies with counter-rotating components have big velocity dispersion:
Measures are less precise.

CONCLUSIONS
Future perspectives

What we know:

- GR admits solitonic solutions for the dragging terms
- Strong dragging implies non-negligible deviations from Newton
- Deviations on mass density and rotation speeds can explain a fraction of the galactic DM
- The dragging speed can be measured with at least three independent methods

GR is gravity. Can be applied to the other DM evidences:

- Dragging metrics of galaxies affects also the gravitational lensing
- Cosmological SNIa redshifts can be affected by retarded potentials and backreaction
- Virial of clusters / elliptical galaxies have GR terms, e.g. with dragging
- Etc: any metric deformation in GR, without presence of matter!

Galoppo+ 2022

Re 2020, Re 2021, Vigneron&Buchert 2019, Buchert 2008

CONCLUSIONS

Thanks for your attention!