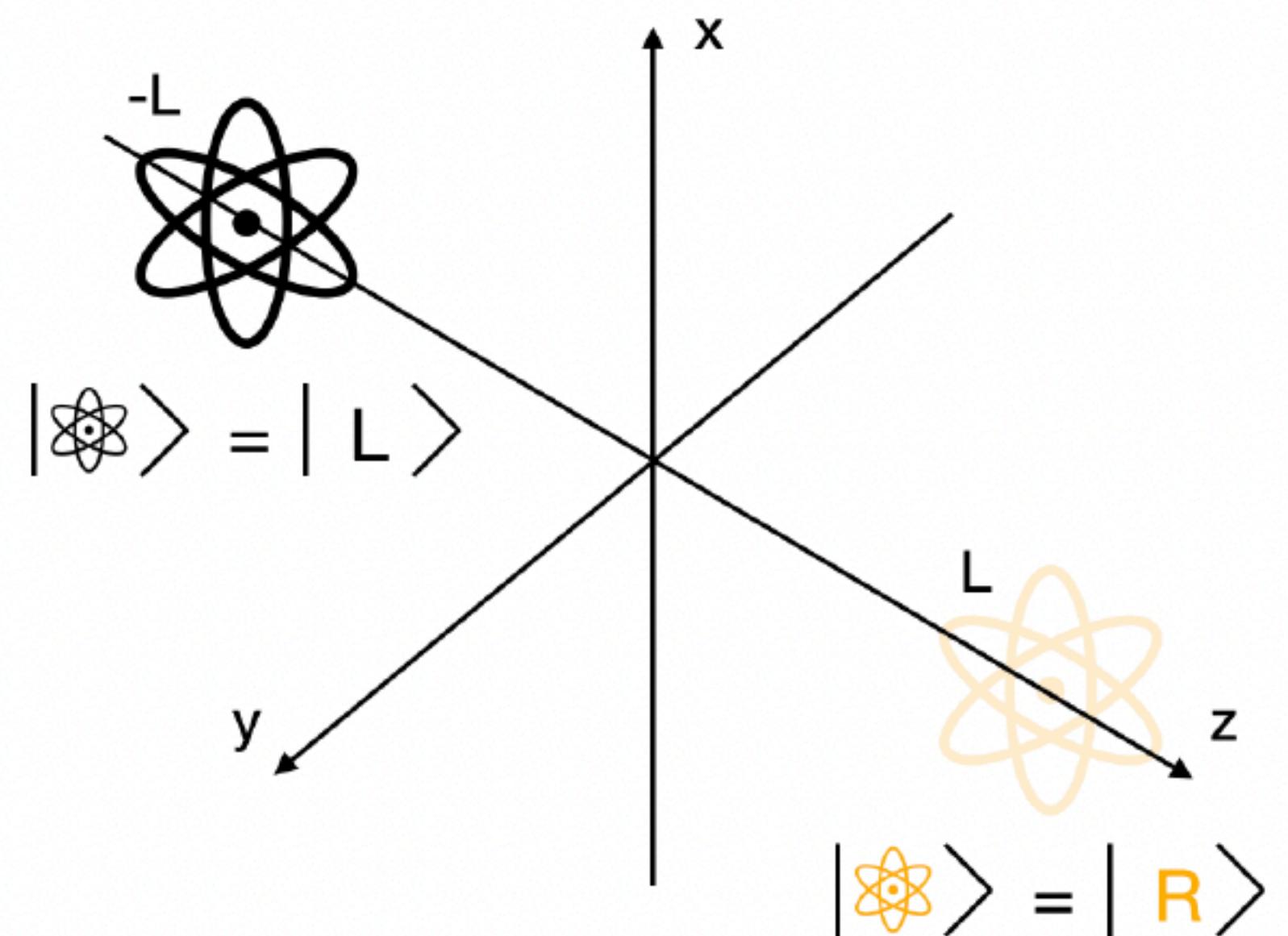


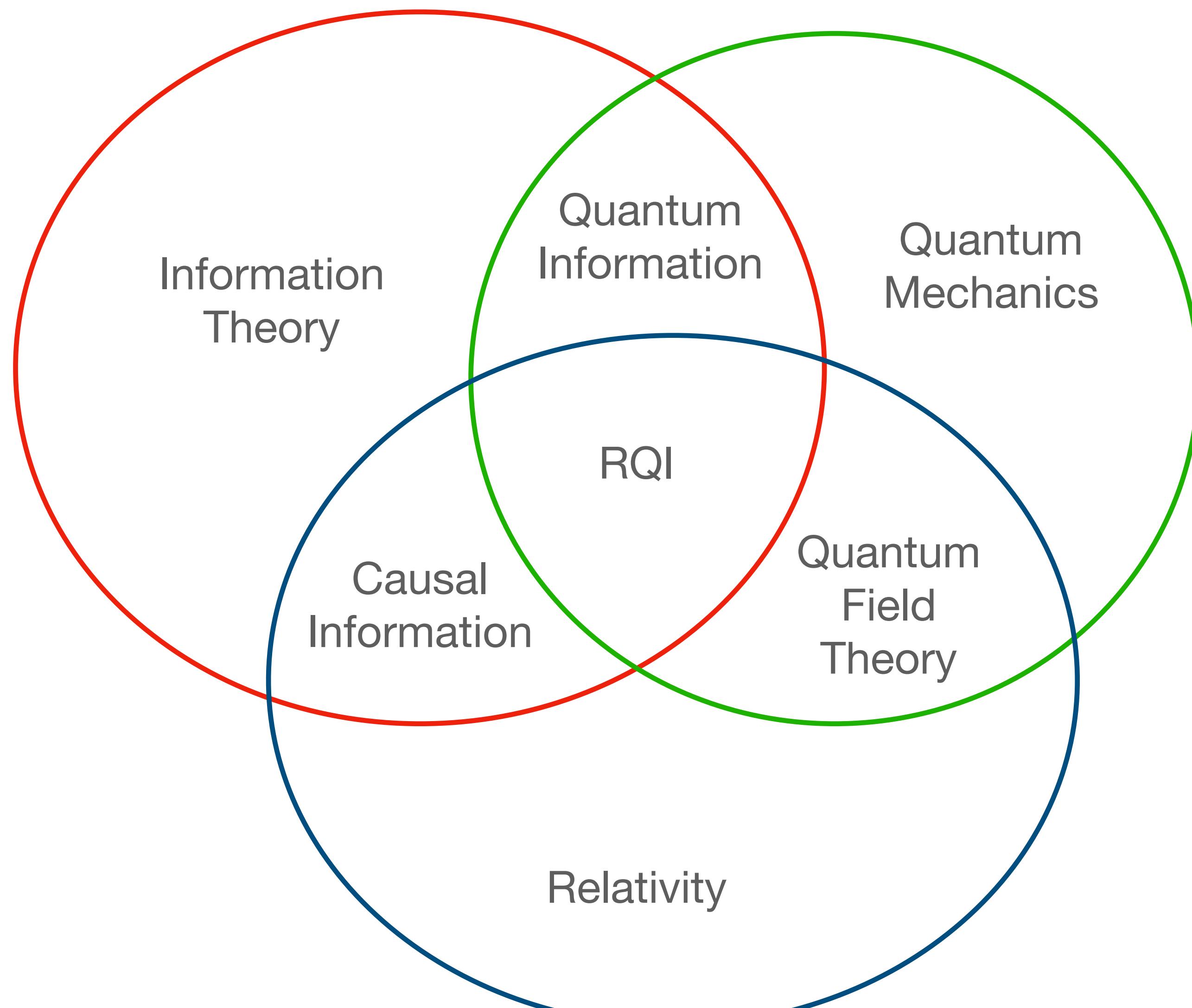
# Self gravity affects quantum states



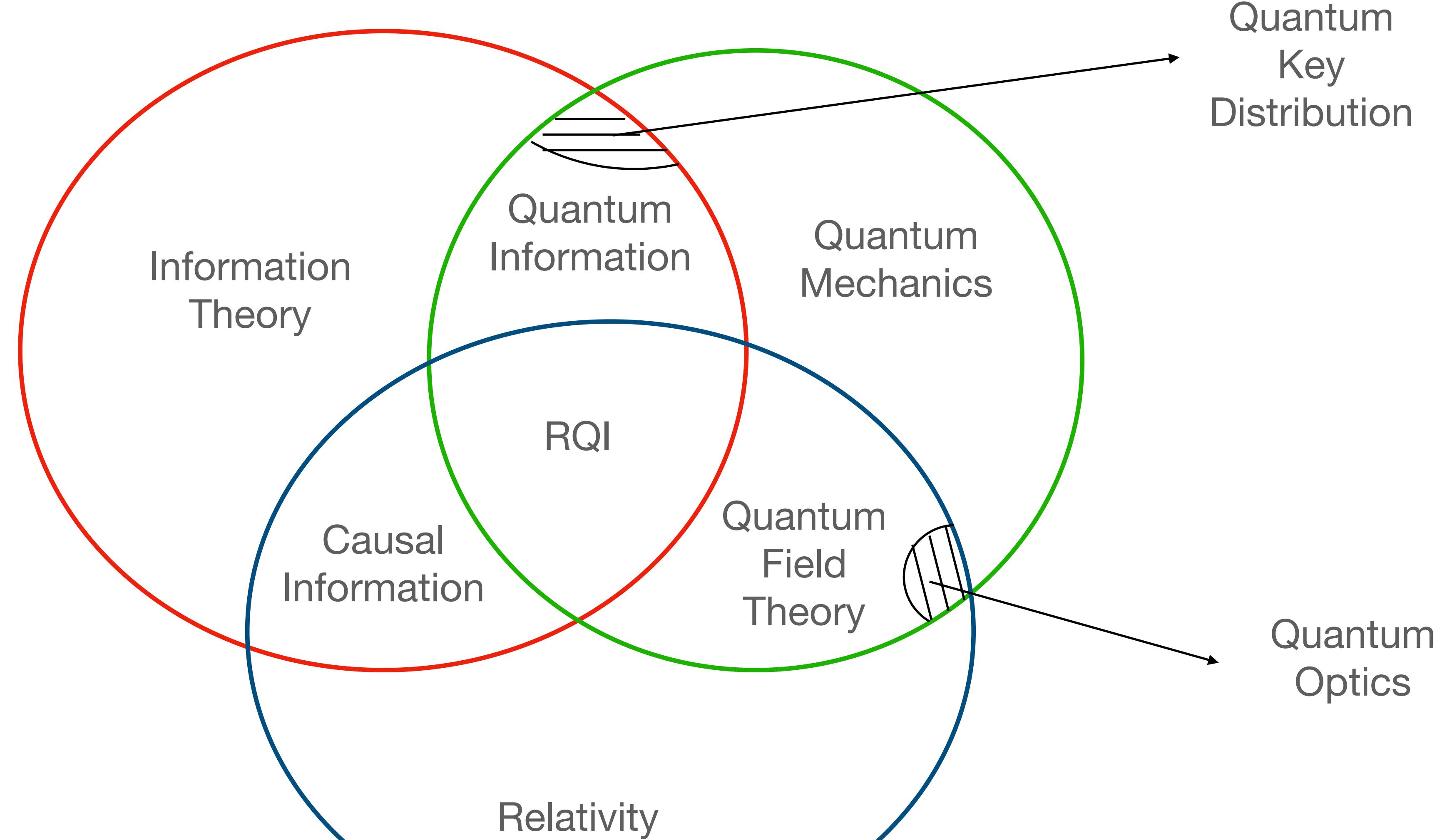
Dr. David Edward Bruschi  
PGI-12, Forschungszentrum Jülich  
Germany

VII.IX.MMXXIII  
Tergeste

# Interdisciplinary approach



# Interdisciplinary approach



# Goal

**We want to study the effects of self-gravity on the quantum dynamics of a quantum system**

# Propagation of quantum systems in flat spacetime

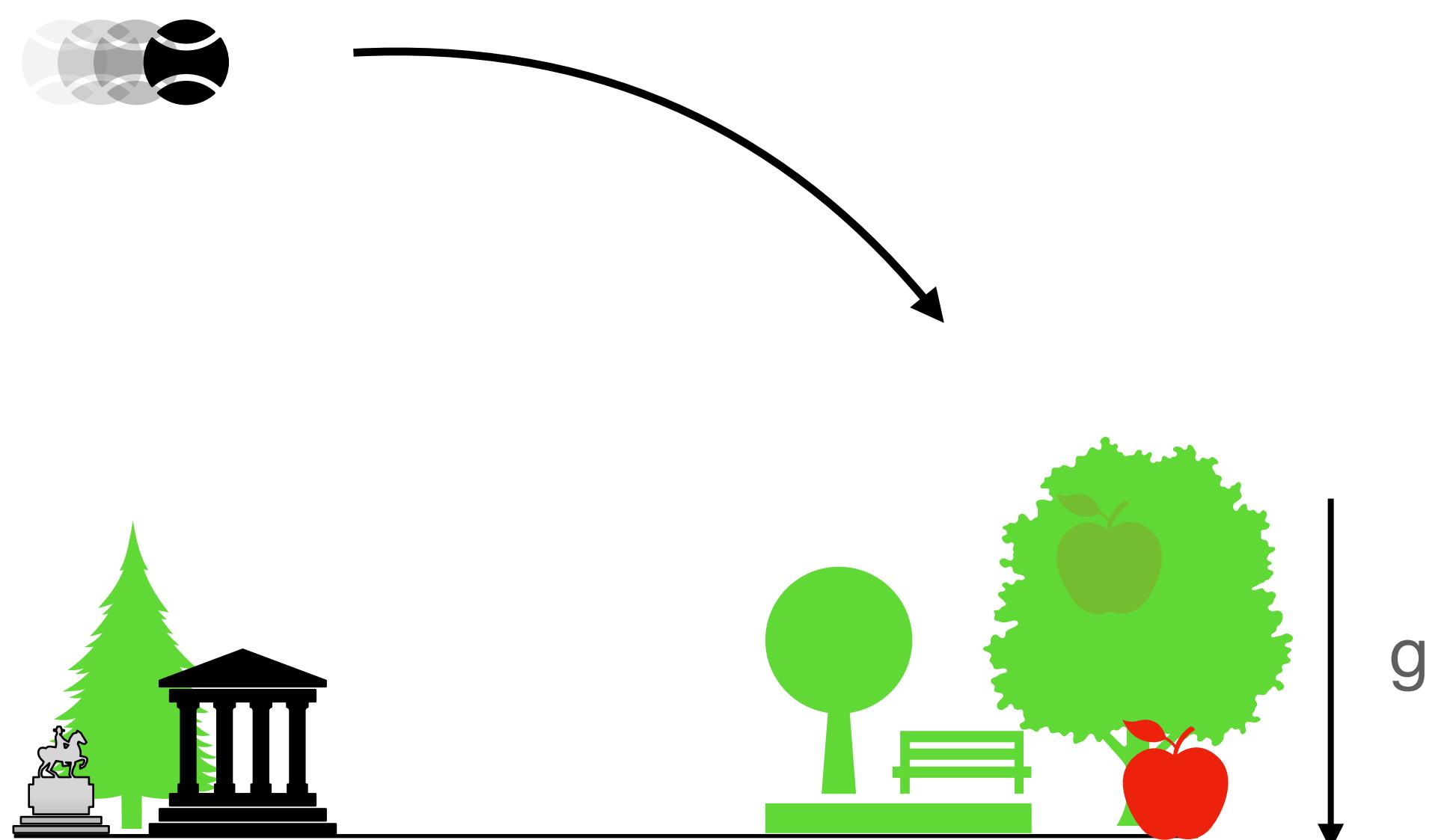


# Propagation of classical systems in curved spacetime

- Quantum Mechanics
- No gravity

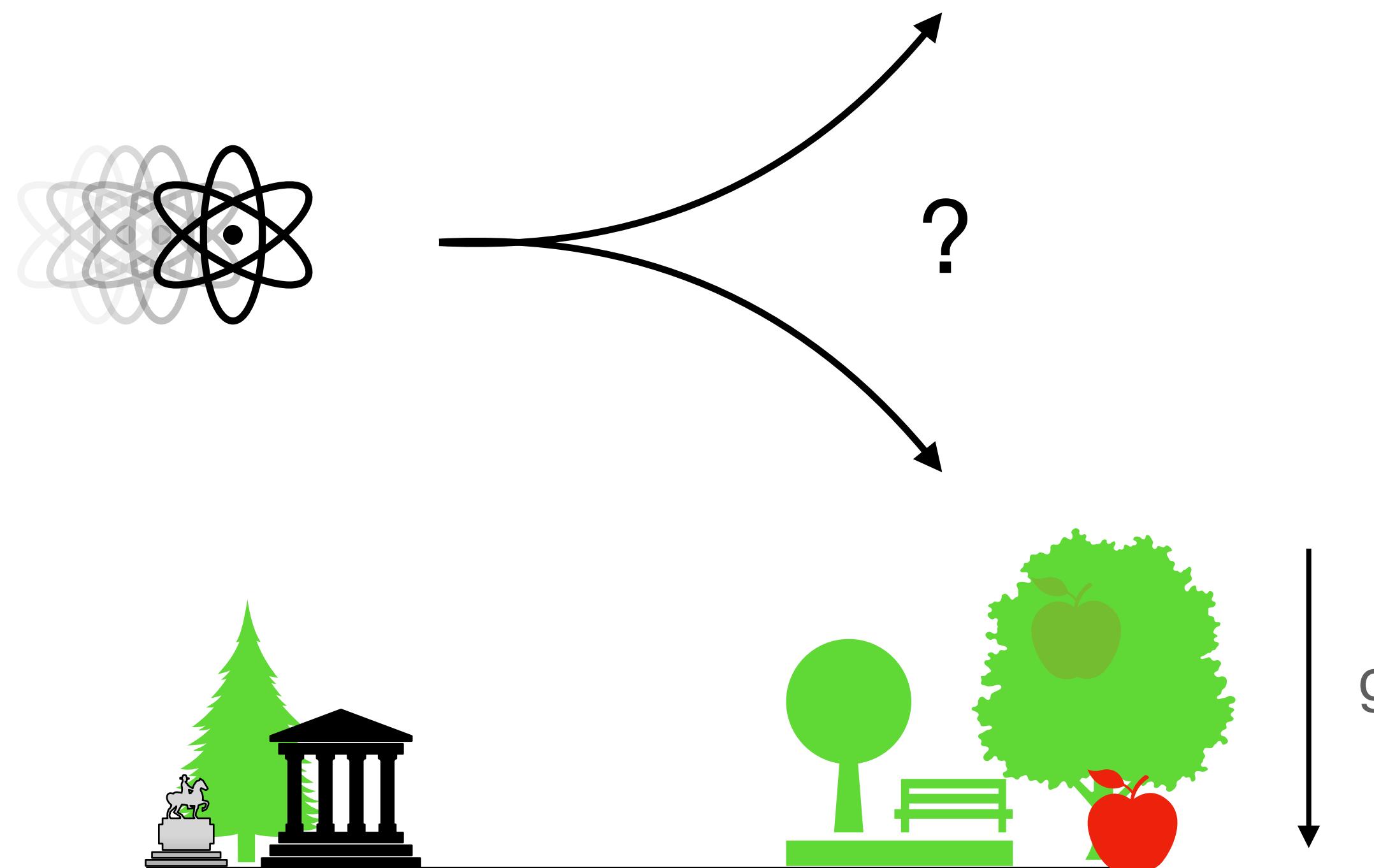


- Classical Mechanics
- Gravity

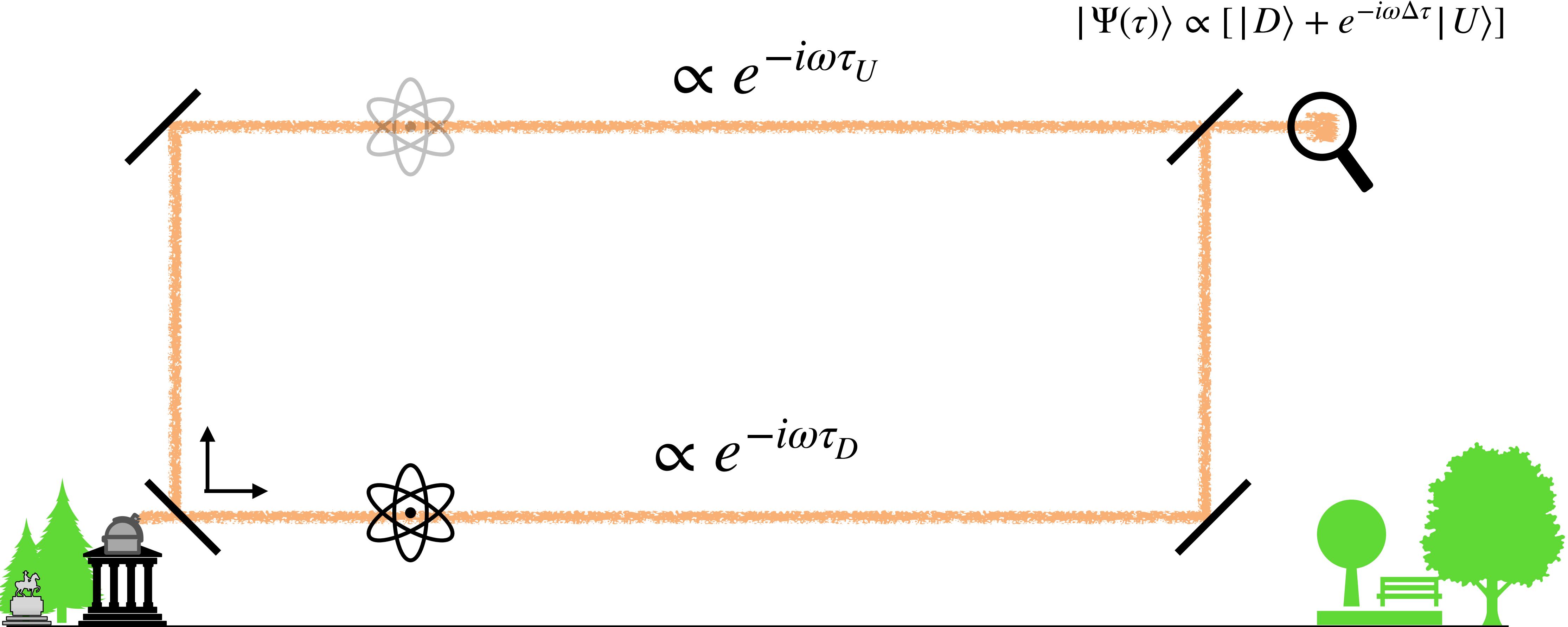


# Propagation of quantum systems in curved spacetime?

**Q: what is the weight of a quantum system?**



# Proposals for tests: interferometers



# Our approach

**We consider a quantum system and a weak classical gravitational field that is quantised.**

# Quantising linearised gravity interacting with matter

Linearised gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu}$$

Perturbative parameter

$$\epsilon \ll 1$$

Gravity/Matter first-order interaction

$$H_I^{(1)} = -\frac{1}{2} \int d^3x h_{\mu\nu} T^{\mu\nu}$$

Matter Stress-Energy Tensor

$$T^{\mu\nu}$$

# Quantising linearised gravity interacting with matter

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Quantize interaction

$$\hat{H}_I^{(1)} = -\frac{1}{2} \int d^3x \hat{h}_{\mu\nu} \hat{T}^{\mu\nu}$$

# Quantised linearised gravity interacting with matter

Gravity/Matter first-order interaction

$$\hat{H}_I^{(1)} = -\frac{1}{2} \int d^3x \hat{h}_{\mu\nu} \hat{T}^{\mu\nu}$$

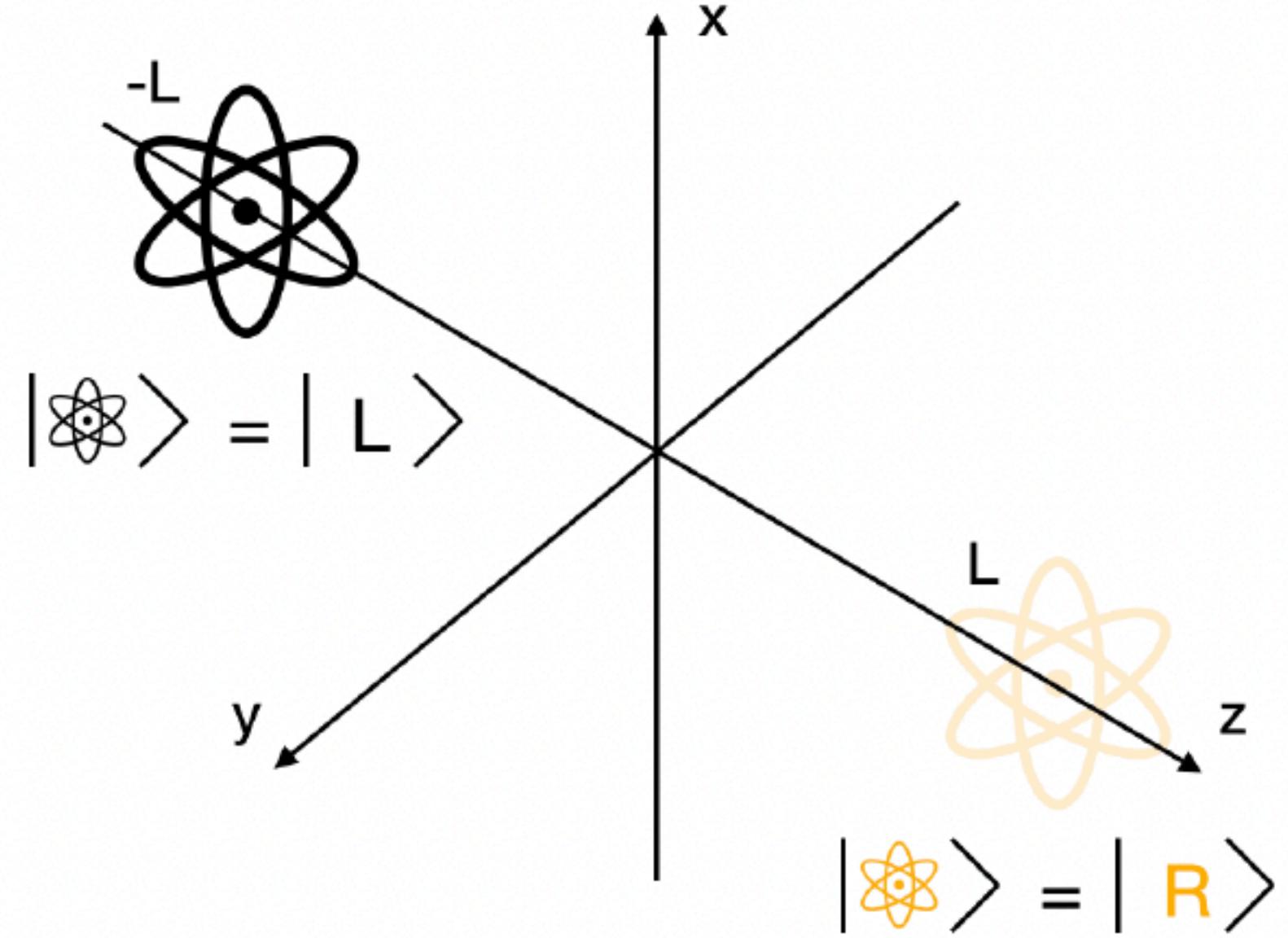
$$\hat{T}_{\mu\nu} := \hbar c \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} - \frac{\hbar c}{2} g_{\mu\nu} \left[ g^{\rho\lambda} \partial_\rho \hat{\phi} \partial_\lambda \hat{\phi} + m^2 \hat{\phi}^2 \right].$$

$$\hat{h}_{\mu\nu} = \mathcal{A} \int d^3k \frac{\sqrt{\hbar}}{\sqrt{2(2\pi)^3 \omega_{\mathbf{k}}^G}} \left[ \hat{P}_{\mu\nu}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{P}_{\mu\nu}^\dagger(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}} \right].$$

$$\mathcal{A} := \sqrt{16\pi G_N/c^2}$$

$$[\hat{P}_{\mu\nu}(\mathbf{k}), \hat{P}_{\mu'\nu'}^\dagger(\mathbf{k}')] = (\eta_{\mu\mu'}\eta_{\nu\nu'} + \eta_{\mu\nu'}\eta_{\mu'\nu})\delta^3(\mathbf{k} - \mathbf{k}'),$$
$$[\hat{P}(\mathbf{k}), \hat{P}^\dagger(\mathbf{k}')]= -\delta^3(\mathbf{k} - \mathbf{k}').$$

# Quantum system



$$\hat{\rho}_S(0) = \alpha |1_R\rangle\langle 1_R| + (1 - \alpha) |1_L\rangle\langle 1_L| + \beta |1_L\rangle\langle 1_R| + \beta |1_R\rangle\langle 1_L|$$

$$\hat{a}_R := \int d^3k F_{\mathbf{k}_0}^*(\mathbf{k}) e^{i \mathbf{L} \cdot \mathbf{k}} \hat{a}_{\mathbf{k}}$$

$$\hat{a}_L := \int d^3k F_{\mathbf{k}_0}^*(\mathbf{k}) e^{-i \mathbf{L} \cdot \mathbf{k}} \hat{a}_{\mathbf{k}}$$

$$|1_R\rangle \equiv |01\rangle := \int d^3k F_{\mathbf{k}_0}(\mathbf{k}) e^{-i \mathbf{L} \cdot \mathbf{k}} \hat{a}_{\mathbf{k}}^\dagger |0\rangle$$

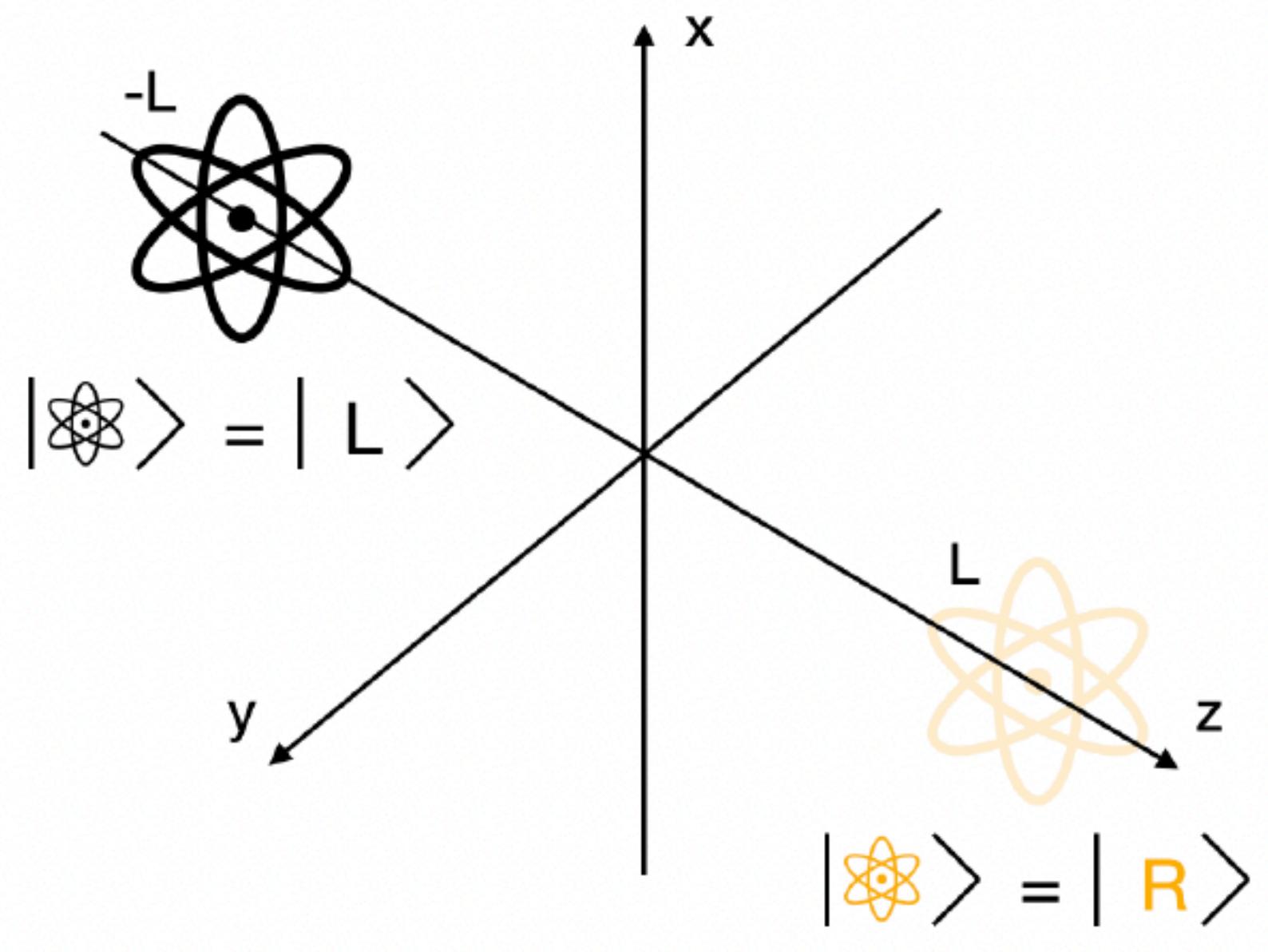
$$|1_L\rangle \equiv |10\rangle := \int d^3k F_{\mathbf{k}_0}(\mathbf{k}) e^{i \mathbf{L} \cdot \mathbf{k}} \hat{a}_{\mathbf{k}}^\dagger |0\rangle$$

# Objective

**Q:** what is the time evolution of the quantum state of the physical system?

# Time evolution

$$\hat{\rho}(t) = \hat{U}(t) (\hat{\rho}_S(0) \otimes \hat{\rho}_G(0)) \hat{U}^\dagger(t)$$



Perturbative time-evolution

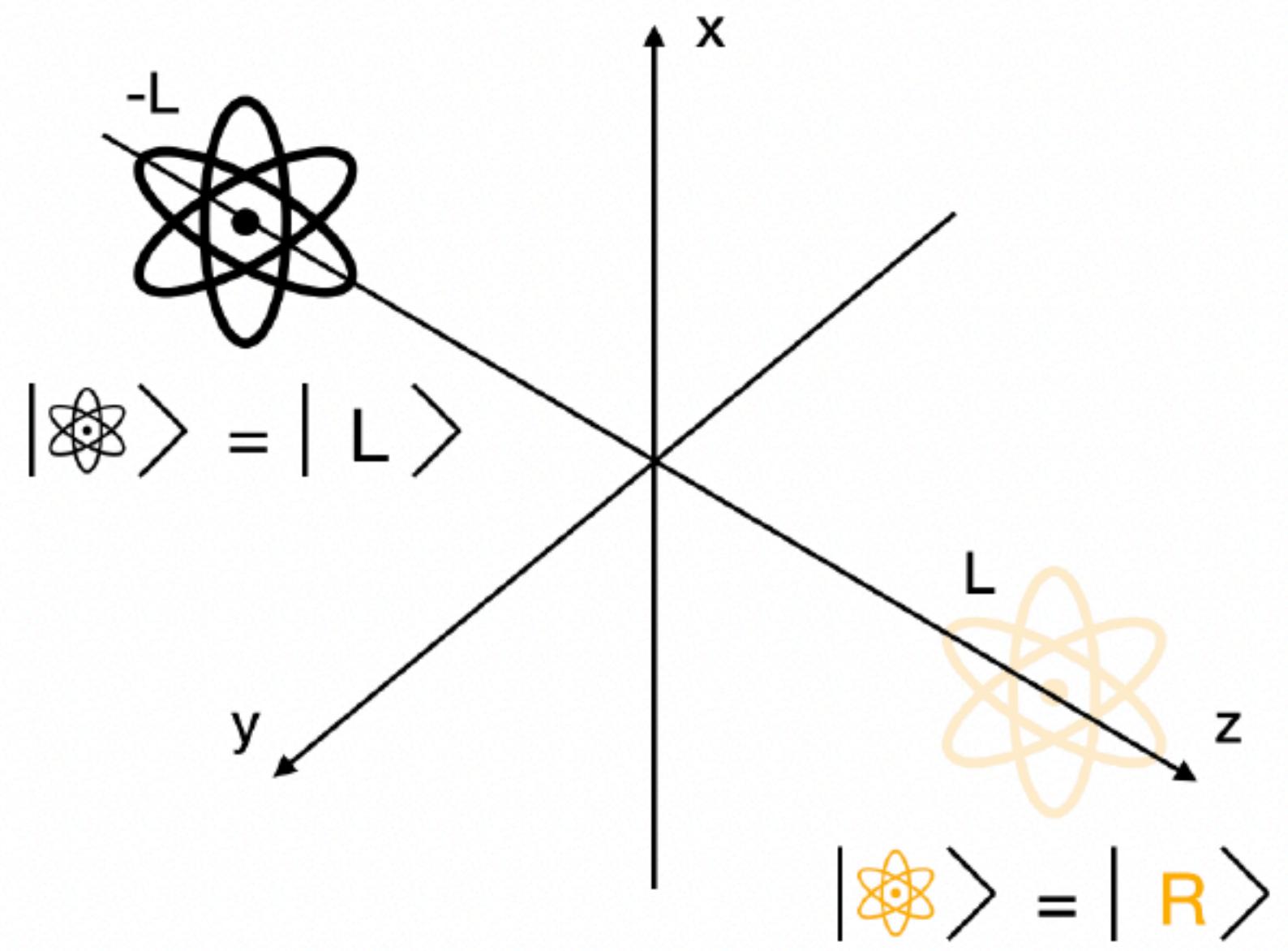
$$\hat{U}(t) = \hat{U}_0(t) \left( 1 - \epsilon \frac{i}{\hbar} \int_0^t dt' \hat{H}_I^{(1)}(t') \right)$$

Reduced state of the system at time t

$$\hat{\rho}_S(t) = Tr_G [\hat{U}(t) (\hat{\rho}_S(0) \otimes \hat{\rho}_G(0)) \hat{U}^\dagger(t)]$$

$$\hat{H}_I^{(1)} \approx - \frac{mc^2 \mathcal{A}}{2\xi} \sqrt{\frac{\hbar}{2c}} \int \frac{d^3 p d^3 p'}{\sqrt{(2\pi)^3 |\mathbf{p}' - \mathbf{p}|}} \left[ \hat{b}(\mathbf{p} - \mathbf{p}') + \hat{b}^\dagger(\mathbf{p}' - \mathbf{p}) \right] \hat{a}^\dagger(\mathbf{p}) \hat{a}(\mathbf{p}').$$

# Time evolution



$$\hat{\rho}(t) = \hat{U}(t)(\hat{\rho}_S(0) \otimes \hat{\rho}_G(0))\hat{U}^\dagger(t)$$

Perturbative time-evolution

$$\hat{U}(t) = \hat{U}_0(t)\left(1 - ie/\hbar \int_0^t dt' \hat{H}_I^{(1)}(t')\right)$$

Reduced state of the system at time t

$$\hat{\rho}_S(t) = Tr_G[\hat{U}(t)(\hat{\rho}_S(0) \otimes \hat{\rho}_G(0))\hat{U}^\dagger(t)]$$

**Optomechanical-like Hamiltonian**

$$\hat{H}_I^{(1)} \approx -\frac{mc^2\mathcal{A}}{2\xi}\sqrt{\frac{\hbar}{2c}} \int \frac{d^3pd^3p'}{\sqrt{(2\pi)^3|\mathbf{p}' - \mathbf{p}|}} \left[ \hat{b}(\mathbf{p} - \mathbf{p}') + \hat{b}^\dagger(\mathbf{p}' - \mathbf{p}) \right] \hat{a}^\dagger(\mathbf{p})\hat{a}(\mathbf{p}').$$

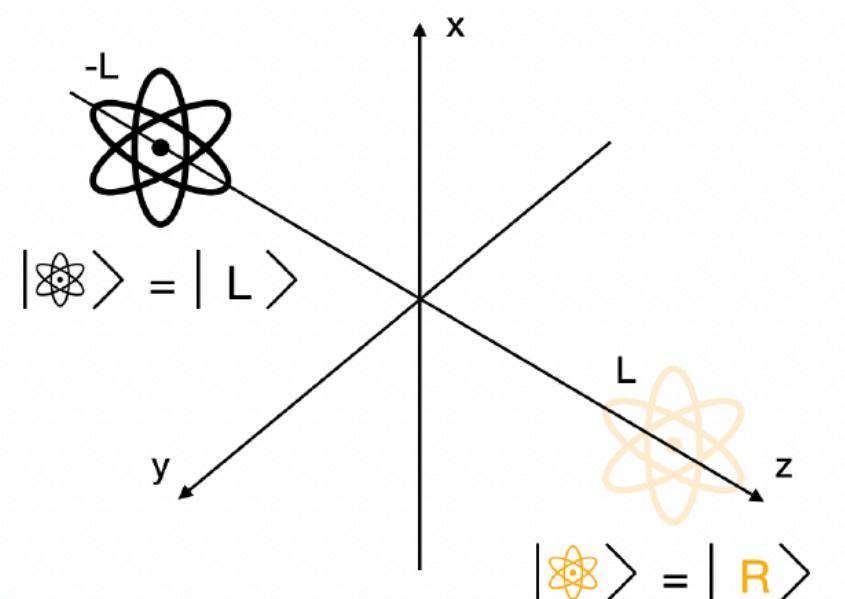
# Time evolution

$$\hat{\rho}_S(t) = \text{Tr}_G [\hat{U}(t) (\hat{\rho}_S(0) \otimes \hat{\rho}_G(0)) \hat{U}^\dagger(t)]$$

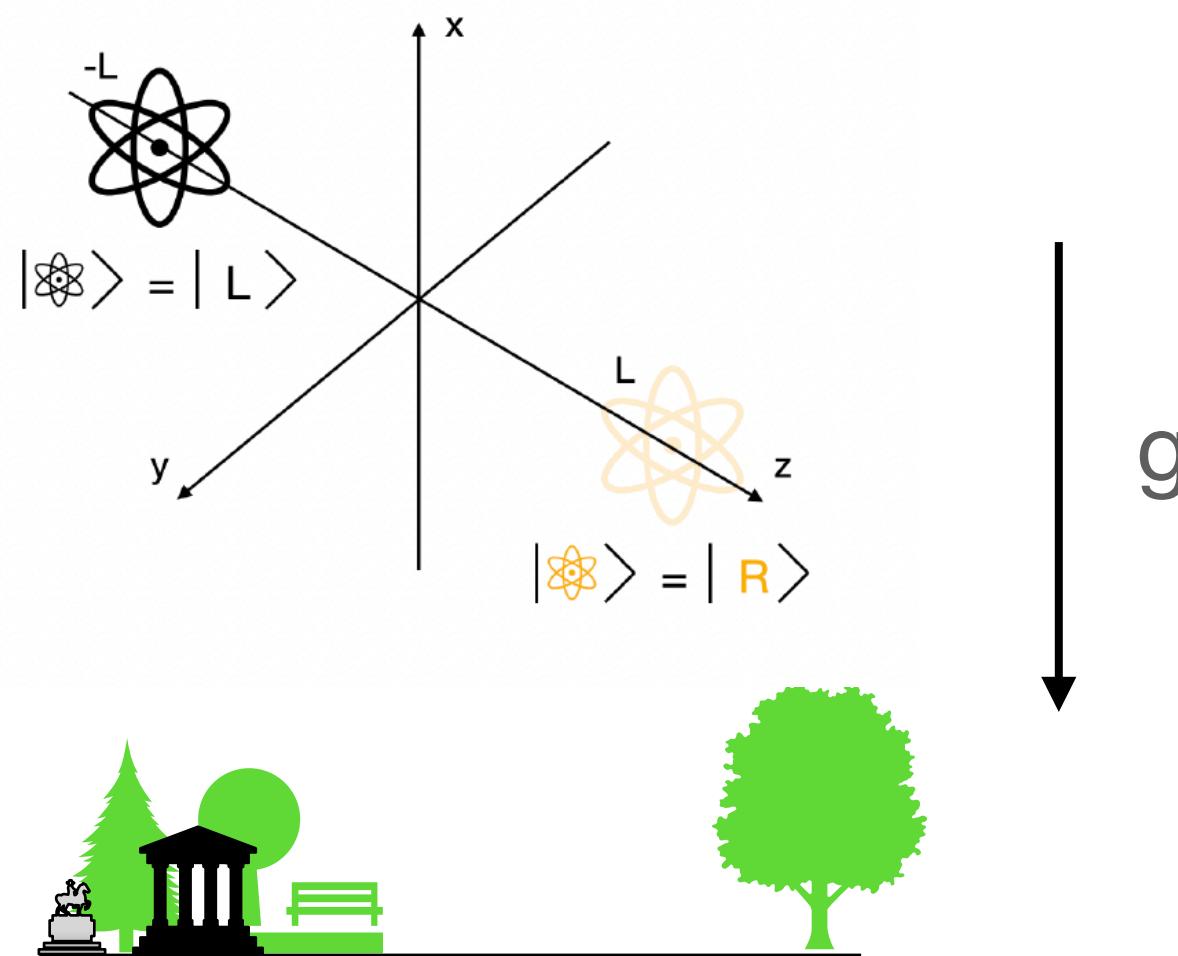
$$\hat{H}_I^{(1)} \approx -\frac{mc^2\mathcal{A}}{2\xi}\sqrt{\frac{\hbar}{2c}} \int \frac{d^3pd^3p'}{\sqrt{(2\pi)^3|\mathbf{p}'-\mathbf{p}|}} \left[ \hat{b}(\mathbf{p}-\mathbf{p}') + \hat{b}^\dagger(\mathbf{p}'-\mathbf{p}) \right] \hat{a}^\dagger(\mathbf{p})\hat{a}(\mathbf{p}').$$

Initial states for the gravitational sector

$$\hat{\rho}_G(0) = |0_G\rangle\langle 0_G|$$



$$\hat{\rho}_G(0) = |\alpha_G\rangle\langle \alpha_G|$$



$$\hat{b} |\alpha\rangle = \alpha |\alpha\rangle$$

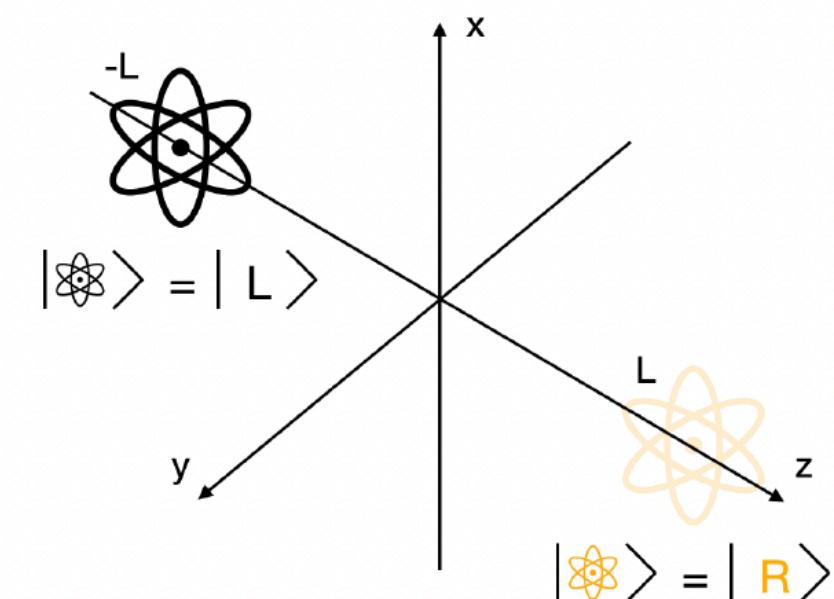
# Time evolution

$$\hat{\rho}_S(t) = Tr_G [\hat{U}(t) (\hat{\rho}_S(0) \otimes \hat{\rho}_G(0)) \hat{U}^\dagger(t)]$$

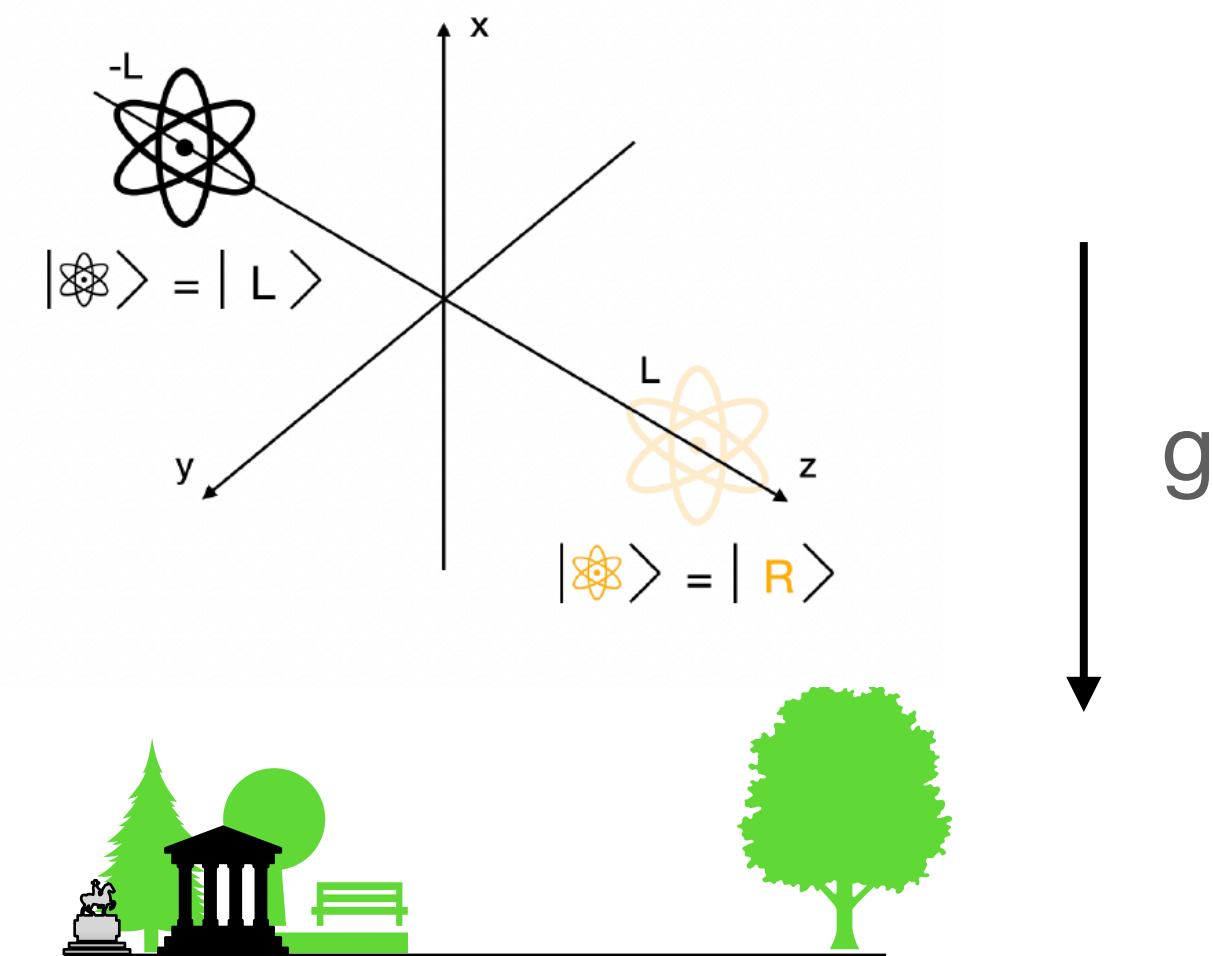
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Initial states for the gravitational sector

**SECOND order effects in the coupling**



**FIRST order effects in the coupling**



# Time evolution

$$\hat{\rho}_S(t) = \text{Tr}_G [\hat{U}(t) (\hat{\rho}_S(0) \otimes \hat{\rho}_G(0)) \hat{U}^\dagger(t)]$$

$$\hat{H}_I^{(1)} \approx -\frac{mc^2\mathcal{A}}{2\xi}\sqrt{\frac{\hbar}{2c}} \int \frac{d^3pd^3p'}{\sqrt{(2\pi)^3|\mathbf{p}'-\mathbf{p}|}} \left[ \hat{b}(\mathbf{p}-\mathbf{p}') + \hat{b}^\dagger(\mathbf{p}'-\mathbf{p}) \right] \hat{a}^\dagger(\mathbf{p})\hat{a}(\mathbf{p}').$$

Non-unitary evolution of the reduced state of the system



$$\hat{\rho}_S(0)$$

$$\hat{\rho}_S(t)$$

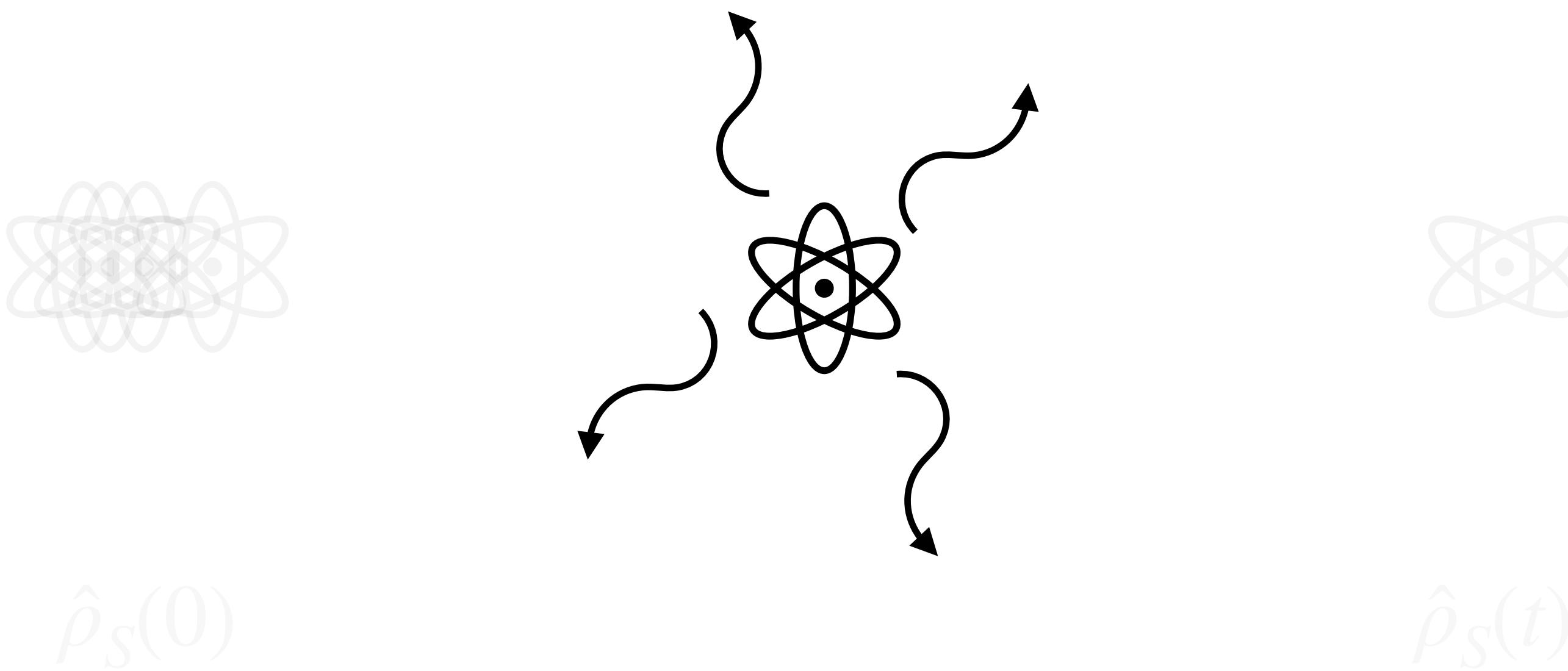
$$\text{Tr}((\hat{\rho}_S(t))^2) \neq \text{Tr}((\hat{\rho}_S(0))^2)$$

# Time evolution

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$$\hat{H}_I^{(1)} \approx -\frac{mc^2\mathcal{A}}{2\xi}\sqrt{\frac{\hbar}{2c}} \int \frac{d^3pd^3p'}{\sqrt{(2\pi)^3|\mathbf{p}'-\mathbf{p}|}} \left[ \hat{b}(\mathbf{p}-\mathbf{p}') + \hat{b}^\dagger(\mathbf{p}'-\mathbf{p}) \right] \hat{a}^\dagger(\mathbf{p})\hat{a}(\mathbf{p}').$$

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Non-unitary evolution of the reduced state of the system



$$Tr((\hat{\rho}_S(t))^2) \neq Tr((\hat{\rho}_S(0))^2)$$

# Conclusions

Time evolution of a relativistic quantum system:

- Linearized gravity - quantised
- Initial “realistic” state of quantum system (i.e., field excitation)
- Self gravity affects evolution of quantum state of system
- Effects depend on the state of the gravitational field - e.g., presence of massive objects
- Work (still) in progress

# Grazie

On the weight of entanglement [Physics Letters B 54, 182-186 (2016)]

Self gravity affects quantum states [arXiv:2006.11768] || being upgraded