Probing the Big Bang with Quantum Fields

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SIGRAV XXV Trieste - 4th September 2023

INFN Istituto Nazionale di Fisica Nucleare

Ashtekar, De Lorenzo, MS, Advances in Theoretical and Mathematical Physics 25, 7 (2021), arXiv: 2107.08506
Ashtekar, Del Rio, MS, General Relativity and Gravitation 54, 45 (2022), arXiv: 2205.00298

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Spacelike Singularities

Singularities in General Relativity

singularity theorems by Hawking and Penrose

Definition [geodesic completeness]:

Let (M, g) be a spacetime with metric g and connected semi-Riemannian manifold M. The manifold is complete if every geodesic γ_τ can be uniquely extended to arbitrary values of its affine parameter τ.

spacelike singularities: geodesics end abruptly while tidal forces diverge

FLRW Space-Time

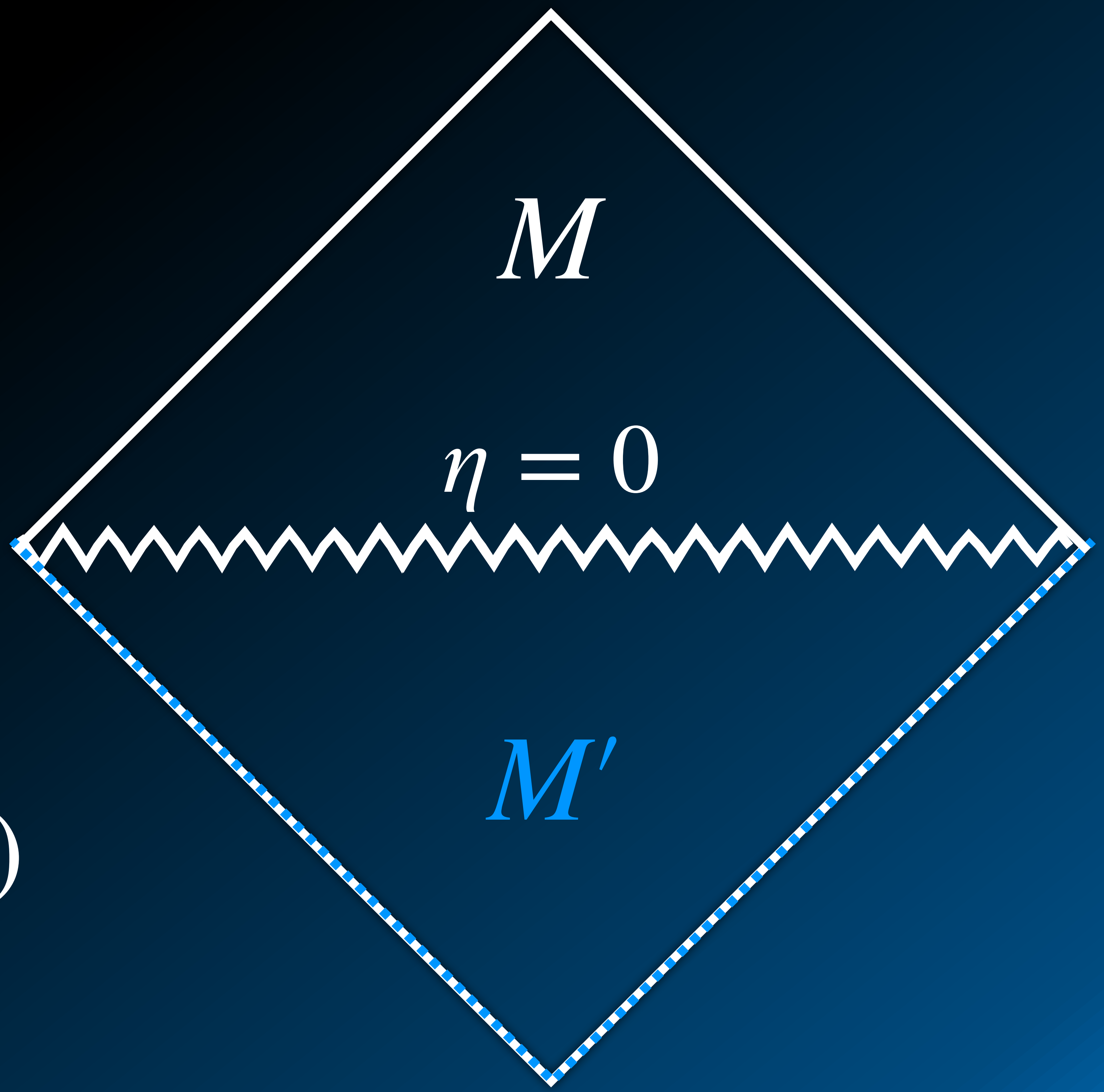
cosmological space-time: FLRW space-time

$$g = a^2(\eta)(-\mathrm{d}\eta \otimes \mathrm{d}\eta + \mathrm{d}\vec{x} \otimes \mathrm{d}\vec{x})$$

scale factor $a(\eta) = a_\alpha \eta^{1+\alpha}$, singularity at $\eta = 0$

conformal extension: $\eta \in (0,\infty) \to \eta \in (-\infty, \infty)$

physical manifold $M \to M^\circ$, now $\mathring{g}$ is $\mathscr{C}^0$ at $\eta = 0$



Singularity Probing

goal: revisiting the status of dynamical singularities through QFT probes

example: Big Bang singularity for FLRW space-time

Will test fields $\phi(x)$ and observables constructed from them, e.g. $\langle \phi(x)\phi(x') \rangle$, $\langle \phi^2(x) \rangle$, $\langle T_{ab}(x) \rangle$ remain regular in the QFT sense across the singularity?

Quantum Fields in FLRW Space-Time

QFT in Cosmological Space-Times

mode sum decomposition for solving $(\Box - \zeta R)\phi = 0$

$$\hat{\phi}(x) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \left[\hat{A}(k) e(k;\eta) + \hat{A}^\dagger(k) e^*(k;\eta) \right] e^{ikx}$$

mode equation $\quad e''(k;\eta) + 2\frac{a'(\eta)}{a(\eta)} e'(k;\eta) + k^2 e(k;\eta) = 0$

normalization $\quad e(k;\eta) e'^*(k;\eta) - e'(k;\eta) e^*(k;\eta) = \frac{i}{a^2(\eta)}$

QFT in Minkowski Space-Time

($\alpha = -1$: $a(\eta) = 1$, volume element $\mathrm{d}^4V^\circ = \mathrm{d}\eta\mathrm{d}x\mathrm{d}y\mathrm{d}z$)

quantum field distribution in Minkowski space-time (M°, g°)

$$\hat{\phi}^\circ(x) = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \left[\hat{A}(k)\frac{e^{-ik\eta}}{\sqrt{2k}} + \hat{A}^\dagger(-k)\frac{e^{ik\eta}}{\sqrt{2k}}\right] e^{ikx}$$

$\hat{\phi}^\circ$ is a (tempered) operator-valued distribution obeying $P\hat{\phi}^\circ(x) = 0$:

$\hat{\phi}^\circ(f) = \int_{M^\circ} \mathrm{d}V^\circ \hat{\phi}^\circ(x) f(x)$ is a self-adjoint operator on the Fock space satisfying $\int_{M^\circ} \mathrm{d}V^\circ \hat{\phi}^\circ(x)(Pf(x)) = 0$ for all test functions $f(x) \in \mathcal{S}$.

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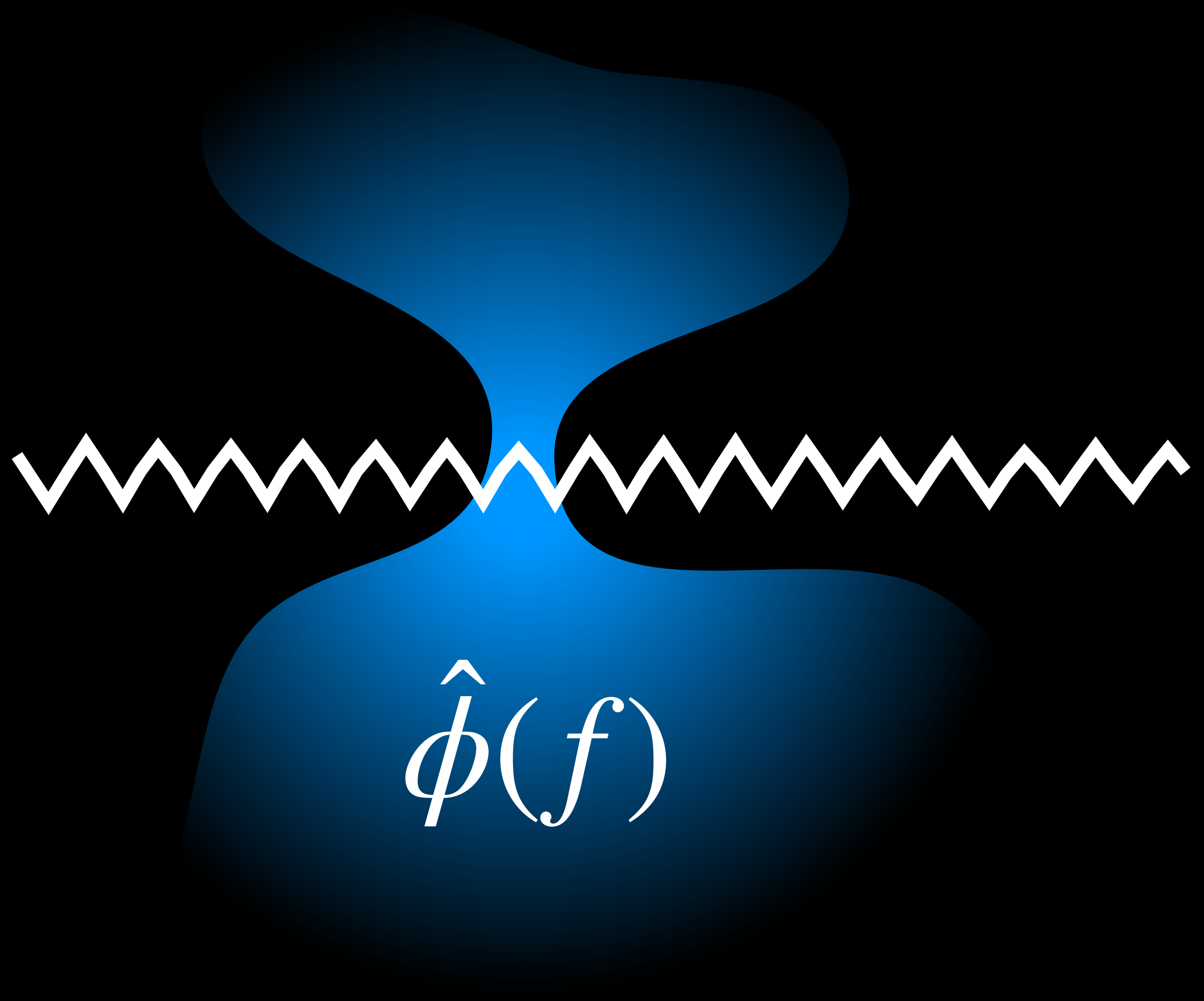
Distributional Nature of Quantum Fields

distributional character is conceptually important:



$$[\hat{\phi}^\circ(x), \hat{\phi}^\circ(x')] = i\hbar(G_{\mathrm{ad}}(x,x') - G_{\mathrm{ret}}(x,x'))\hat{\mathrm{id}}$$

$$\langle \hat{\phi}^\circ(x), \hat{\phi}^\circ(x') \rangle = \frac{\hbar}{4\pi^2} \frac{1}{|\vec{x} - \vec{x}'|^2 - |t - t' - i\varepsilon|^2}$$

well-defined in QFT sense: $\phi(f)$ bounded for $\mathrm{supp}(f) \cap \Sigma_{\eta=0} \neq \emptyset$

QFT features a distributional deformation of the classical bracket

Radiation-Filled Universe

($\alpha = 0$: $a(\eta) = a_0\eta$, $R = 0$, volume element $\mathrm{d}^4V = \mathrm{d}^4V^\circ a_0^4\eta^4$)

quantum field $$\hat{\phi}(f) = \int_{M^\circ} \mathrm{d}^4V\hat{\phi}(x)f(x) = \int_{M^\circ} \mathrm{d}^4V^\circ a^4(\eta)\frac{\hat{\phi}^\circ(x)}{a(\eta)}f(x)$$

2-point distribution $$\langle\hat{\phi}(x)\hat{\phi}(x')\rangle = \frac{\langle\hat{\phi}^\circ(x)\hat{\phi}^\circ(x')\rangle}{a(\eta)a(\eta')} = \frac{\hbar}{4\pi^2 a_0^2\eta\eta'\sigma_\varepsilon(x,x')}$$

with $2\sigma_\varepsilon(x,x') = |\vec{x}-\vec{x}'|^2 - |\eta-\eta'-i\varepsilon|^2$

vacuum polarization $$\langle\hat{\phi}^2(x)\rangle_{\mathrm{ren}} = \lim_{x'\to x}\langle\hat{\phi}(x')\hat{\phi}(x)\rangle - G_{\mathrm{DS}}(x,x') = 0$$

in general $\langle\hat{\phi}^2(x)\rangle_{\mathrm{ren}} \propto R$ while $R \equiv 0$

Stress-Energy Tensor

($\alpha = 0$: $a(\eta) = a_0\eta$, $R = 0$, volume element $\mathrm{d}^4V = \mathrm{d}^4V^\circ a_0^4\eta^4$)

stress-energy tensor $$\langle \hat{T}_{ab}(x)\rangle_{\mathrm{ren}} = \frac{\hbar\,\nabla_a\eta\,\nabla_b\eta}{720\pi^2 a_0^2\eta^6} + \frac{\hbar g^\circ_{ab}}{576\pi^2 a_0^2\eta^6}$$

(counterterms calculated by the DeWitt-Schwinger method)

homogeneous distribution $$\underline{\eta}^{-\alpha} : f(x) \to -\frac{(-1)^{\alpha-1}}{(\alpha-1)!}\int \mathrm{d}\eta\,\ln|\eta|\,\frac{\mathrm{d}^\alpha f}{\mathrm{d}\eta^\alpha}(x)$$

diverges as a function but is well defined as OVD of form η^{-2}

Discussion

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- distributional character of fields is crucial as they are „more tolerant“ when encountering singularities (singularities are tamed)
- classical fields $\phi(x)$ diverge while the one-particle Hilbert space norm $||\phi||_2$ remains finite
- foreshadowing: symplectic product on classical phase space is conserved
- similar results can be found for the gravitational singularity in Schwarzschild space-time
- possible hints for quantum gravity: self-consistent theory that allows matter and geometry to interact quantum mechanically, geometry might also have a distributional character at the micro level