A distance functional between cosmological Celestial Spheres

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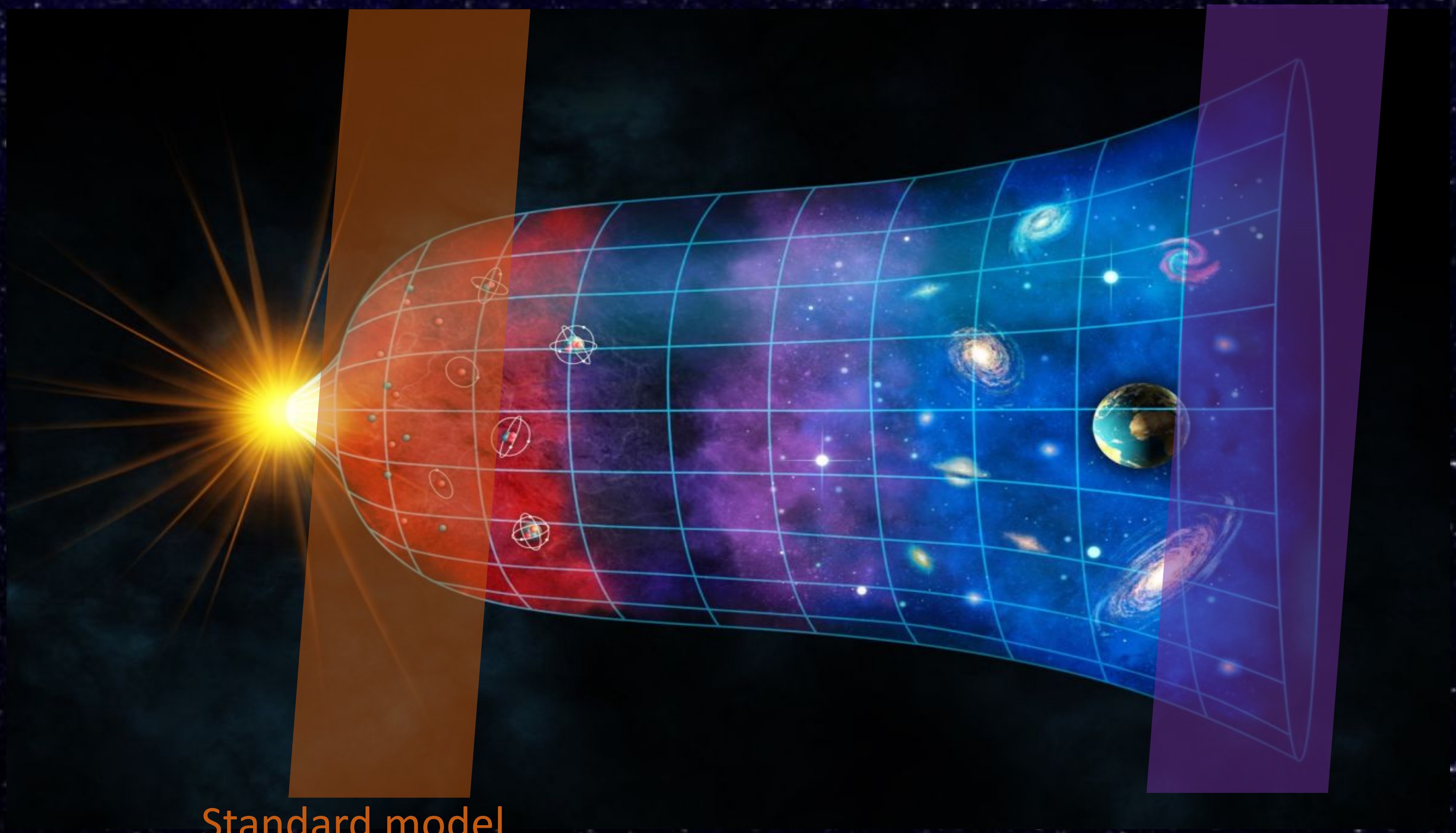
The picture provided by the ΛCDM model and by the FLRW spacetime, is quite successful in providing a rather accurate physical and geometrical representation of the universe at present era and over spatial scales ranging from $\approx 100\ h^{-1}$ Mpc to the visual horizon of our past light cone.

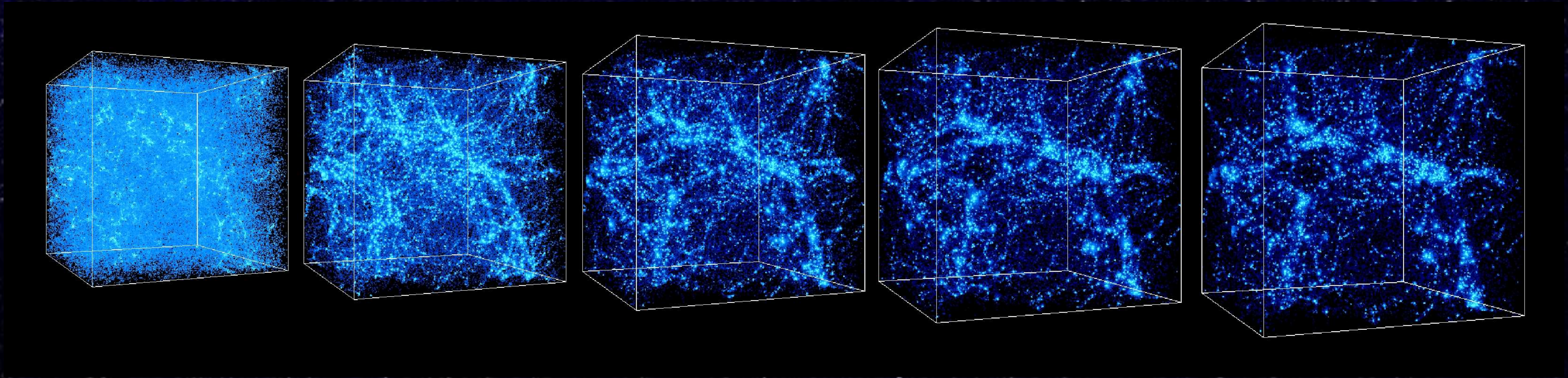
However extending the validity of the FLRW solutions (even in their perturbatuve form) over scales $\lesssim 100\ h^{-1}$ Mpc is quite problematic



As we probe spatial regions in the range $\leq 100h^{-1}$Mpc, the actual distribution of matter becomes extremely anisotropic with a high density contrast.

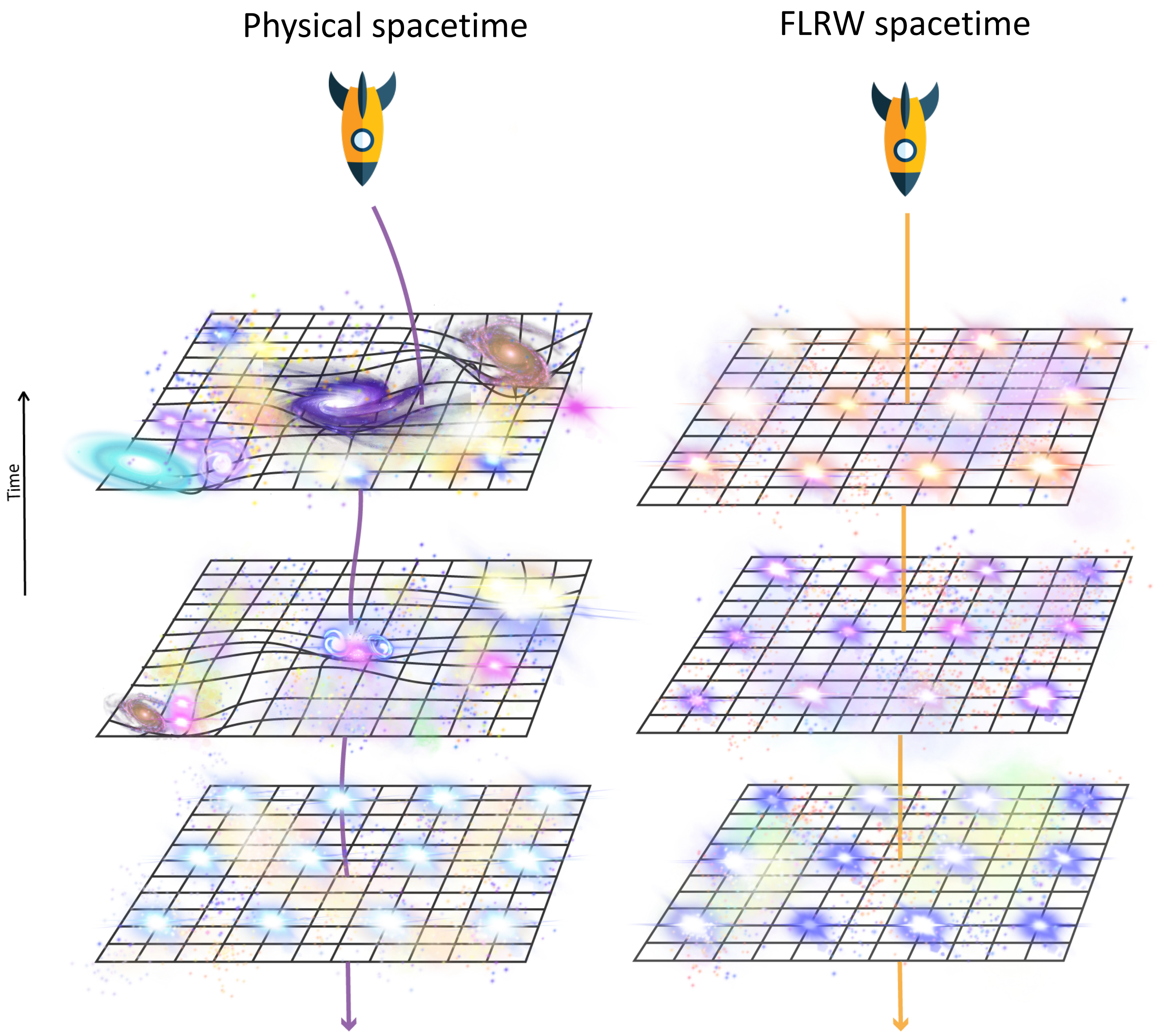
Gravitational clustering gives rise to a complex network of structures, characterized by the presence of a foam like web of voids and galaxy filaments often extending well into the $100h^{-1}$Mpc range

At this scales we may have non-perturbative correction terms due to the coupling between gravitationally bound structures and the emergent spacetime geometry.



…this leads to the question

What is the role of these pre-homogeneity regions ?

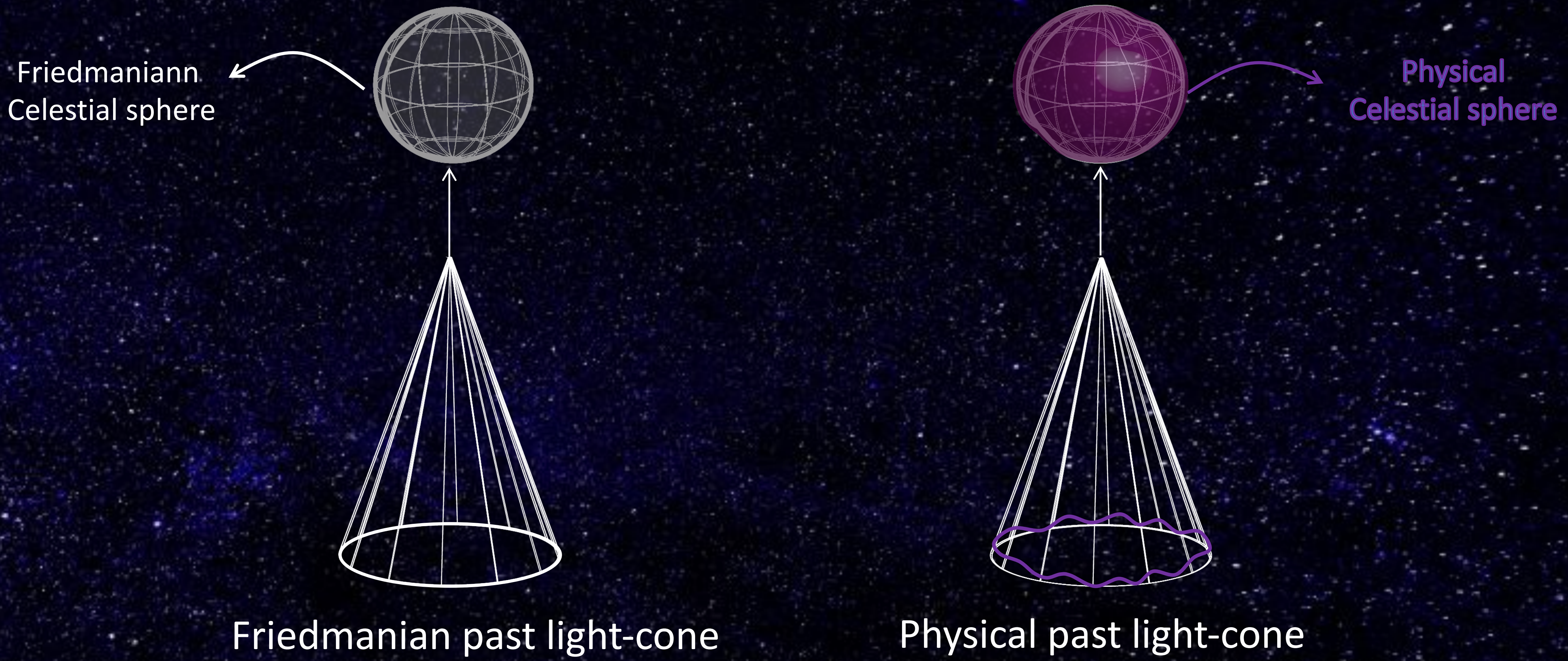


We would like to be able to quantify the effect of the prehomogeneity region on the overall dynamics of the Universe

The general Idea is to consider the point of view of two different observers: the Friedmaniann observer and the Physical observer.

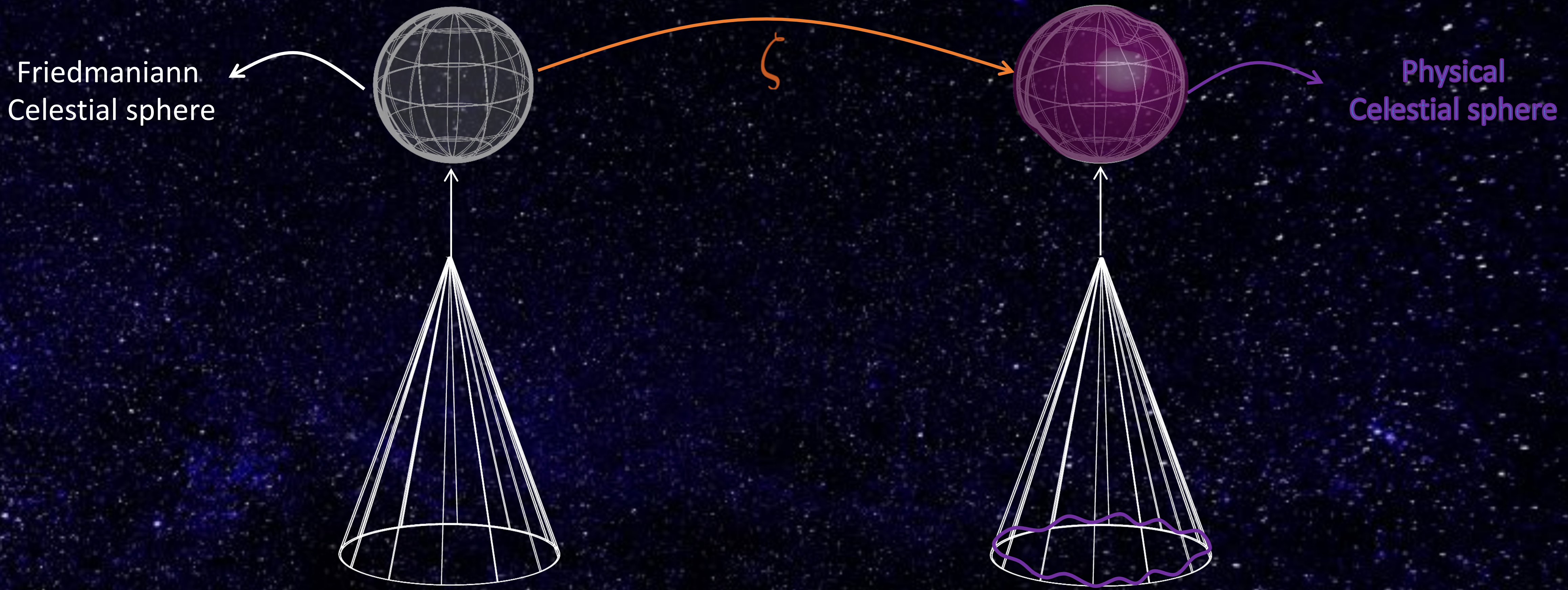
How much do they differ from each other?

…we need to adress this problem in the context of spacetime geometry



We want to compare the two Celestial Spheres

These Celestial spheres are Riemaniann surfaces (a round 2-sphere for FLRW; a bumpy surface in the physical case) , so from the matematical point of view we are comparing Metric Surfaces: the best way to do tha is by using the harmonic map theory, adopted usually in image visualization problems.

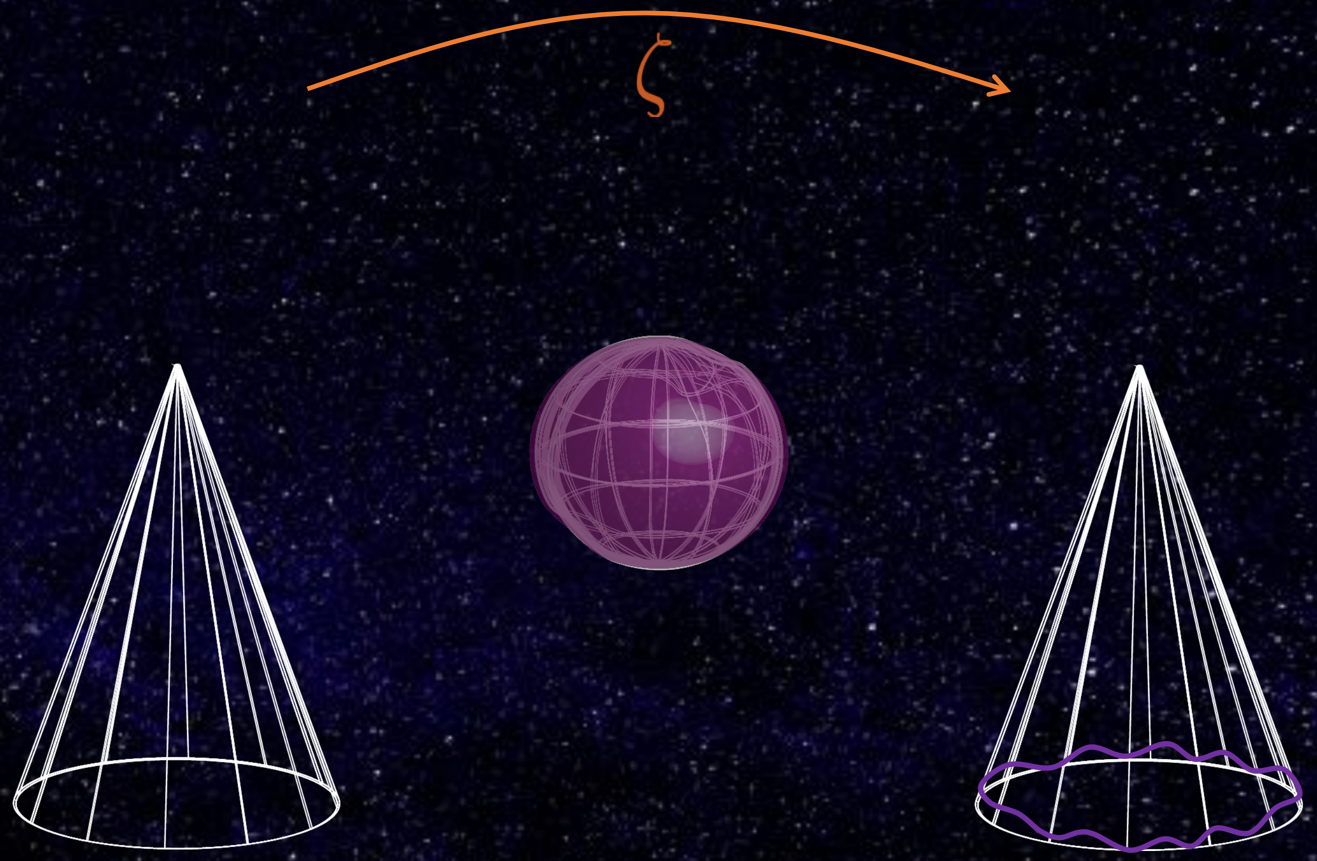


ζ Contains informations about the difference between the two Celestial-Spheres which arise for two main reasons:

- The respective Hubble flows are different from each other.
- On scale $< L_0$ local inhomogeneities couple with the geometry (no longer RW) which influence the motion of the observer (peculiar velocities affect the choice of the three referece anchors determining ζ)

ζ is a Lorentz transformation   Represented as an element of $\zeta \in PSL(2, \mathbb{C})$

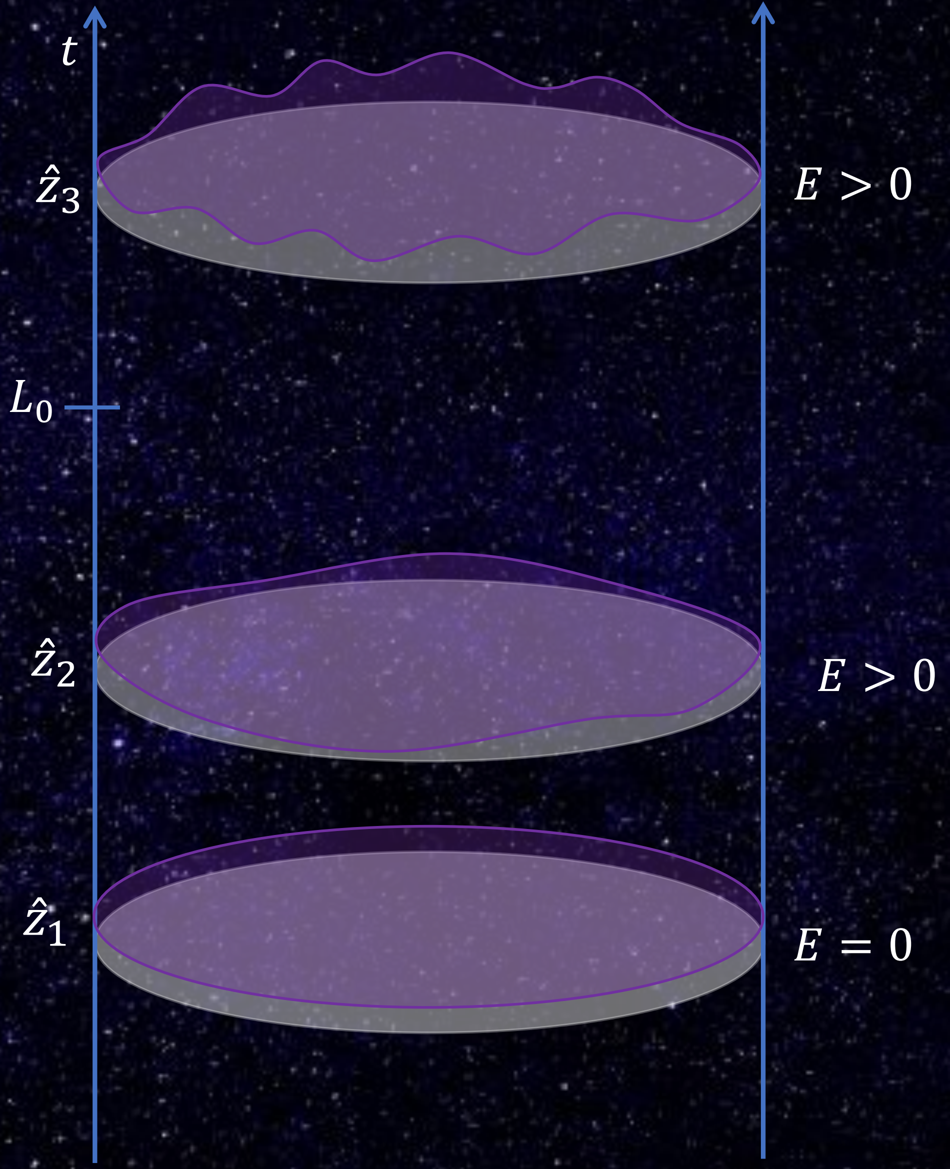
The group of conformal trasformation between Celestial Spheres



In the simplest case, when both the celestial spheres are spheres, the map induce a conformal diffeomorfism between the two

All the difference between the two 2-spheres can be encoded in a conformal factor: Φ

Taking advantage of this conformal transformation we can define…



FLRW Celestial Spheres

Physical Celestial Spheres

The Comparison Functional:

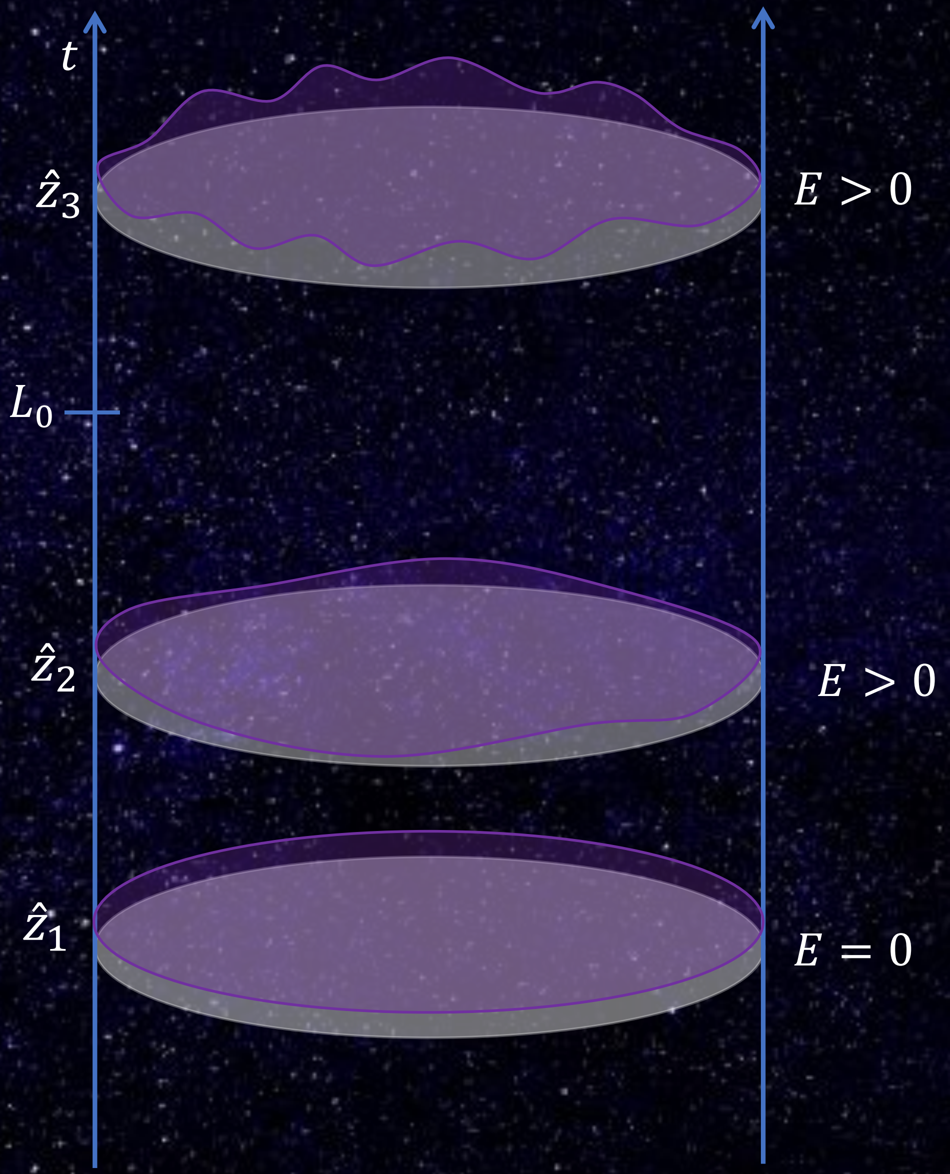
$$E_{\widehat{\mathbb{CS}}_{\hat{z}},\, \mathbb{CS}_z}[\zeta_{(\hat{z})}] := \int_{\widehat{\mathbb{CS}}_{\hat{z}}} (\Phi(y) - 1)^2 \, d\mu_{\hat{h}_{(\hat{z})}}(y).$$

Conformal factor

FLRW Celestial Spheres

area element on $\widehat{\mathbb{CS}}_{\hat{z}}$

At a given redshift $\hat{z}$ compares the FLRW celestial sphere and the Physical celestial sphere



FLRW Celestial Sphere

Physical Celestial Sphere

A really crucial property of the comparison functional is its relation with the area distance:

$$E_{\widehat{\mathbb{CS}}_{\hat{z}},\,\mathbb{CS}_{z}}[\zeta_{(\hat{z})}] = \int_{\widehat{\mathbb{CS}}_{\hat{z}}} \left[D(\zeta_{(\hat{z})}(y)) - \widehat{D}_{\hat{z}}(y)\right]^2 d\mu_{\mathbb{S}^2}(y)$$

Physical area distance

FLRW area distance

solid angle measure

This equation tell us that the comparison functional describes the *mean square fluctuations*, at the chosen redshift $\hat{z}$, of the physical area distance with respect to the Friedmaniann area distance.

The comparison functional can be written in terms of measurable quantities so, in principle:.

It is measurable!

This suggest us that we can address the issue of finding the optimal functional: the one that minimize the fluctuations between the physical area distance and the Friedmaniann area distance.

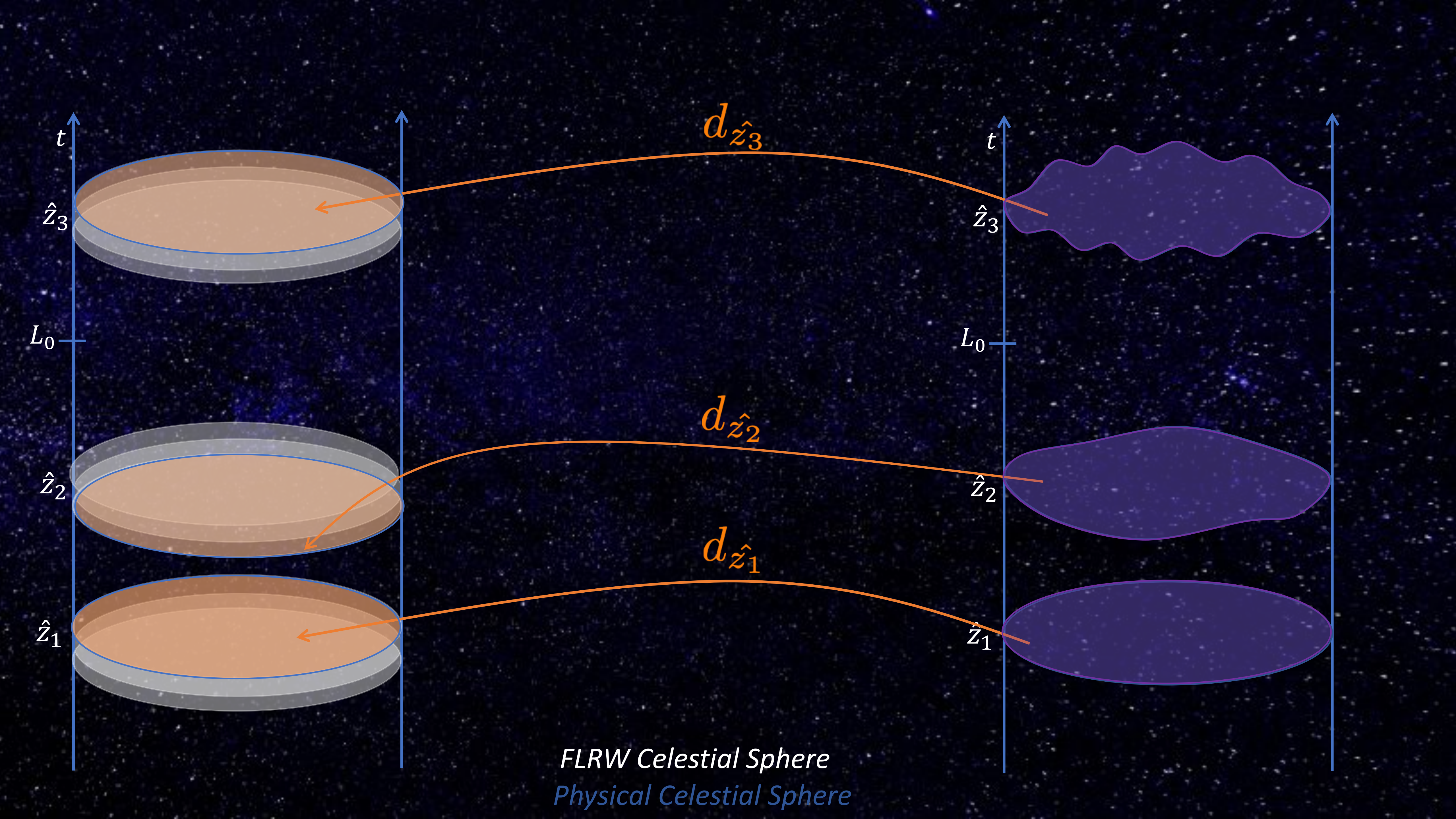
We minimize the Comparison Functional over all the possible peculiar velocities fluctuations around the map ζ :

$$d_{\hat{z}}\left[\widehat{\mathbb{C}\,\mathbb{S}}_{\hat{z}},\ \mathbb{C}\,\mathbb{S}_{z}\right] := \inf_{\zeta_{(\hat{z})}\,\in\,\mathrm{PSL}(2,\mathbb{C})} E_{\widehat{\mathbb{C}\,\mathbb{S}}_{\hat{z}},\ \mathbb{C}\,\mathbb{S}_{z}}\left[\zeta_{(\hat{z})}\right]$$

defining the Distance Functional

It has a number of properties:

- It is scale dependent ($\hat{z}$)
- It is a functional distance between the two Celestial Spheres



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$\hat{z}_1$
FLRW Celestial Sphere
Physical Celestial Sphere

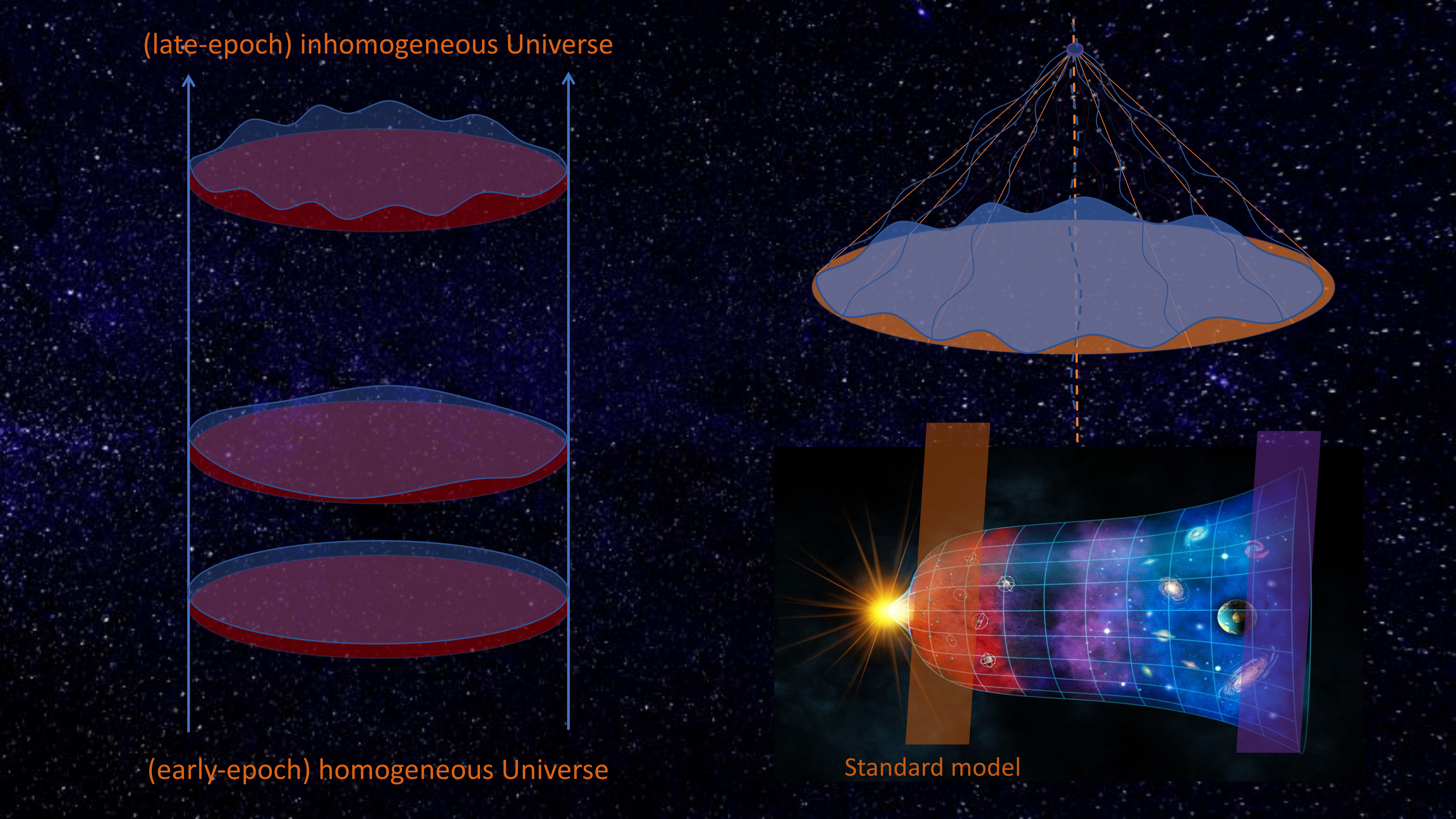
The scale-dependent distance functional has memory of the inhomogeneous region probed by our past light cone also when the information reaching us comes from large $\hat{z}$ sources.

For single sources this memory is manifest from lensing events etc… However $d_{\hat{z}}$ organizes these local memories in a unique global functional.

We can associate to this memory field a corresponding red-shift dependent positive contribution to the FLRW cosmological constant $\widehat{\Lambda}$ given by:

$$\widehat{\Lambda}_{\widehat{S}} := \frac{18}{\pi}\,\frac{d_{\widehat{z}}\left[\widehat{\mathbb{C}\,\mathbb{S}}_{\widehat{z}},\ \mathbb{C}\,\mathbb{S}_z\right]}{\widehat{r}_{\widehat{z}}^{\;4}} = \frac{18}{\pi}\left(\int_0^{\widehat{z}}\frac{d\widehat{z}\,'}{H(\widehat{z}\,')}\right)^{-4} d_{\widehat{z}}\left[\widehat{\mathbb{C}\,\mathbb{S}}_{\widehat{z}},\ \mathbb{C}\,\mathbb{S}_z\right].$$

to whatever degree one accepts the role of (area) distance fluctuations in precision cosmology, one has to grant this expression an equal degree of acceptance. Weak lensing, and galaxy surveys , delivering accurate measurements of our celestial sphere, will allow us to understand to what extent $\hat{\Lambda}_{\hat{S}}$ contributes to the FLRW cosmological constant .



(late-epoch) inhomogeneous Universe
(early-epoch) homogeneous Universe
Standard model