A quantum-spacetime model with kinematical IR/UV mixing and its phenomenology

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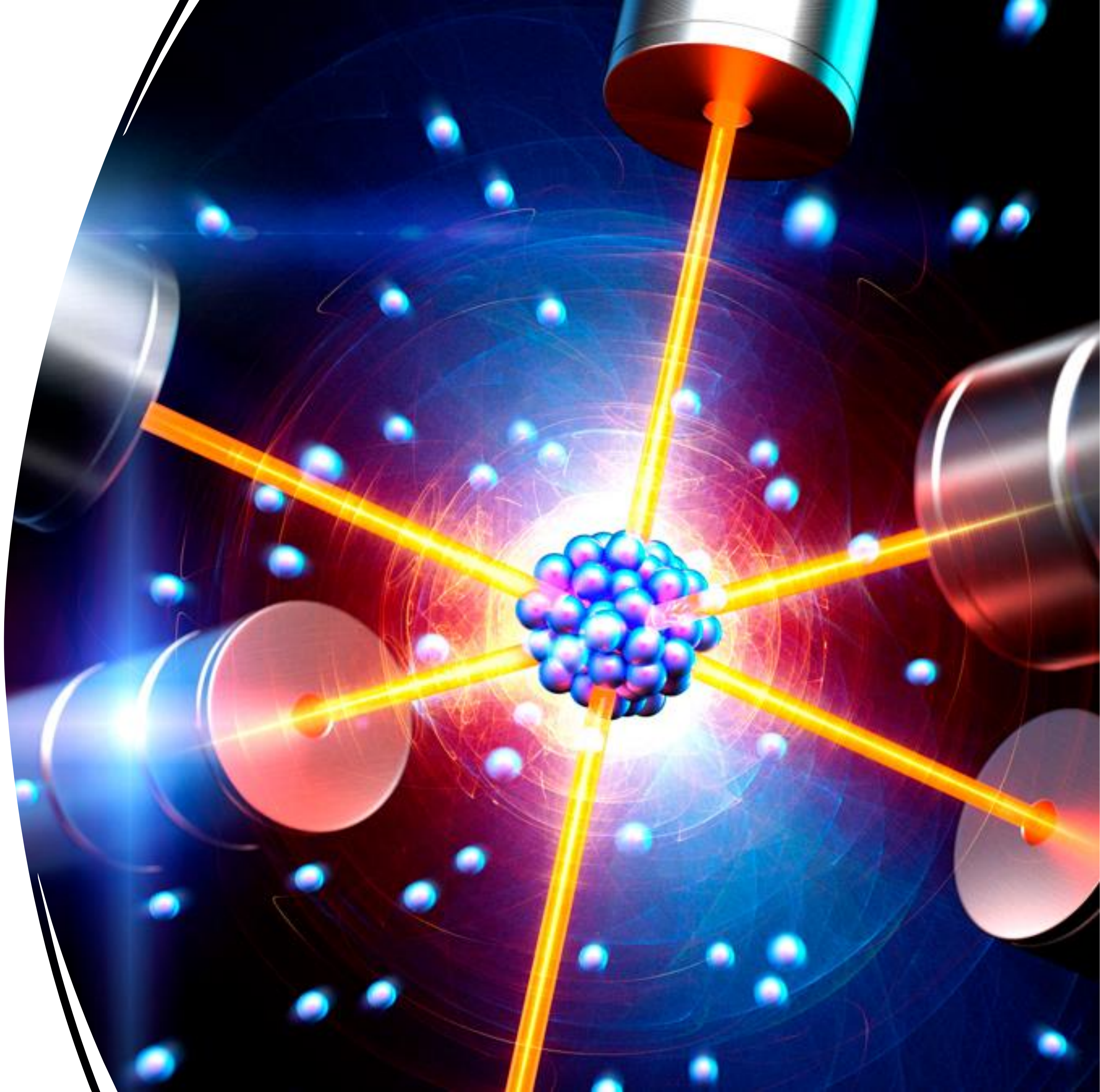
In collaboration with G. Amelino-Camelia, G. Fabiano, F. Mercati, soon on the Arxiv.



Introduction

- In many quantum gravity models an effective descriptions of spacetime in the UV is characterized by coordinates non commutativity:

$$[x_\mu, x_\nu] \propto \mathrm{i}\, \ell\, f_{\mu\nu}(x)$$

Where ℓ is an UV scale.

- IR/UV mixing ⟹ new physics both in the UV and in the IR regime.

THEORETICAL ASPECTS

Dynamical IR/UV mixing in Moyal spacetime

- Consider the Moyal non-commutative spacetime :

$$[x_\mu, x_\nu] = \mathrm{i}\ \theta_{\mu\nu}$$

- In ϕ^4 field theories there is an IR/UV mechanism that affects θ dependent corrections to the propagator:

$$\int_0^\Lambda dk \cos\left(\frac{1}{2}k\,\tilde{p}\right)\frac{\mathrm{k}^3}{\mathrm{k}^2+\mathrm{m}^2} \cong \frac{1}{2}\left(\frac{2}{|\tilde{p}|}\right)^2 - \frac{1}{2}m^2 \ln\left(\frac{1+\left(\frac{2}{|\tilde{p}|}\right)^2}{\mathrm{m}^2}\right)$$

where $\tilde{p}_\mu = \theta_{\mu\nu}p^\nu$ and $\Lambda \gg |\tilde{p}|^{-1}$

- The UV portion of the loop integration introduces an IR divergence $\sim |\tilde{p}|^{-1}$.

"On the IR/UV mixing and experimental limits on the parameters of canonical non commutative spacetime", Amelino-Camelia, Mandanici, Yoshida, *JHEP* 01 (2004) 037

κ-lightlike non-commutative spacetime

- Spacetime non-commutativity:

$$[x^0, x^1] = \mathrm{i}\ell(x^0 - x^1); \qquad [x^0, x^i] = \mathrm{i}\ell x^i; \qquad [x^1, x^i] = \mathrm{i}\ell x^i;$$

- Deformed relativistic symmetries described by an Hopf algebra.

"Doubly Special Relativity with light-cone deformation", Blaut,Daszkiewicz, Kowalski-Glikman, *Mod.Phys.Lett.A* 18 (2003) 1711

κ-lightlike Hopf algebra

- Deformed alegbra of symmetry and deformed notion of spatial isotropy.

- Casimir element

$$C = \frac{2}{\ell}(P_0 - P_1)e^{\frac{\ell(P_0+P_1)}{2}} \sinh\left(\frac{\ell(P_0 + P_1)}{2}\right) - (P_2^2 - P_3^2)e^{\ell(P_0+P_1)}$$

- Coprodcuts

1. $\Delta P_0 = P_0 \otimes I + I \otimes P_0 + \left(1 - e^{-\ell(P_0+P_1)}\right) \otimes \frac{P_1 - P_0}{2}$
2. $\Delta P_1 = P_1 \otimes I + I \otimes P_1 + \left(1 - e^{-\ell(P_0+P_1)}\right) \otimes \frac{P_0 - P_1}{2}$
3. $\Delta P_2 = P_2 \otimes I + \left(1 - e^{-\ell(P_0+P_1)}\right) \otimes P_2$
4. $\Delta P_3 = P_3 \otimes I + \left(1 - e^{-\ell(P_0+P_1)}\right) \otimes P_3$

Kinematical IR/UV mixing

- Dispersion relation inspired by the Casimir element at first order in ℓ

$$m^2 = (p_0^2 - p_1^2)\left(1 + \frac{\ell(p_0+p_1)}{2}\right) - (p_2^2 + p_3^2)(1 + \ell(p_0 + p_1))$$

- On shell relation in the limit $p \ll m$

$$p_0 = m - \ell\frac{m^2}{4} + \frac{\vec{p}^2}{2m} - \frac{\ell}{4}m\,p_1 + \frac{\ell}{4}(p_2^2 + p_3^2) + \ell\,\frac{p_1}{8m}(p_1^2 + 3p_2^2 + 3p_3^2) + O(\ell^2, m^{-2})$$

- In the energy-momentum dispersion relation appears the term $-\frac{\ell}{4}m\,p_1$, that is dominant in the IR regime ⟹ kinematical IR/UV mixing.

Coproduct-inspired total momentum

- Total momentum inspired by the coproducts:

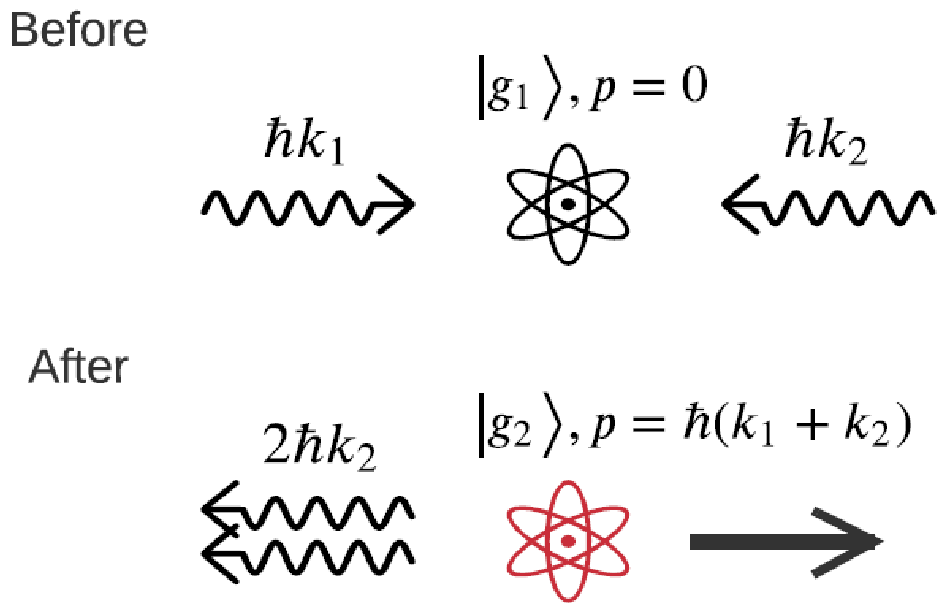
$$1.\ (k \oplus q)_0 = k_0 + q_0 - \frac{\ell}{2}(k_0 + k_1)(q_0 - q_1)$$
$$2.\ (k \oplus q)_1 = k_1 + q_1 + \frac{\ell}{2}(k_0 + k_1)(q_0 - q_1)$$
$$3.\ (k \oplus q)_2 = k_2 + q_2(1 - \ell\,(k_0 + k_1))$$
$$4.\ (k \oplus q)_3 = k_3 + q_3(1 - \ell\,(k_0 + k_1))$$

- Non commutative energy-momentum composition laws.

PHENOMENOLOGICAL APPLICATIONS

Cold atoms phenomenology: Raman transitions

- Momentum is imparted to a cold atom though a process involving the absorption of a photon of frequency ν and the stimulated emission, in the opposite direction, of a photon of frequency ν '.



Cold atoms phenomenology: $\frac{h}{m}$ measurement

- By imposing energy-momentum conservation one can calculate:

$$\frac{h}{m} = \frac{\Delta\nu}{2\nu^{\star}(\nu^{\star} + \frac{p_i}{h})}$$

Where:

1. $\Delta\nu = \nu - \nu'$;
2. $\nu^{\star}$ is the resonance frequency;
3. p_i is the initial momentum of the atom.

- Indirect measurement of the ratio $\frac{h}{m}$ by direct measurements of p_i, $\Delta\nu$, $\nu^{\star}$, which are well under control in the experiment.

Cold atoms phenomenology: $\frac{h}{m}$ measurement (deformation)

- By using deformed conservation laws , at first order in ℓ we obtain:

$$\frac{h}{m} = \frac{\Delta\nu}{2\nu^{\star}\left(\nu^{\star} + \frac{p_i}{h}\right)}(1 + \ell\beta)$$

- Where β is a deformation function and we model the process as an interaction where the initial state is characterized by the atom and the absorbed photon and the final state is the accelerated atom and the emitted photon.

"Constraining the Energy-Momentum Dispersion Relation with Planck-Scale Sensitivity Using Cold Atoms", Amelino-Camelia, Mercati, Tino, *Phys.Rev.Lett.* 103 (2009) 171302

"A Bound on Planck-scale modifications of the energy-momentum composition rule from atomic interferometry ", Arzano, Kowalski-Glikman, Walkus, *EPL* 90 (2010) 3, 30006

Cold atoms phenomenology: $\frac{h}{m}$ measurement (deformation)

- Leading order corrections depending on the ordering in the interaction process:

$A + \gamma \rightarrow A' + \gamma'$ $$\beta \approx -\frac{3}{4}(m)\left(\frac{m}{p_i + h\nu_*}\right)\cos(\phi)\sin(\theta)$$

$\gamma + A \rightarrow \gamma' + A'$ $$\beta \approx \frac{1}{4}(m)\left(\frac{m}{p_i + h\nu_*}\right)\cos(\phi)\sin(\theta)$$

$\gamma + A \rightarrow A' + \gamma'$ $$\beta \approx -\frac{1}{4}(m)\left(\frac{m}{p_i + h\nu_*}\right)\cos(\phi)\sin(\theta)$$

$A + \gamma \rightarrow \gamma' + A'$ $$\beta \approx -\frac{1}{4}(m)\left(\frac{m}{p_i + h\nu_*}\right)\cos(\phi)\sin(\theta)$$

Results

- Average over channels and angles

$$\langle \beta \rangle \approx \frac{1}{16\pi} \int_{S^2} d\Omega \sum_i \beta_i = 0$$

- Variance

$$\ell \Delta \beta \approx \frac{1}{16\pi} \int_{S^2} d\Omega \sum_i \beta_i^2 = \frac{\ell}{4} \frac{m^2}{p_i + h\nu_*}$$

- Amplification factor stemming from kinematical IR/UV mixing term in the dispersion relation.

- First DSR result with such an amplification factor ($\sim 10^8$)

Conclusions

- We obtained a relativistic model that presents a kinematical IR/UV mixing mechanism.

- Interesting phenomenological opportunities to obtain constraints on the deformation scale by probing the IR regime with high precision.

Thank you!