

Hawking Radiation without Lorentz Invariance

Francesco Del Porro

Based on:

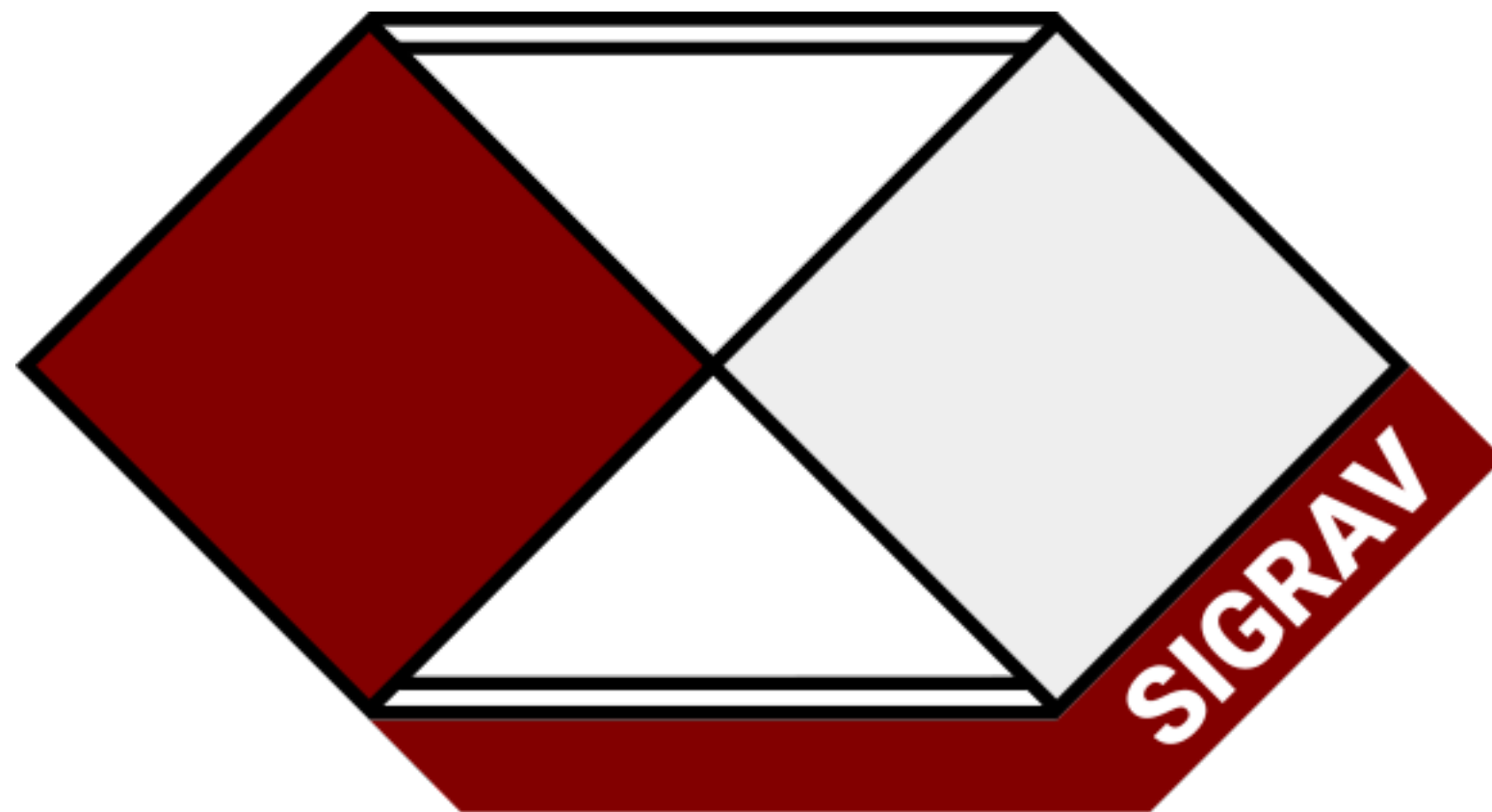
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ArXiv:2309.xxxx

In collaboration with:

S. Liberati, M. Herrero-Valea and M. Schneider



SISSA



Outline

- Lorentz Violating gravity: why and how
- Black Holes in LV gravity
- Hawking radiation in from UH
- The role of KH
- Conclusion

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Lorentz Violating gravity

- A “good way” to break LLI is to assume an inhomogeneous scaling behavior between time and space:

$$\vec{x} \rightarrow b\vec{x}, \quad \tau \rightarrow b^z\tau$$

- One can introduce a Stueckelberg vector field, the Aether, that parametrizes the time direction:

$$U_\mu = \frac{\partial_\mu \tau}{\sqrt{g^{\alpha\beta} \partial_\alpha \tau \partial_\beta \tau}} \quad S[g, \tau] = -\frac{1}{16\pi G} \int_{\mathcal{M}} \sqrt{-g} (R + c_\theta \theta^2 + c_\omega \omega_{\mu\nu} \omega^{\mu\nu} + c_\alpha a_\mu a^\mu)$$

Matter Fields

- The theory allows the presence of higher (spatial) derivative operators:

$$S_m[\phi] = \frac{1}{2} \int_{\mathcal{M}} \sqrt{-g} \phi \left[\nabla_\mu \nabla^\mu - \sum_{j=2}^n \frac{\alpha_{2j}}{\Lambda^{2j-2}} (\Delta)^j \right] \phi \quad \Delta = \nabla_\mu \gamma^{\mu\nu} \nabla_\nu$$

Matter Fields

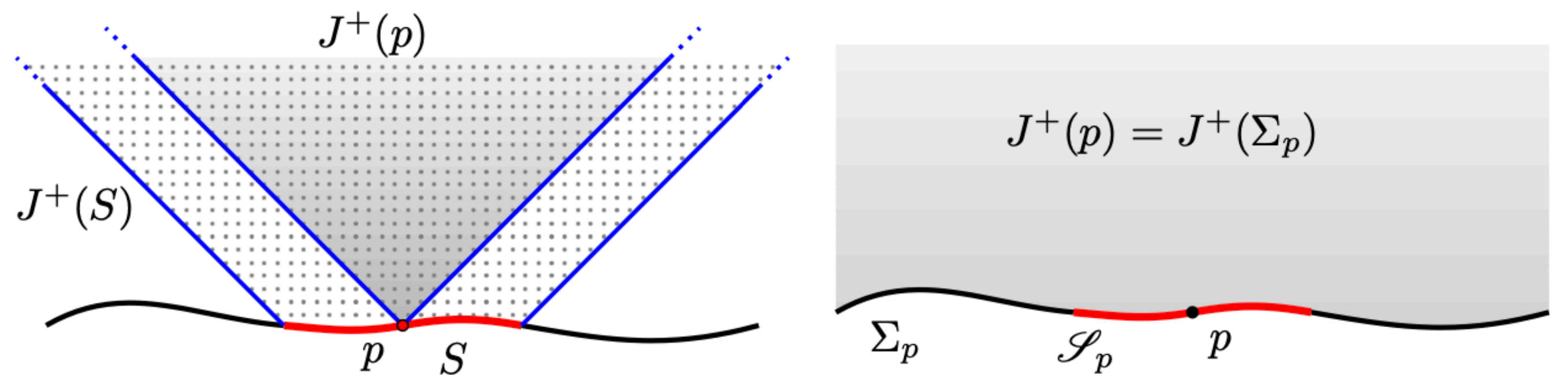
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Superluminal particles:

$$\omega^2 = q^2 + \alpha_4 \frac{q^4}{\Lambda^2} + \dots + \alpha_{2n} \frac{q^{2n}}{\Lambda^{2(n-1)}}$$

Different notion of causality:



Outline

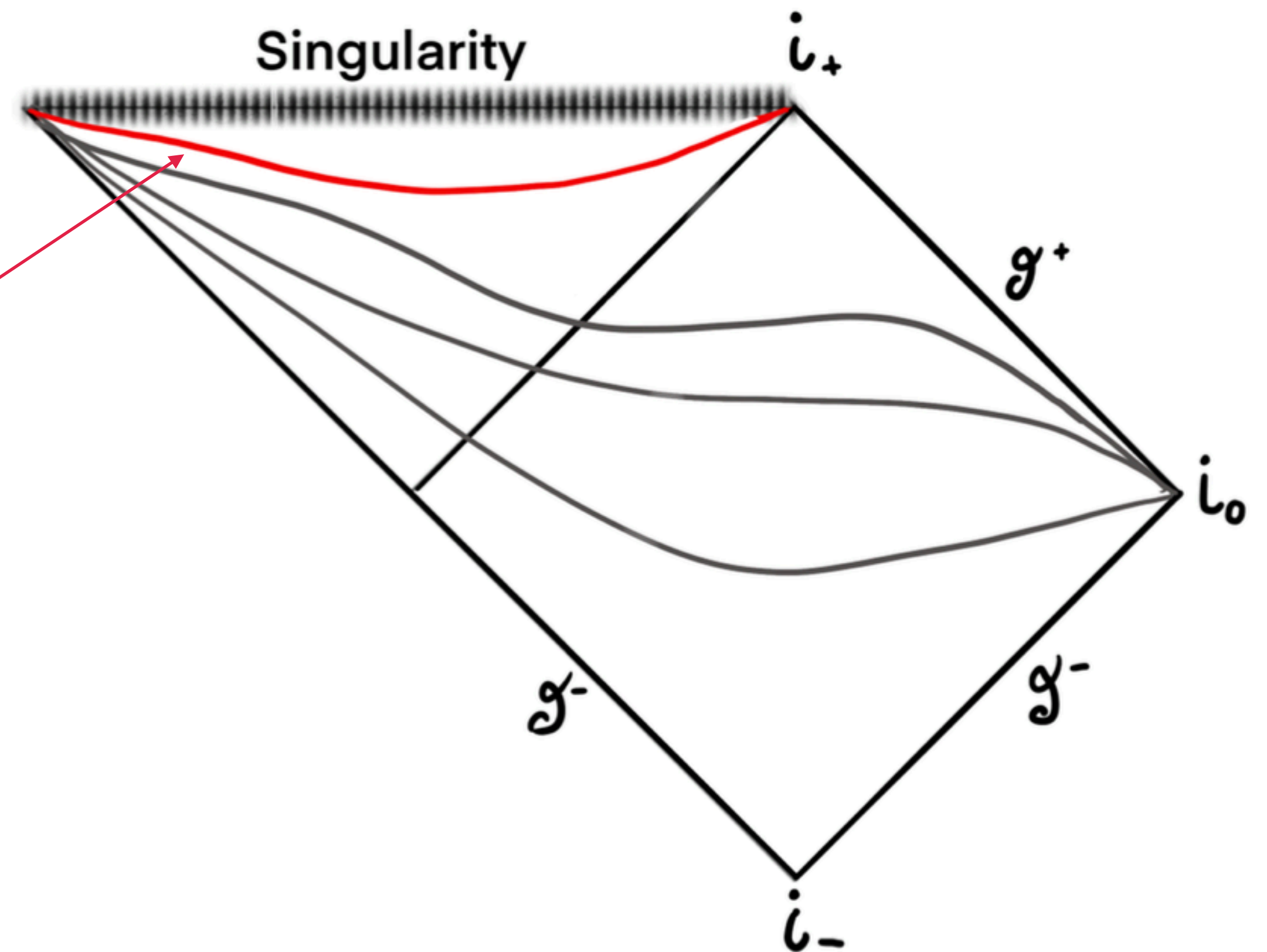
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Horizons

- Killing Horizons are no more causal boundaries! What is a Black Hole?

Horizons

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If U_μ becomes orthogonal to a compact surface, we have a Universal Horizon

$$UH = \{(\chi \cdot U) = 0, \quad (\chi \cdot a) \neq 0\}$$

UH in Schwarzschild

- We will consider a Schwarzschild solution of the theory:

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2 dS_2$$

$$U_\mu dx^\mu = \left(1 - \frac{M}{r}\right) dt + \frac{M}{r - 2M} dr$$

$$\chi^\mu \partial_\mu = \frac{\partial}{\partial t}$$

- The Universal Horizon is located at:

$$UH = \left\{1 - \frac{M}{r} = 0\right\} \implies \{r = M\}$$

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Does the UH radiate?

- The derivation of Hawking Radiation is based on the fact that Killing Horizons are causal boundaries. We may expect similar properties from the UH...

- Let us take a massless scalar field on a BH geometry:
$$\left[\nabla_{\mu} \nabla^{\mu} - \sum_{j=2}^n \frac{\alpha_{2j}}{\Lambda^{2j-2}} (\Delta)^j \right] \phi = 0$$

- Solving with a WKB ansatz, and decoupling the angular part, we get the dispersion relation:

$$\phi = \phi_0 e^{i\mathcal{S}_0} = \phi_0 \exp \left[-i \int (\omega U_{\mu} dx^{\mu} + q S_{\mu} dx^{\mu}) \right] \implies \omega^2 = q^2 + \sum_{j=2}^n \frac{\alpha_{2j}}{\Lambda^{2j-2}} q^{2j}$$

Near-UH Solutions

- Since $[U^\mu \partial_\mu, S^\nu \partial_\nu] \neq 0$, neither ω nor q are conserved quantities. The Killing energy Ω , it is:

$$\Omega = \omega(U \cdot \chi) + q(S \cdot \chi)$$

Near-UH Solutions

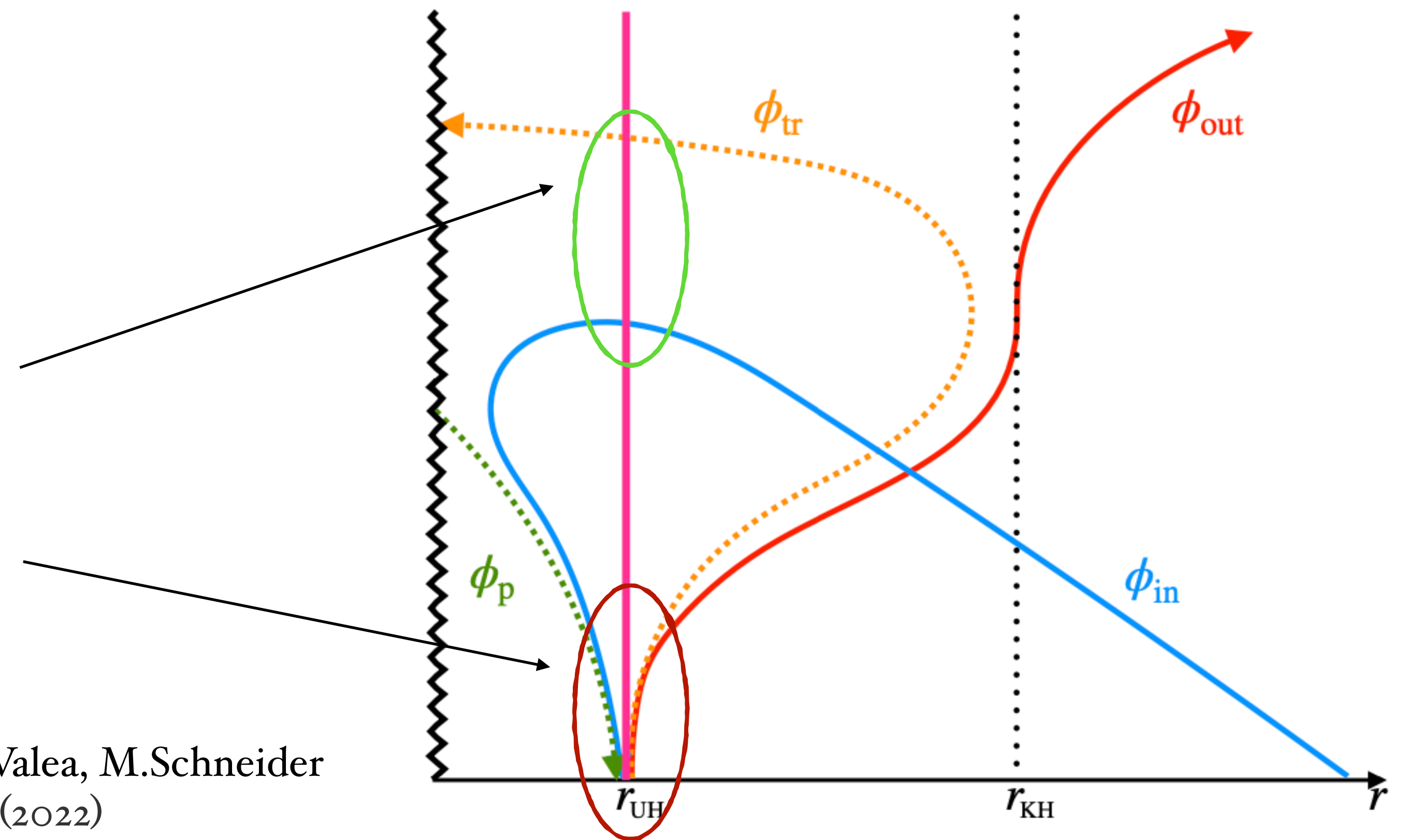
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$$(U \cdot \chi) \simeq 0$$

- $|q|$ is regular at the UH: soft modes
- $|q| \rightarrow +\infty$ at the UH: hard modes

FDP, S.Liberati, M.Herrero-Valea, M.Schneider
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Near-UH Solutions

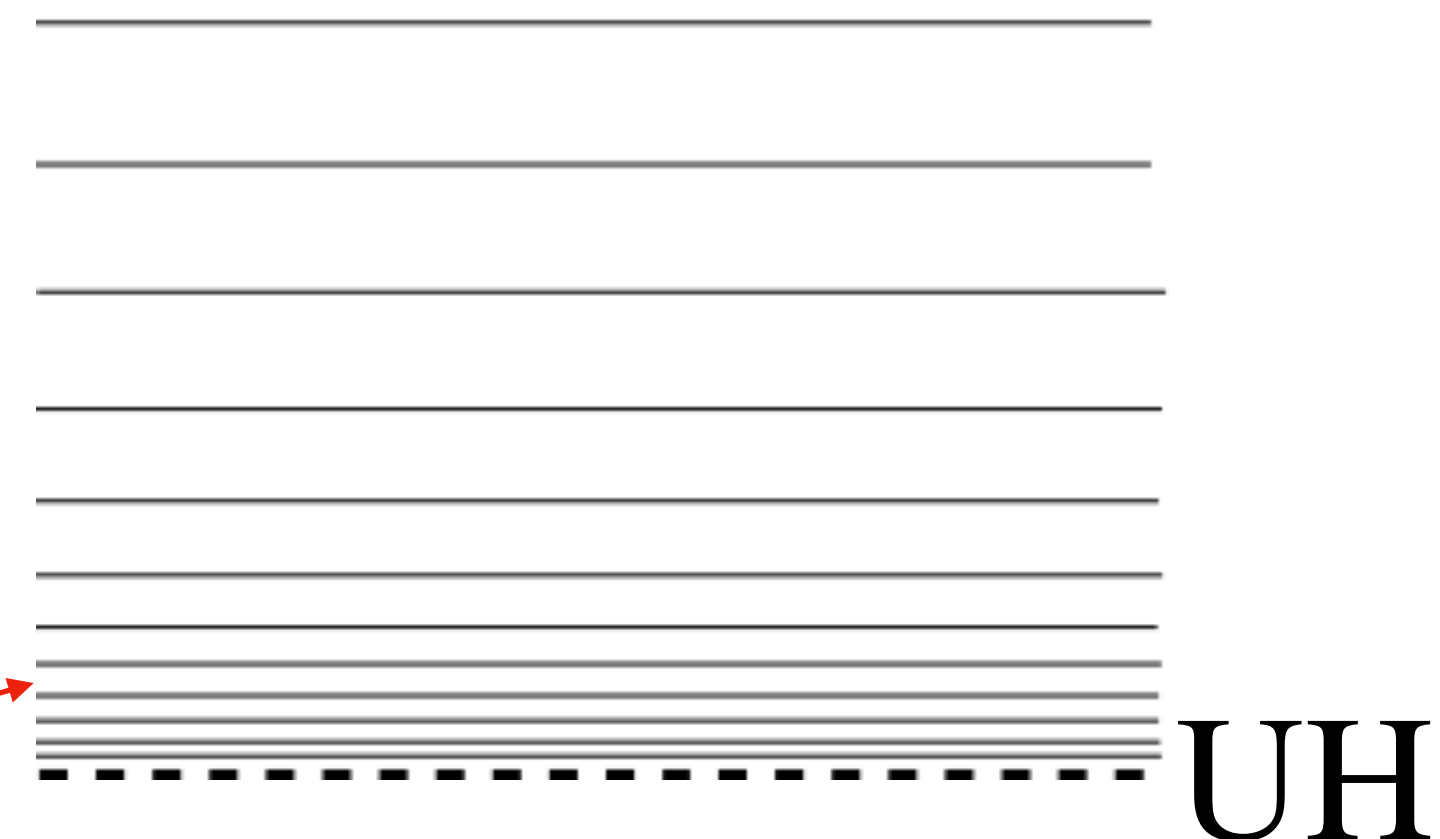
- We are interested in studying the hard mode which eventually reaches the asymptotic region, for which:

$$|q| \sim \frac{1}{(U \cdot \chi)^{\frac{1}{n-1}}} \quad |\omega| \sim \frac{1}{(U \cdot \chi)^{\frac{n}{n-1}}}$$

- This gives the form of the outgoing mode, which is non-analytical at the Horizon:

$$\phi_{\Omega}^{out} = A \exp \left[\frac{i}{2\kappa} \Omega \log(U \cdot \chi) \right] = A e^{-i\Omega\tau}$$

Phase contours

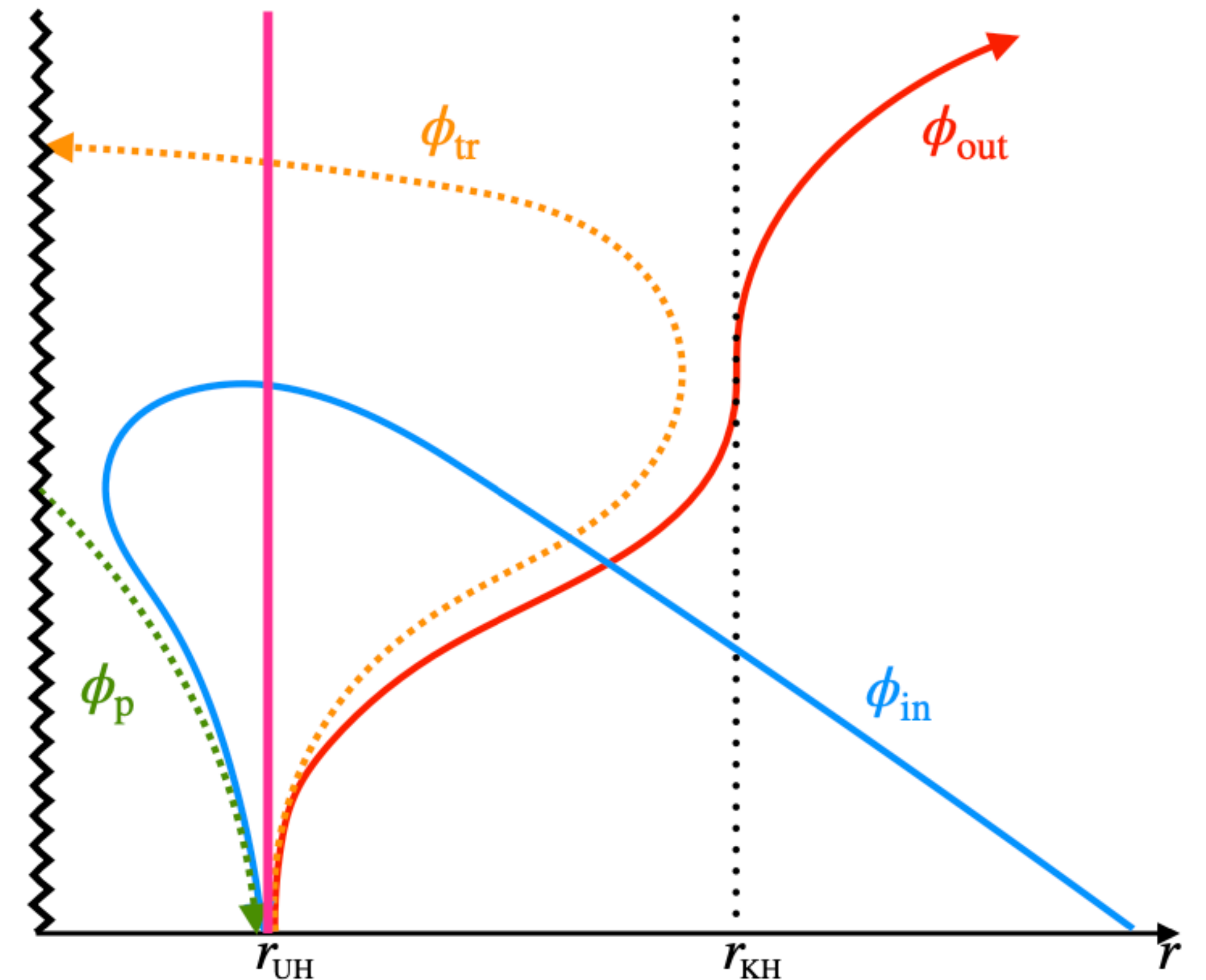


UH temperature

M.Schneider, FDP, M.Herrero-Valea,
S.Liberati in ArXiv:2207.08938

- We can evaluate the thermal properties through the tunneling amplitude:

$$\text{Im}\mathcal{S}_0 = \text{Im} \left[-i \int_{r_{\text{UH}}-\epsilon}^{r_{\text{UH}}+\epsilon} (\omega U_\mu dx^\mu + q S_\mu dx^\mu) \right]$$



UH temperature

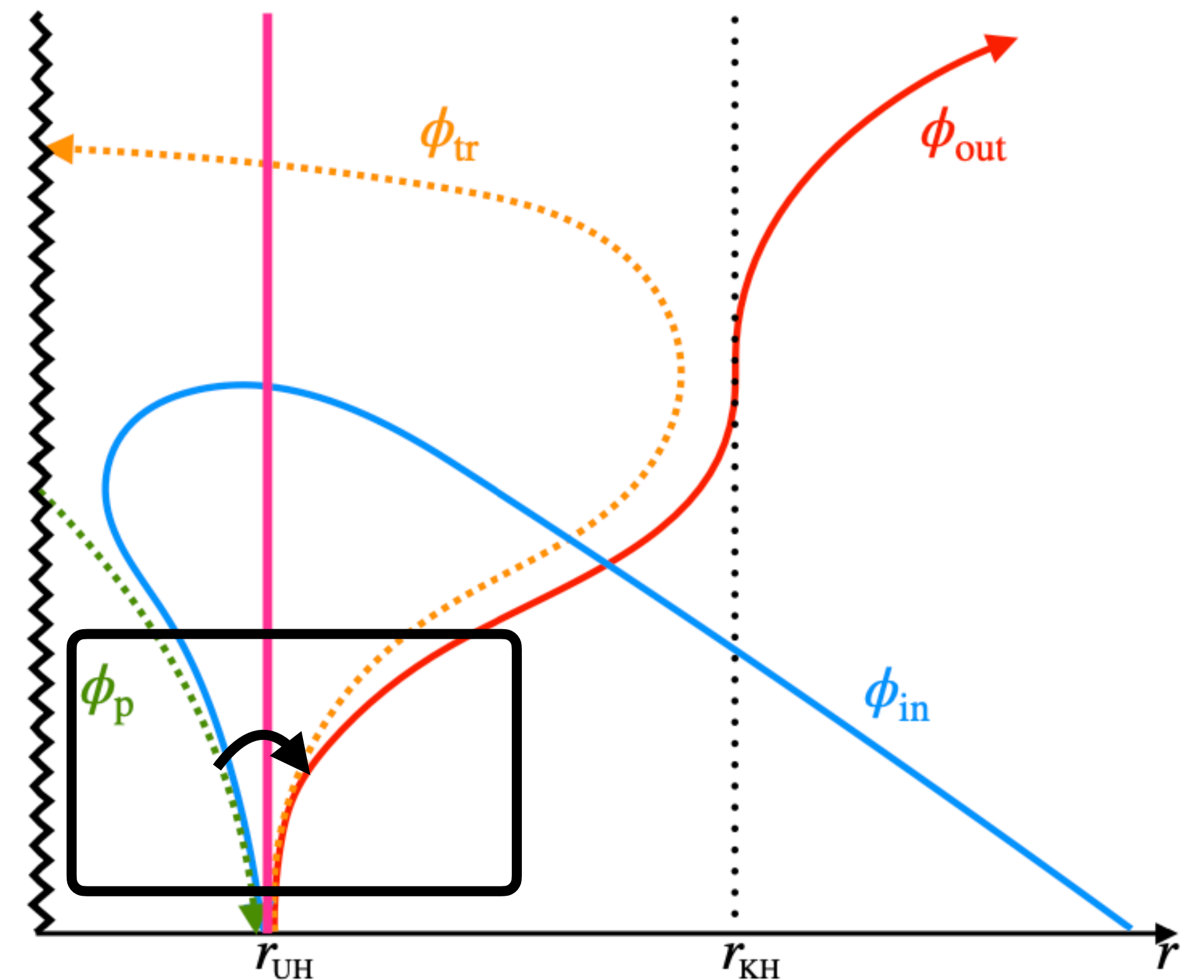
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- The amplitude of tunneling out is given by

$$\Gamma = e^{-2\text{Im}\mathcal{S}_0} = e^{-\Omega\pi/\kappa_{\text{UH}}} \implies T_{\text{UH}} = \frac{\kappa_{\text{UH}}}{\pi}$$



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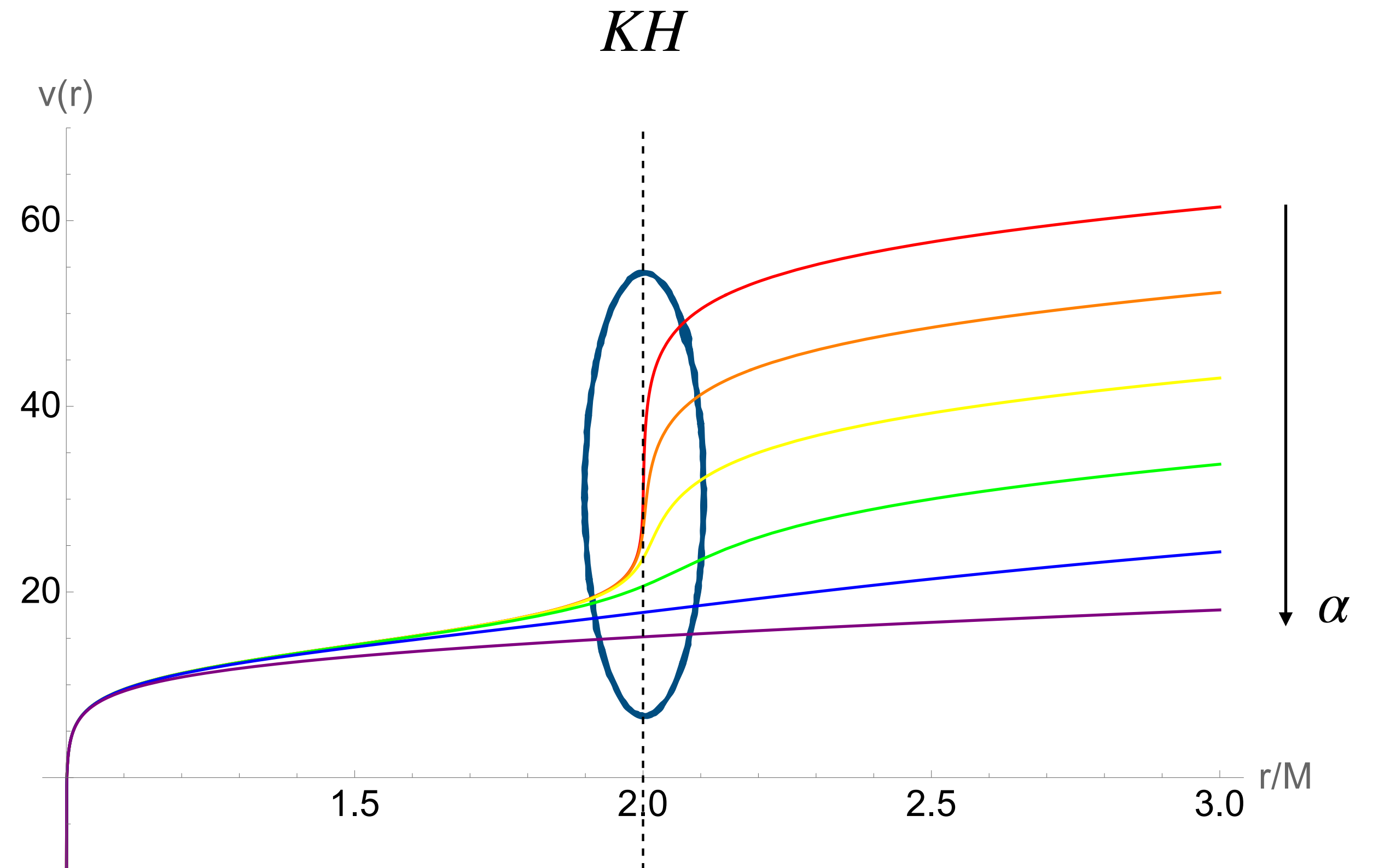
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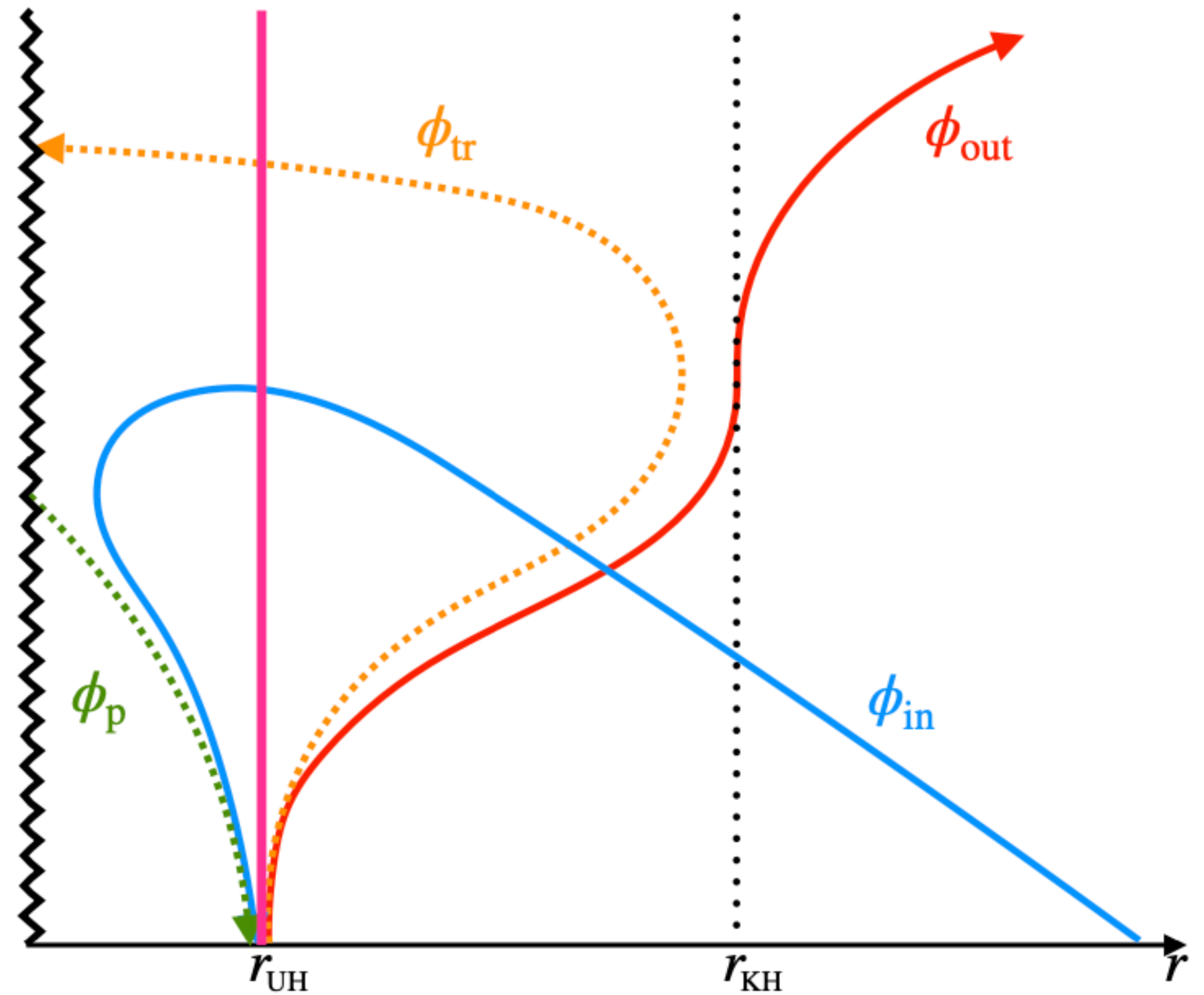
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The rays for which
 $\alpha = \Omega/\Lambda \ll 1$ linger at the
KH for long time



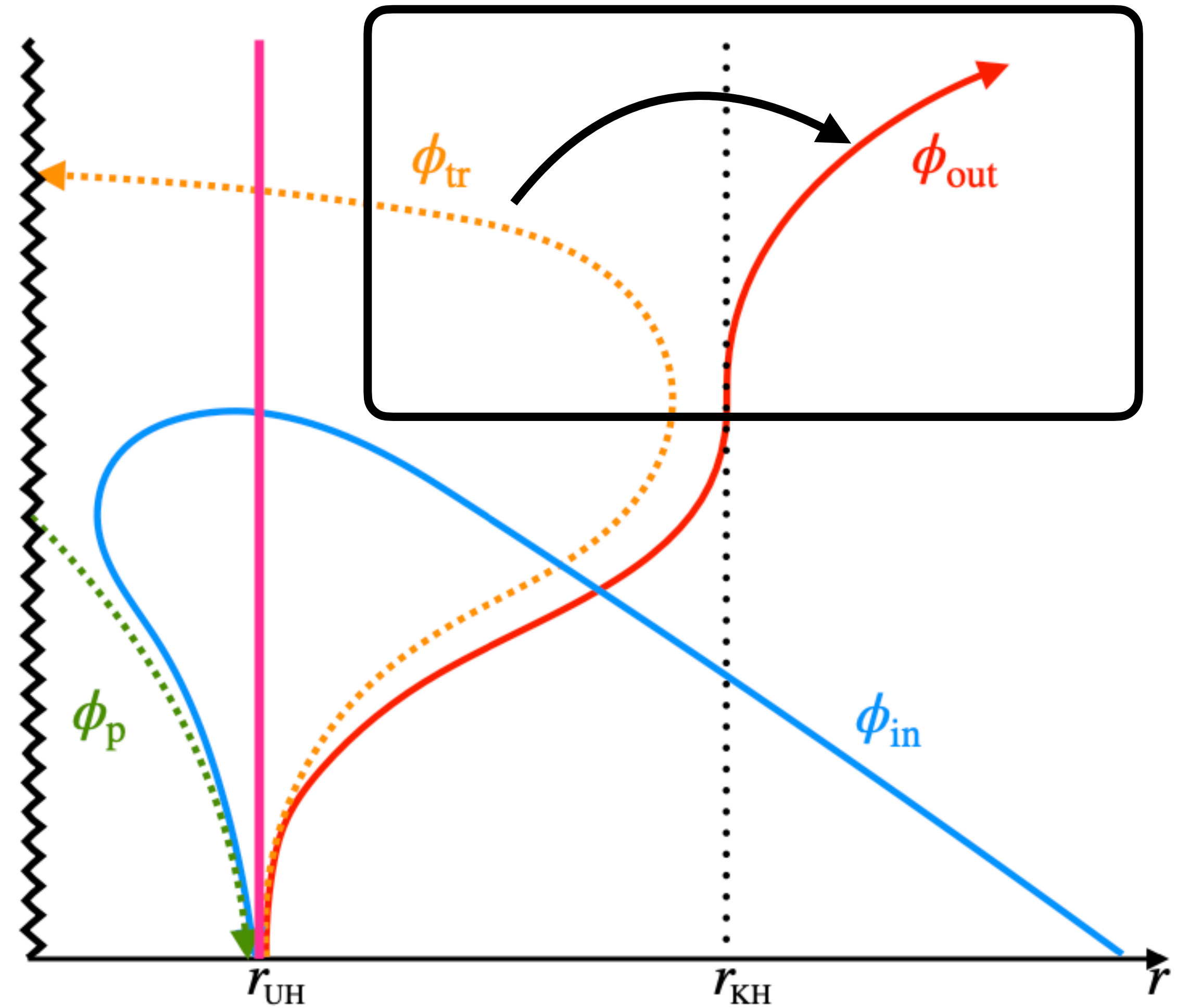
Thermal spectrum?

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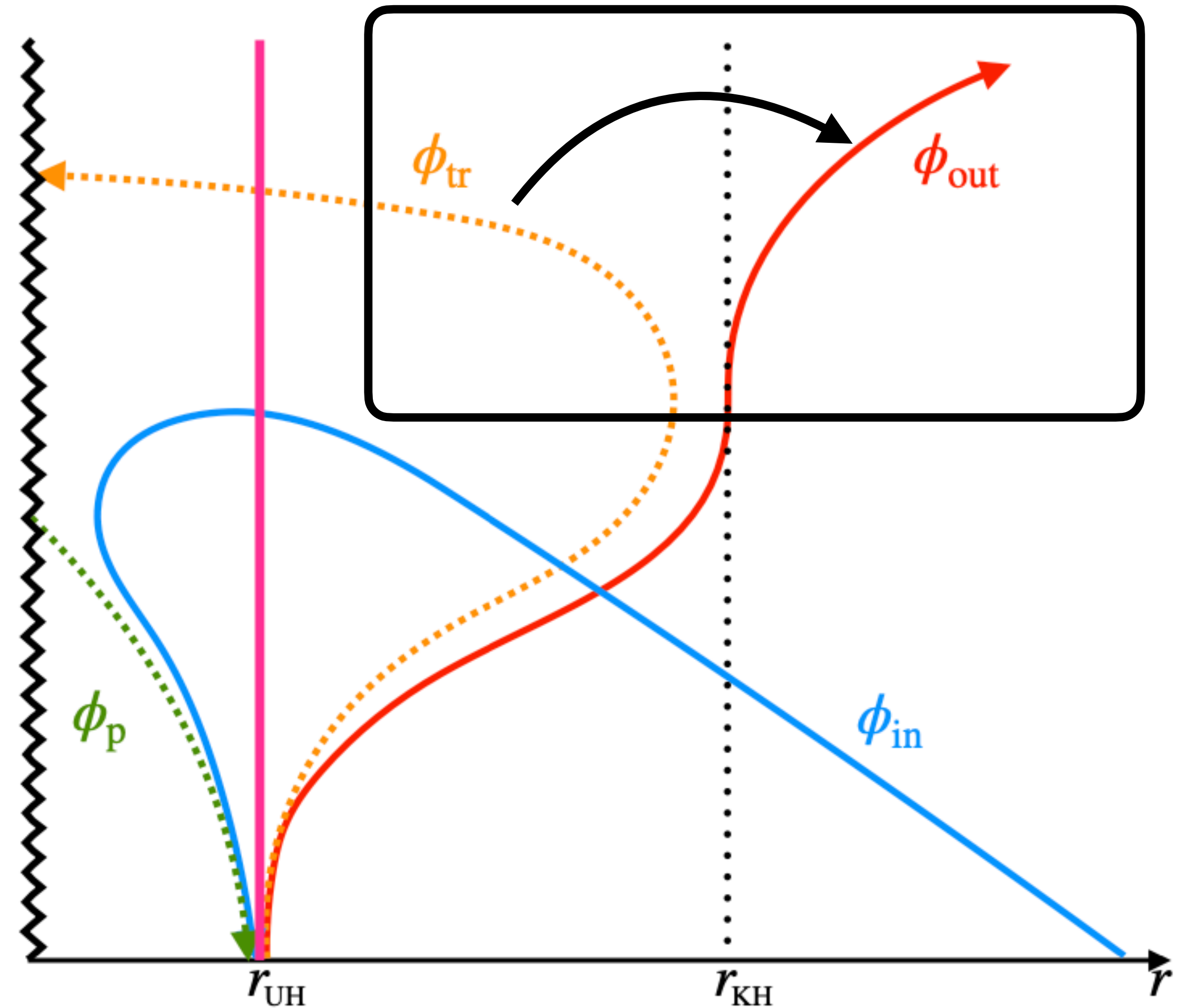
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- The tunneling amplitude is:

$$\Gamma = e^{-\frac{\Omega}{T_\alpha}} \quad T_\alpha = \frac{T_h}{1 - 3\alpha^2}$$



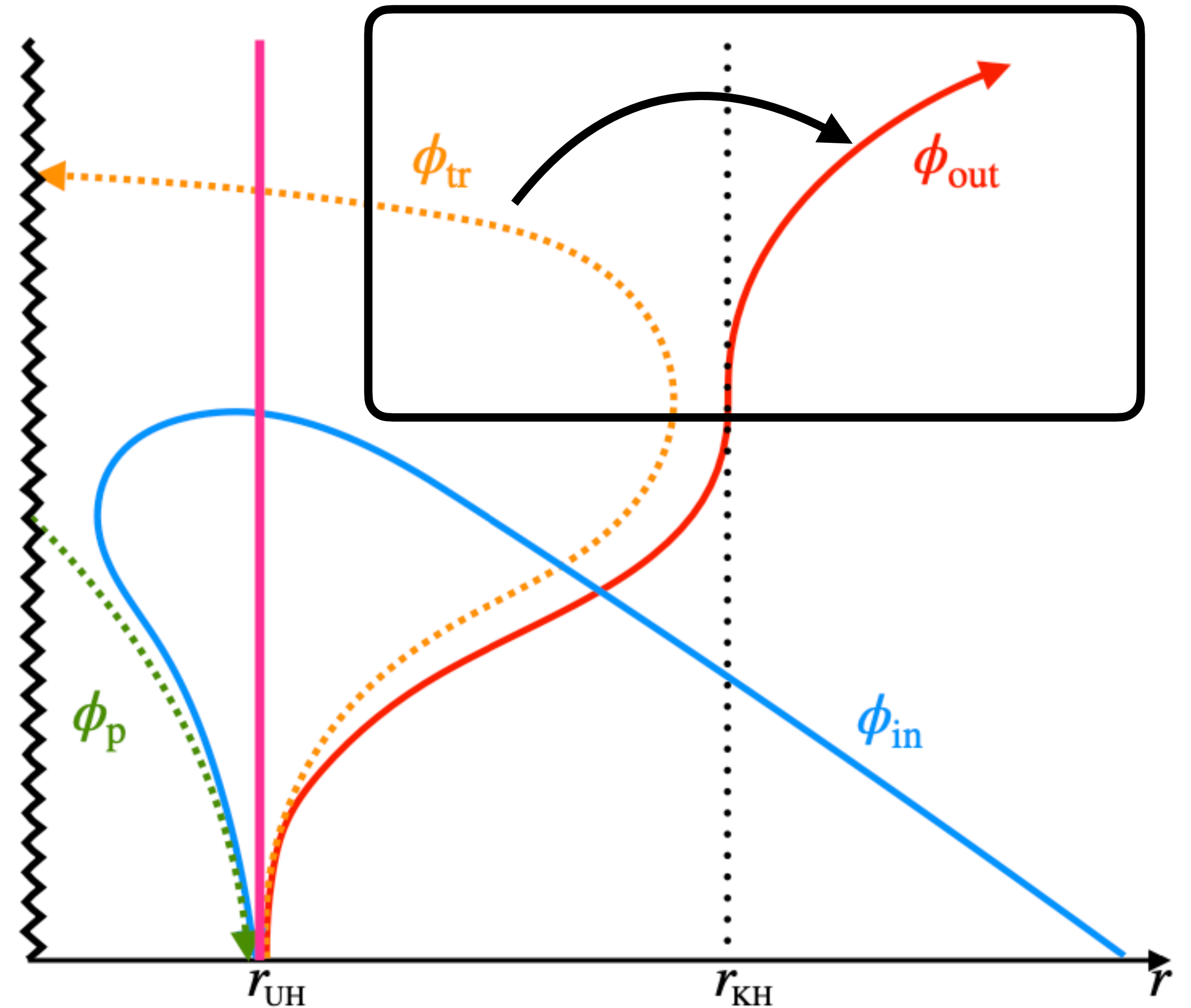
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Deviation from thermality!!



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Conclusions and further developments

- LV Black Holes have a causal boundary with similar thermal properties to the GR ones
- Along their propagation, particles feel differently the KH depending on their energies
- Low-energies particles are reprocessed by the KH, recovering the relativistic limit
- Further developments: what arrives at infinity? Is there a global state?

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Thank you!

