Two-body problem in theories with kinetic screening

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XXV SIGRAV Conference on General Relativity and Gravitation
07.09.2023.

ERC-2018-COG GRAMS 815673

Based on:
MB, Barausse [2305.07725]

Scalar fifth forces

- Is the phenomenon of *gravity* = GR + additional attractive universal long-range interaction mediated by a scalar (fifth force)?
- Motivations
  - i Dark energy driven by a scalar field Brax (2018)
  - ii Behavior of DM in galaxies (e.g. superfluid DM Berezhiani, Khoury [1507.01019])
  - iii Pheno perspective: new gravitational probes allow us to constrain fifth forces
- Simplest example: massless scalar (Brans-Dicke)
  - ⋆ Conformal coupling $\Phi g_{\mu\nu}$, $\Phi \approx 1 + \alpha\varphi/M_{\rm Pl} \to \alpha T\varphi/M_{\rm Pl}$
  - ⋆ Cassini bounds Bertotti, Iess, Tortora (2003): $\alpha < 10^{-3}$
- Screening mechanism is needed
  - ⋆ Is screening effective beyond staticity and spherical symmetry?

k-essence

- k-essence action

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R + K(X) \right] + S_m(\psi_i, \Phi g_{\mu\nu})$$

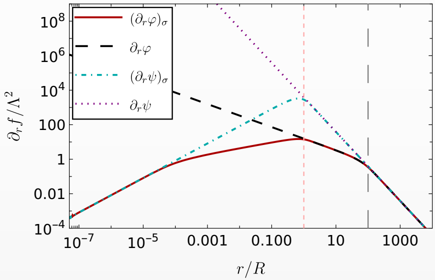
$$K = -\frac{1}{2} X + \Lambda^4 \sum_{n=2}^{N} \frac{c_n}{2n} \left(\frac{X}{\Lambda^4} \right)^n, X = g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

- Cosmological context $\Lambda \sim \sqrt{H_0 M_{\rm Pl}} \sim \mathrm{meV}$
  - ⋆ $K(\varphi, X) \subset$ Horndeski class
  - ⋆ Only unconstrained sector after GW170817 and requiring GW $\nrightarrow$ DE Creminelli+ ['17, '18, '19]
- (Shift-symmetric) k-mouflage $K(X)$: turns off the fifth force when $X \gtrsim \Lambda^4$
- Radiative stability for large X de Rham, Ribeiro [1405.5213]

Review: Joyce+ [1407.0059]

Screening in isolation

- For $c_N < 0$: $\left(\frac{\partial_r \varphi}{\Lambda^2}\right)^{2N-1} \approx \left(\frac{r_{\rm sc}}{r}\right)^2$
- Screening radius
  $r_{\rm sc} \approx \frac{1}{\Lambda}\sqrt{\frac{m\alpha}{4\pi M_{\rm Pl}}} =$
  $10^{12}\mathrm{km}\,\alpha^{1/2}\left(\frac{\Lambda}{\mathrm{meV}}\right)^{-1}\left(\frac{m}{M_\odot}\right)^{1/2}$
- Scalar regular at the origin e.g. $N = 2$: $\varphi \approx \mathrm{const} + \mathscr{O}(r^{1/3})$
- Screening is a non-perturbative phenomenon in Λ:
  $\varphi(r \to \infty) \approx -\frac{m\alpha}{4\pi M_{\rm Pl}}\frac{1}{r} + \mathscr{O}(r^{-5})$
- Deep inside the source breakdown of screening (attractive forces cancel)



$10^{12}\mathrm{km} \approx 0.04\mathrm{pc}$

on point-particle screening e.g. Brax, Burrage, Davis [1209.1293]

Helmholtz decomposition: static limit

- PN expansion of the full theory $v/c \ll 1$:
  $\partial_i(K_X \partial^i \varphi) = \frac{\alpha}{2M_{\rm Pl}} T$
- At the Newtonian order $\varphi(t,\boldsymbol{r}) \approx \varphi_{\rm static}(\boldsymbol{r}, \boldsymbol{r}_1(t), \boldsymbol{r}_2(t))$
- GR (Newtonian gravity) and the scalar force decouple
  $\boldsymbol{F} = \boldsymbol{F}_{\rm N} + \boldsymbol{F}_5$
- Helmholtz decomposition: $\chi \equiv K_X \boldsymbol{\nabla}\varphi$, $\chi = -\frac{1}{2}\boldsymbol{\nabla}\psi + \boldsymbol{B}$
- Longitudinal (irrotational) component:

$$\psi = -\frac{1}{4\pi M_{\rm Pl}} \int d^3\boldsymbol{r}' \frac{\alpha T(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

- Divergenceless (solenoidal) part:

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \frac{1}{4\pi} \int d^3\boldsymbol{r}' \frac{\boldsymbol{C}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}, \boldsymbol{C} = K'' \boldsymbol{\nabla} X \times \boldsymbol{\nabla}\varphi$$

Screening in isolation revisited

- Solenoidal source: $\boldsymbol{C} = K'' \boldsymbol{\nabla} X \times \boldsymbol{\nabla} \varphi$
- In spherical symmetry: $\boldsymbol{\nabla}\varphi,\ \boldsymbol{\nabla} X \propto \hat{\boldsymbol{r}} \implies \boldsymbol{C} = 0$
- We need to invert χ^2 : $K'(X)^2 X = \frac{1}{4} X_\psi$
  - ⋆ Well-posedness found us $1 + \frac{2K_{XX}X}{K_X} > 0$
- Works for other highly-symmetric configurations
  $\exists! \boldsymbol{v} |\ \boldsymbol{\nabla}\varphi,\ \boldsymbol{\nabla} X \propto \boldsymbol{v}$

Introduced in:
Bekenstein, Magueijo [astro-ph/0602266], Brax, Valageas [1408.0969]
cf. [2305.07725] for the covariant formulation

What about binaries?

- Two-parameter problem (*) $\{\kappa, q\}$

$$\nabla \cdot \left(\nabla \phi \sum_{n=1}^{N} X^{n-1} \right) = 4\pi\kappa \left[\delta^{(3)} \left(\mathrm{r} - \frac{1}{2}\hat{\boldsymbol{z}} \right) + \frac{1}{q}\delta^{(3)} \left(\mathrm{r} + \frac{1}{2}\hat{\boldsymbol{z}} \right) \right]$$

$$\kappa = \frac{m\alpha}{4\pi M_{\rm Pl}\Lambda^2}\frac{1}{D^2} \propto \left(\frac{r_{\rm sc}}{D}\right)^2, \, q = \frac{m_a}{m_b}$$

- Analytical control has been lacking for a two-body problem in kinetic/Vainshtein screening
  - ⋆ $\varphi \approx \varphi(\mathrm{CM}) + \Delta\varphi$ Andrews, Chu, Trodden [1305.2194]
  - ⋆ EOB ansatz Kuntz [1905.07340]
- Superposition approximation suggests: $X_\psi \gg B$

(*)

$$\frac{x_i}{D} \to x_i \,, \quad \frac{\varphi}{D\Lambda^2} \to \phi \,, \quad \frac{X}{\Lambda^4} \to X$$

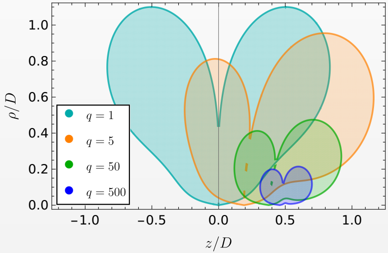
Irrotational approximation for a binary (1/2)

- First iteration: $\boldsymbol{B} = 0$; solve for $X(X_\psi)$
- Second iteration: is $\boldsymbol{C} \approx N_K G_{\boldsymbol{\nabla}}$ important?

$$N_K = -\frac{1}{8\pi}\frac{K_{XX}}{K_X}\frac{dX}{dX_\Psi}|\boldsymbol{\nabla} X_\Psi|\sqrt{X_\Psi},$$

$$G_{\boldsymbol{\nabla}} = \sqrt{1 - \frac{(\boldsymbol{\nabla} X_\Psi \cdot \boldsymbol{\nabla}\Psi)^2}{(\boldsymbol{\nabla} X_\Psi)^2 X_\Psi}},$$

- Two regimes where $\boldsymbol{B}$ should be suppressed $\left\{\kappa \propto \left(\frac{r_{\rm sc}}{D}\right)^2\right\}$
  - ⋆ $[q \gg 1]$ Support for N_K, $G_{\boldsymbol{\nabla}}$ around different particles
  - ⋆ $[\kappa \ll 1]$ Support for N_K shrinks



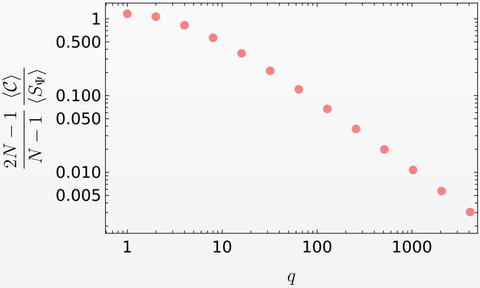
Irrotational approximation for a binary (2/2)

- What about $\kappa \gg 1$ and $q \approx 1$?
- Consider k-poly in the deep screening regime

$$N_K \approx -\frac{\kappa}{8\pi}\frac{N-1}{(2N-1)}\frac{|\nabla \hat{X}_\psi|}{\sqrt{\hat{X}_\Psi}}\,,\; X_\Psi = \kappa^2 \hat{X}_\Psi(q)$$

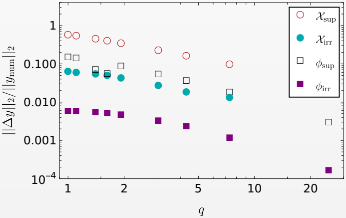
$$S_\Psi = -\frac{1}{2}\kappa\left[\delta^{(3)}\left(\mathrm{r} - \frac{1}{2}\hat{\boldsymbol{z}}\right) + \frac{1}{q}\delta^{(3)}\left(\mathrm{r} + \frac{1}{2}\hat{\boldsymbol{z}}\right)\right]$$

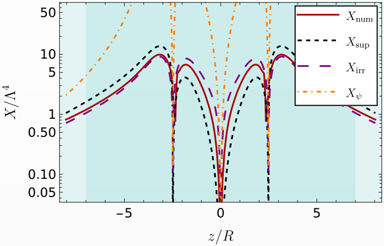
- Ratio of the sources doesn't depend on $\kappa \propto \left(\frac{r_{\mathrm{sc}}}{D}\right)^2$

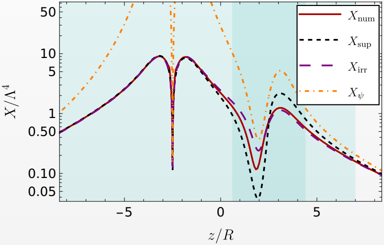


Numerical results

- $N = 2$ k-poly model
- Elliptical BVP (via FDM)
- For $\kappa \propto \left(\frac{r_{sc}}{D}\right)^2 < 1$ (non-)linear superposition
- Deep screening: solenoidal component suppressed, irrotational approximation induces errors up to $\sim 10\%$ on X (on average) when $q \approx 1$

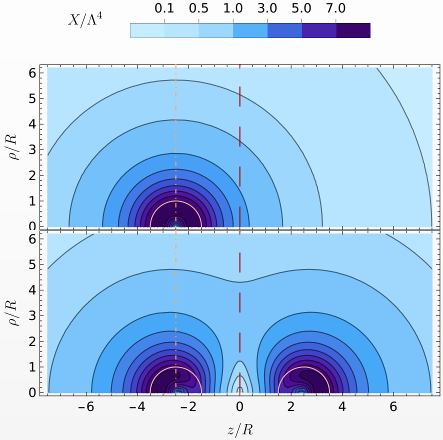






Descreened bubbles

- Saddle point: attractive forces cancel, breakdown of screening
- $\frac{\delta}{D} \simeq \frac{1}{\kappa} \frac{q^{3/2}}{(1+\sqrt{q})^4}$
- Earth-Moon ($\delta \approx 0.2\mathrm{km}$), Sun-Earth ($\delta \approx 1\mathrm{km}$), Sun-Jupiter ($\delta \approx 2800\mathrm{km}$) [$\Lambda \sim \mathrm{meV}$]
- More effective if there is anti-screening at the saddle point (e.g. MOND-like phenomenology Bekenstein, Magueijo [astro-ph/0602266])



Conclusions: In screening, more is different

- Helmholtz decomposition separates a problem in a trivial (irrotational) and the non-trivial (solenoidal) part
- Two-body problem: solenoidal component either suppressed or allows for decent quantitative description
- Around the saddle point breakdown of screening
- Helmholtz decomposition could be useful beyond leading PN contribution

Supplementary material

Fith force screening

- General scalar theory in the decoupling limit $\varphi = \bar{\varphi} + \bar{\pi}$
  $\mathscr{L} = -\frac{1}{2}\bar{Z}^{\mu\nu}\partial_\mu\pi\partial_\nu\pi - \frac{1}{2}\bar{m}^2\pi^2 + \bar{g}\,T + \ldots$,
  $\bar{Z} = \bar{Z}(\partial\bar{\phi}, \partial^2\bar{\phi}, \ldots)$
- Fifth-force potential of a point source (all functions of $\bar{\varphi}$):
  $\hat{\pi} \sim \frac{\bar{g}}{\sqrt{\bar{Z}}}\frac{\exp\left(-\frac{\bar{m}}{\sqrt{\bar{Z}}}r\right)}{r}$
- Varieties of screening:
  - ⋆ $\hat{\pi}$ as a trigger (via potential): weak coupling $\bar{g}$ (symmetron), large mass $\bar{m}$ (chameleon)
  - ⋆ $\partial\hat{\pi}$ as a trigger (via acceleration) $\bar{Z}$: kinetic screening
  - ⋆ $\partial^2\hat{\pi}$ as a trigger (via curvature) $\bar{Z}$: Vainshtein mechanism

Review: Joyce+ [1407.0059]

Theoretical consistency in the IR

- Screening $\implies$ superluminality $c_s \gtrsim 1$ around spacelike backgrounds
- Classically not a problem: causality def. w.r.t. effective metric
  Babichev, Mukhanov, Vikman [gr-qc/0607055], Bezares+ [2008.07546]

$$G^{\mu\nu}\nabla_\mu\nabla_\nu\varphi = 0\,, G^{\mu\nu} = g^{\mu\nu} + \frac{2K''}{K'}\nabla^\mu\varphi\nabla^\nu\varphi$$

- Well-posed IVP for class of K
  - ⋆ As long $1 + \frac{2K_{XX}X}{K_X} > 0$ e.g. $c_2 = 0$, $c_3 < 0$, $c_{n>3} = 0$
  - ⋆ Even if not true, not necessarily a fundamental problem
    Burgess, Williams [1404.2236], Lara, Bezares, Barausse [2112.09186]

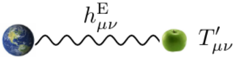
Theoretical consistency in the UV?

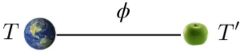
- Positivity bounds on the EFTs Adams+ [hep-th/0602178]
  - ⋆ Assuming local, causal, Lorentz-invariant UV completion
  - ⋆ $2 \to 2$ scattering in the IR: $\partial_s^2 A_{\rm EFT}|_{s=0,t=0} > 0$
- Quadratic screening is not positive
  - ⋆ Lorentz-violating UV completion? Classicalization? Dvali+ [1010.1415]
- Indications of "positive" odd $K(X) \propto X^N$
  - ⋆ higher-n positivity Chandrasekaran, Remmen, Shahbazi-Moghaddam [1804.03153], positivity around Lorentz-violating backgrounds Davis, Melville [2107.00010]

Einstein/Jordan frame

Jordan/Einstein frame

Einstein frame: $\mathcal{L} = -(\partial h_E)^2 - (\partial\phi)^2 - \frac{1}{2M_{\rm Pl}} h^E_{\mu\nu} T^{\mu\nu}_E - \frac{\beta}{M_{\rm Pl}} \phi T_E$





$$\Phi_{\rm E} = -\frac{GM}{r} \qquad \phi = -\beta\frac{GM}{r}$$

Matter is not conserved

$$\nabla^{\rm E}_\mu T^{\mu\nu}_{\rm E} = \frac{\alpha}{M_{\rm Pl}} T_{\rm E} \nabla^{\rm E}_\nu \phi$$

Particles do not follow geodesics: fifth force!

$$\ddot{\vec{x}} = -\vec{\nabla}\Phi_E - \beta\vec{\nabla}\phi = -\vec{\nabla}\Phi \qquad h^{\rm E}_{00} = -2\Phi_{\rm E}$$

$$h_{00} = -2\Phi \qquad \Phi = \Phi_E + \beta\phi = -(1+2\alpha^2)\frac{GM}{r}$$

$$h_{ij} = -2\Psi\delta_{ij} \qquad \Psi = \Psi_E - \beta\phi = -(1-2\alpha^2)\frac{GM}{r}$$

Fig: Vernizzi (2022) ; Ref: Hui, Nicolis, Stubbs [0905.2966]

Screening in isolation (2/2)

- Screening operates also w. GR: EKG system and polytropic EoS ter Haar+ [2009.03354], Bezares+ [2105.13992]
- Also operates for more generic coupling $\propto \varphi^2 T, XT\ldots$ Lara+ [2207.03437]

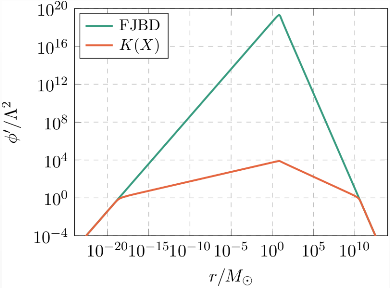


Fig: ter Haar+ [2009.03354]

EFT regime of validity

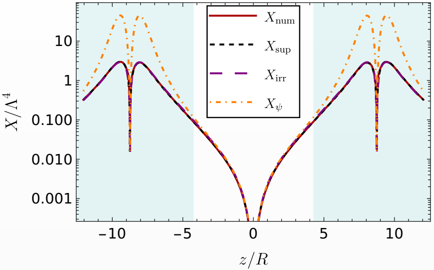
- Kinetic screening radius $r_{\rm sc} \sim \frac{1}{\Lambda}\left(\frac{\alpha M}{M_{\rm Pl}}\right)^{1/2}$
  EFT brakedown $r_{\rm UV}^{\rm poly} \sim \frac{1}{\Lambda}(\Lambda r_{\rm sc})^{-N/(N-1)}$
- Signifies classical nonlinear regime, not the UV scale
- $K(X)$ vs. GR: $(\Lambda, r_{\rm sc})$ vs. $(M_{\rm Pl}, r_{\rm Sch})$
- Careful with DBI $r_{\rm UV}^{\rm DBI} \sim \frac{1}{\Lambda}(\Lambda r_{\rm sc})^{2/3}$

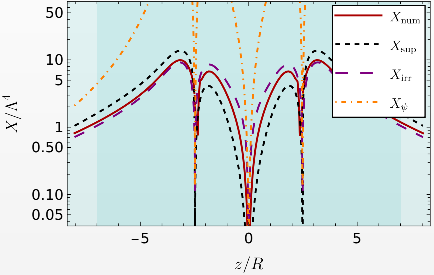
Ref: de Rham, Ribeiro [1405.5213]

Covariant Helmholtz decomposition

- k-essence EoM: $\nabla_\mu \chi^\mu = \frac{1}{2}\frac{\alpha}{M_{\rm Pl}} T$, $\chi_\mu \equiv K'(X)\nabla_\mu \varphi$
- Hodge decomposition: $\chi_\mu = -\frac{1}{2}\nabla_\mu \psi + B_\mu$
- Longitudinal component: $\Box \psi = -\frac{\alpha}{M_{\rm Pl}} T$
- Divergenceless part:
  $\Box B^\mu - R^\mu_\nu B^\nu = J^\mu$, $J_\mu = 2\nabla^\nu [K''(X)\nabla_{[\nu} X \nabla_{\mu]} \varphi]$,$\nabla_\mu B^\mu = 0$
- BD limit: $K''(X) = 0 \implies B_\mu = 0$

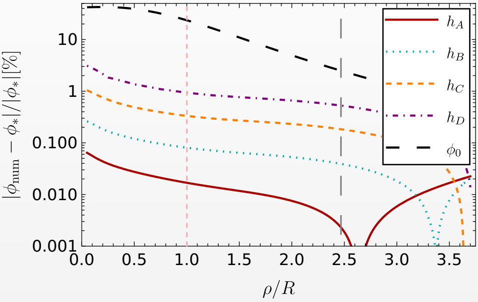
Numerical results (2/2)





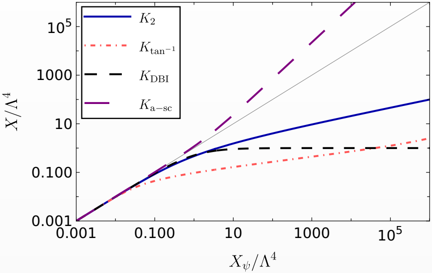
Numerics

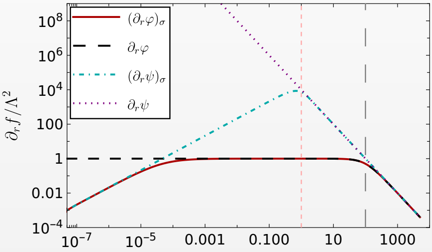
- Ingredients
  - Discretization in cylindrical coordinates (via finite difference method)
  - Symmetry BC + Dirichlet outside of the screening region
  - Gaussian source
  - Trial + linear system + Newton-Raphson
- Convergence tests
- Reproducing one-body problem



(Irrotational approximation for) other theories? (1/2)

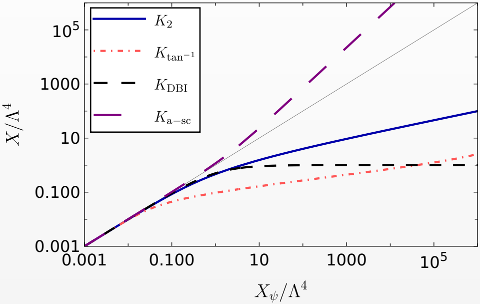
- Engineered model to pass solar system and cosmological constraints $K_{\tan^{-1}}$ Barreira+ [1504.01493]
- Non-linear completion: D-BIonic model
  Burrage, Khoury [1403.6120]
  $K_{\rm D-BI} = \sqrt{1 - X/2} - 1$
- Anti-screening (subluminal c_s) Hertzberg, Litterer, Shah [2209.07525]
  $K_{\rm a-sc} = -\frac{1}{p}\left[(1 + X/2)^p - 1\right]$





(Irrotational approximation for) other theories? (2/2)

- Further suppressed solenoidal component
- $F_{\tan^{-1}} \approx \kappa^{-3} K_\star (1 + K_\star)^3 X_\star^2 \frac{|\nabla \hat{X}_\psi|}{\hat{X}_\psi^{5/2}} \propto (K_\star/\kappa)^4$
- $F_{\mathrm{D-BI}} \approx \frac{1}{4}\sqrt{X(1-X)}$



k-essence beyond staticity (1/2)

- How effective is the screening in a dynamical scenario?
- Clear signs of the radiative screening for the stellar oscillations Bezares+ [2105.13992], Shibata, Traykova [2210.12139]
- Indications of the less effective screening in the gravitational collapse and the binary merger Bezares+ [2105.13992, 2107.05648]
- No hair theorems require dashing of the scalar field when the collapse to BH occurs

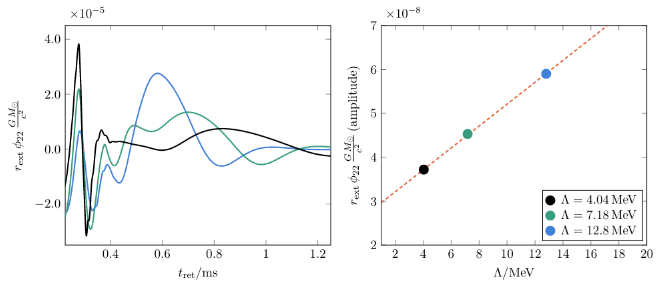


Fig: Bezares+ [2105.13992]

k-essence beyond staticity (2/2)

- Control over $\mathrm{MeV} \to \mathrm{meV}$ extrapolation in numerics
- Analytical approach for binary radiation based on $\varphi \approx \varphi(\mathrm{CM}) + \Delta\varphi$ de Rham, Tolley, Wesley [1208.0580]
- More systematic approach via Helmholtz decomposition?

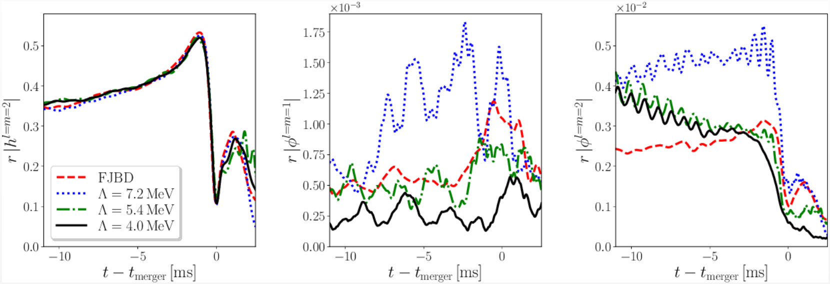


Fig: Bezares+ [2107.05648]