Valentina Danieli

Anharmonic Effects on the Squeezing of Axion Perturbations

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Introduction

- ✓ The history of the Universe undergoes a period of exponential expansion, **inflation**.
- ✓ Quantum fluctuations provide the seeds for structure formation.
- ✓ The CMB sky we see today is classical.

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- ✓ Quantum fluctuations provide the seeds for structure formation.
- ✓ The CMB sky we see today is classical.

Quantum to classical transition

- ✓ Inflation itself provides an explanation due to **squeezing**.
- ✓ Further source of classicalization: **reheating**.

Framework

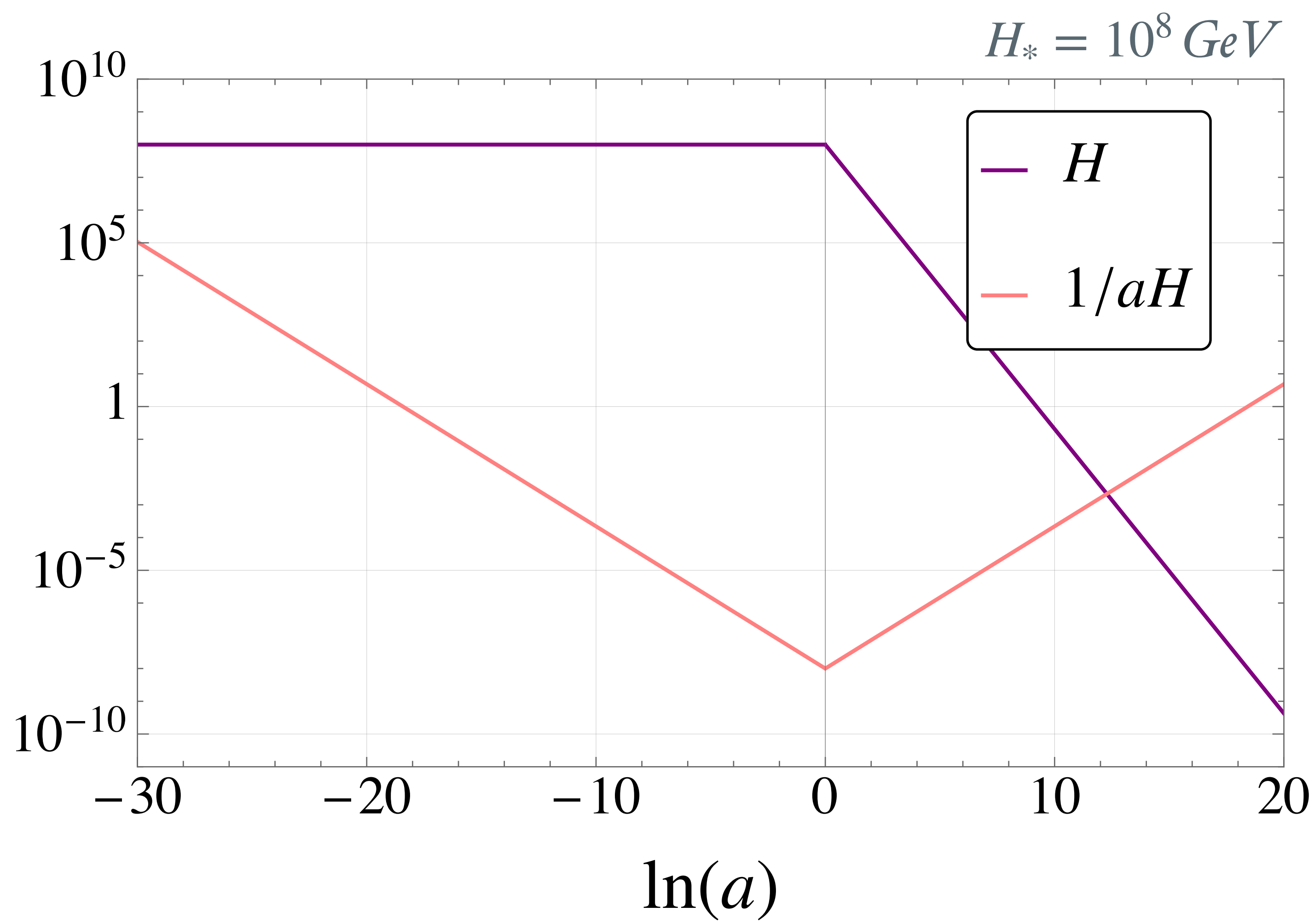
✓ de Sitter (DS) inflation followed by a Radiation Domination (RD) phase

✓ Axions produced via misalignment mechanism with $f > max(T_{rh}, H_I)$

✓ Axion is a spectator field

✓ Instantaneous reheating

$$H = \begin{cases} H_* & a < 0 \\ H_* \, a^{-2} & a > 0 \end{cases}$$



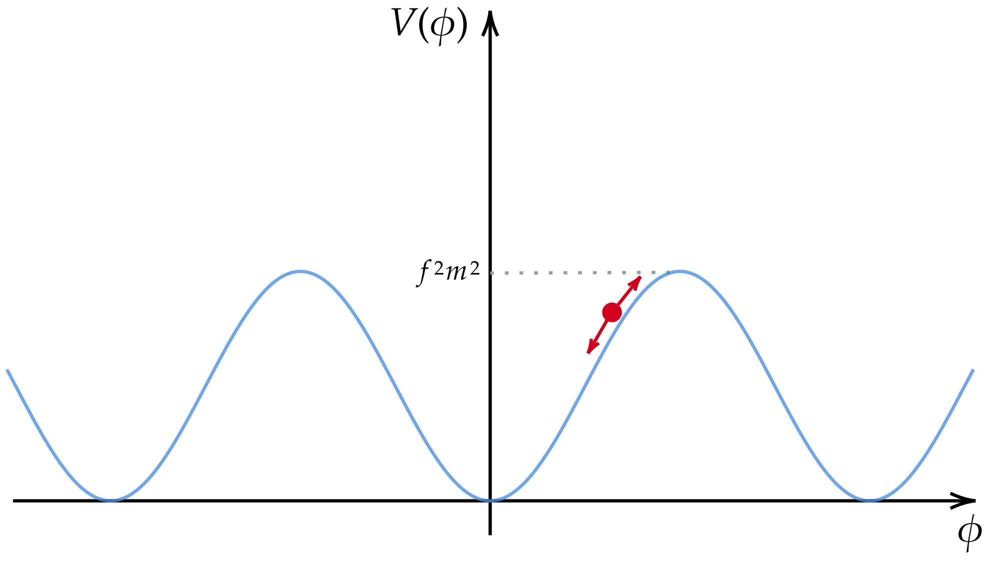
Framework

What about the axion potential?

$$V(\phi) = f^2\, m_\phi^2 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + f m_\phi^2 \sin\left(\frac{\phi_0}{f}\right) = 0$$

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + \left[\frac{k^2}{a^2} + m_\phi^2 \cos\left(\frac{\phi_0}{f}\right)\right]\delta\phi = 0$$

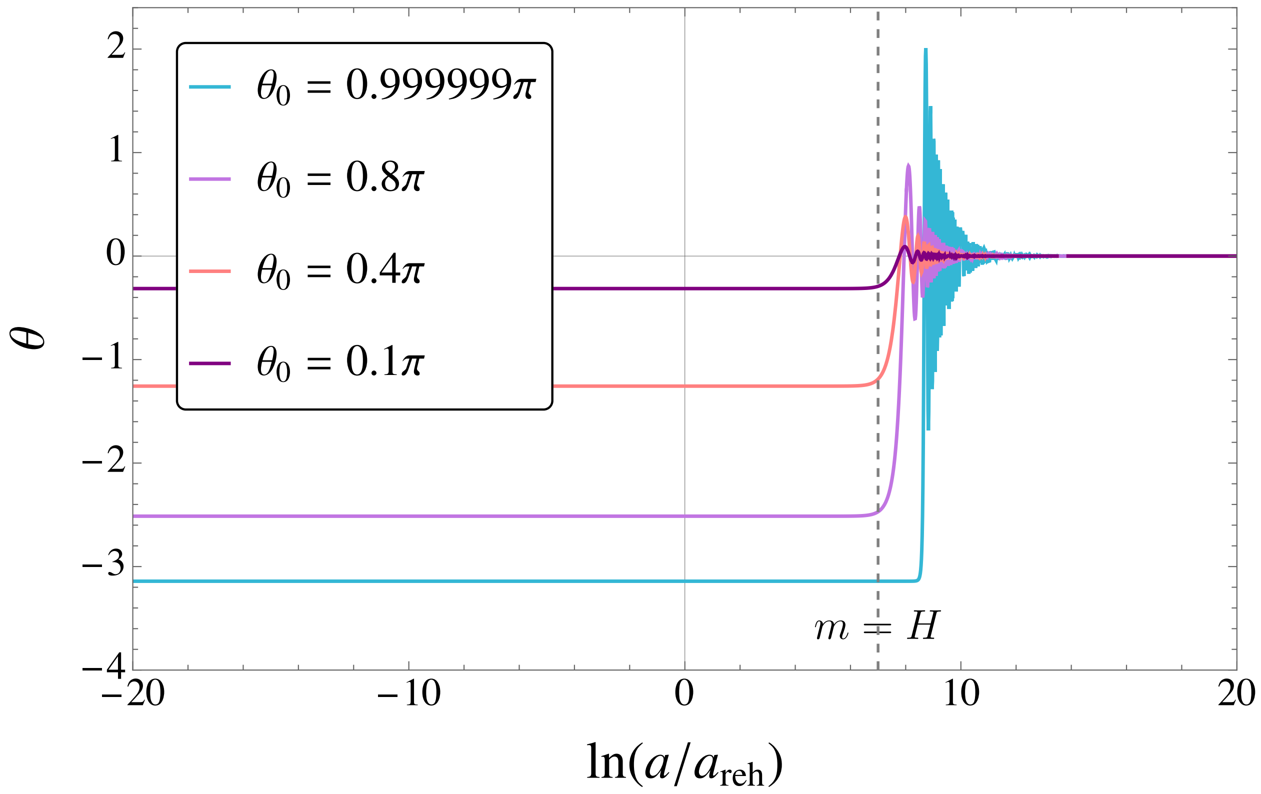


Background Field

Equation of motion

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + f m_\phi^2 \sin\left(\frac{\phi_0}{f}\right) = 0$$

$f = 10^{10}\, GeV$
$m = 10^{2}\, GeV$
$H_* = 10^{8}\, GeV$



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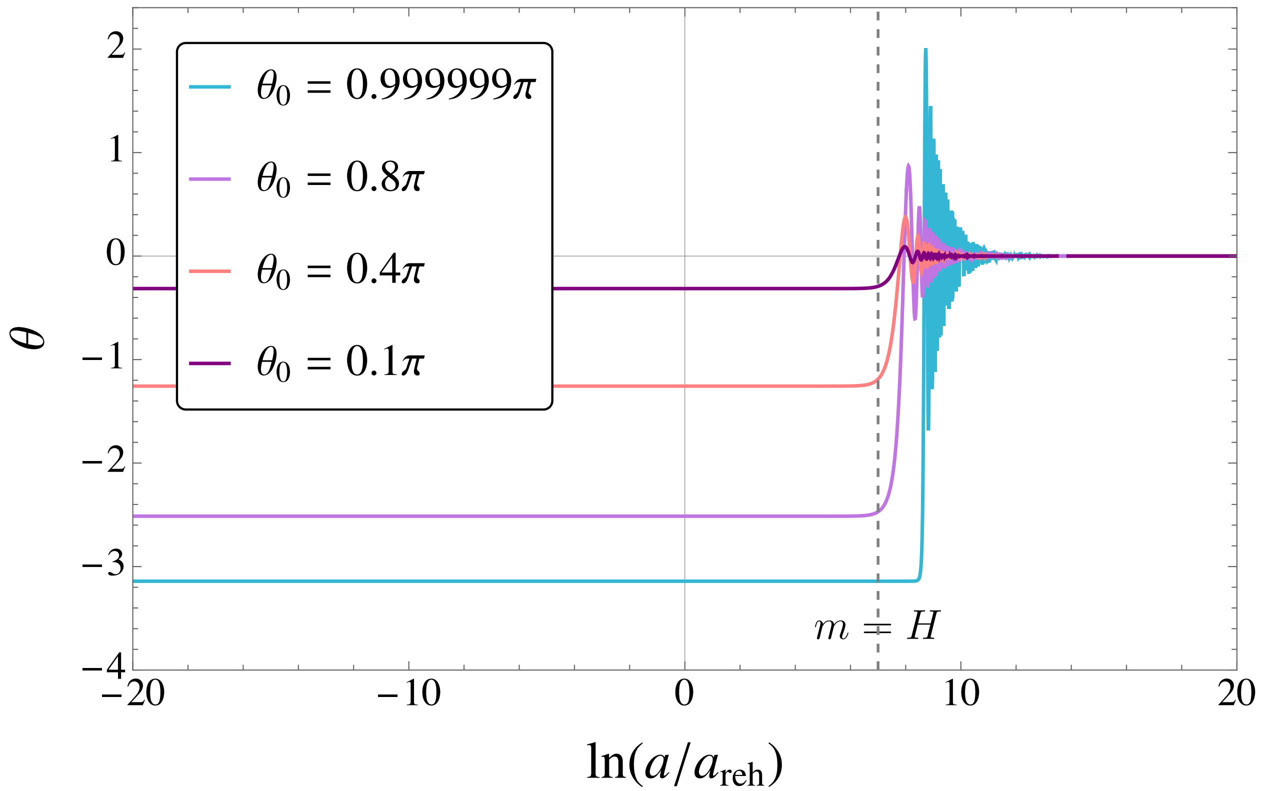
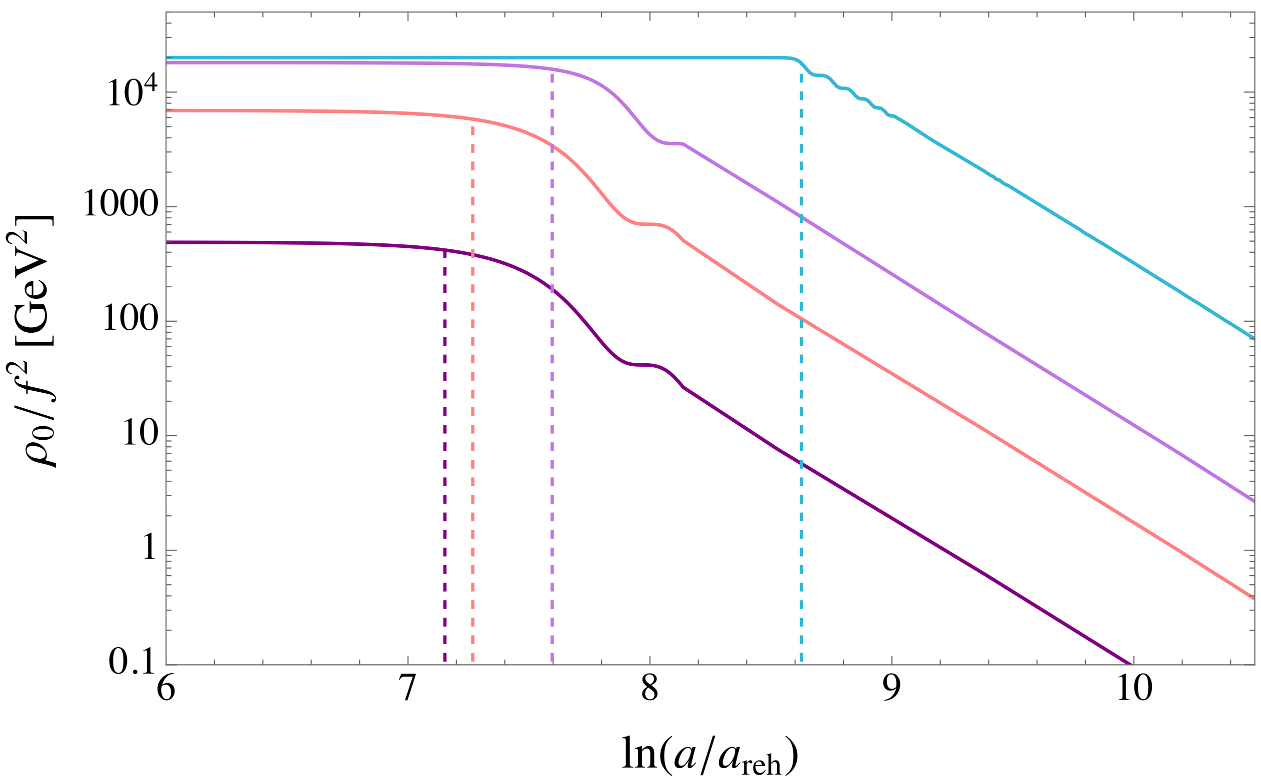
Energy density

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi) = \frac{H^2}{2}\left(\frac{d\phi}{d\eta}\right)^2 + V(\phi)$$





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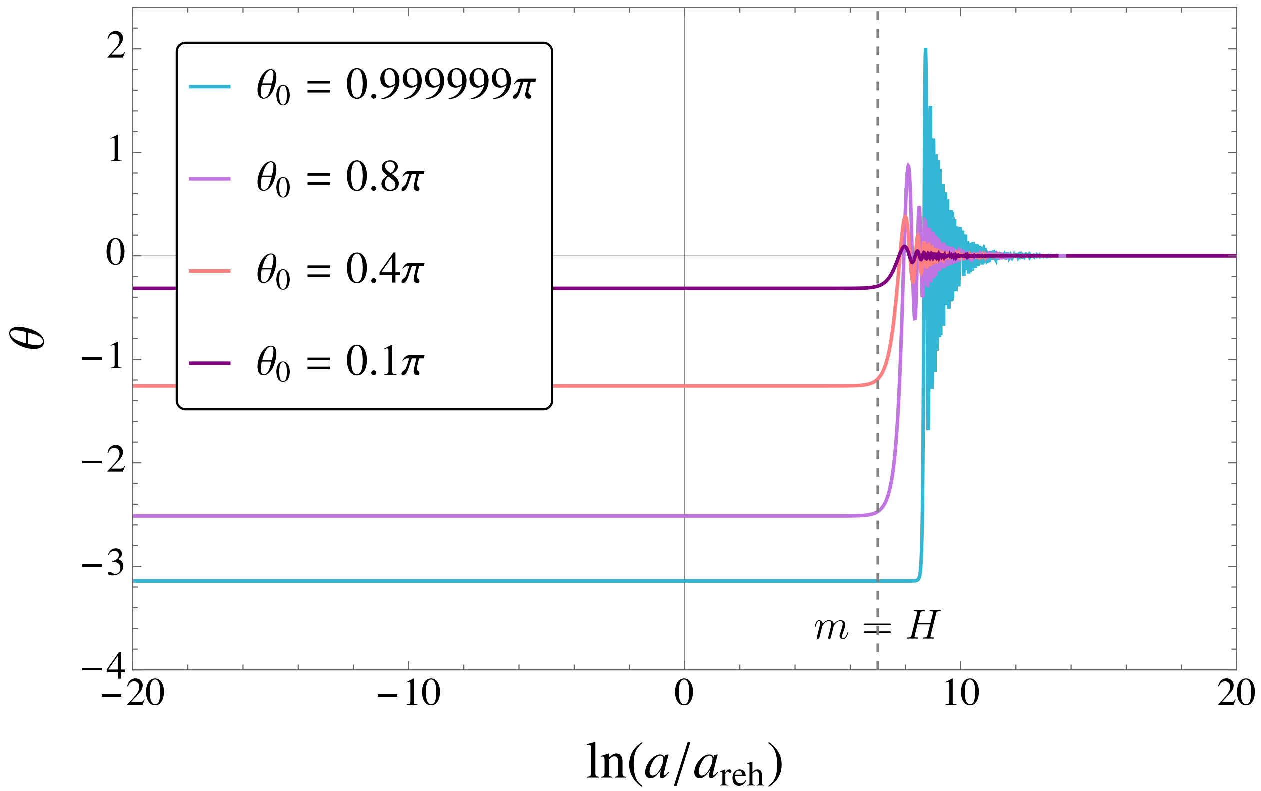
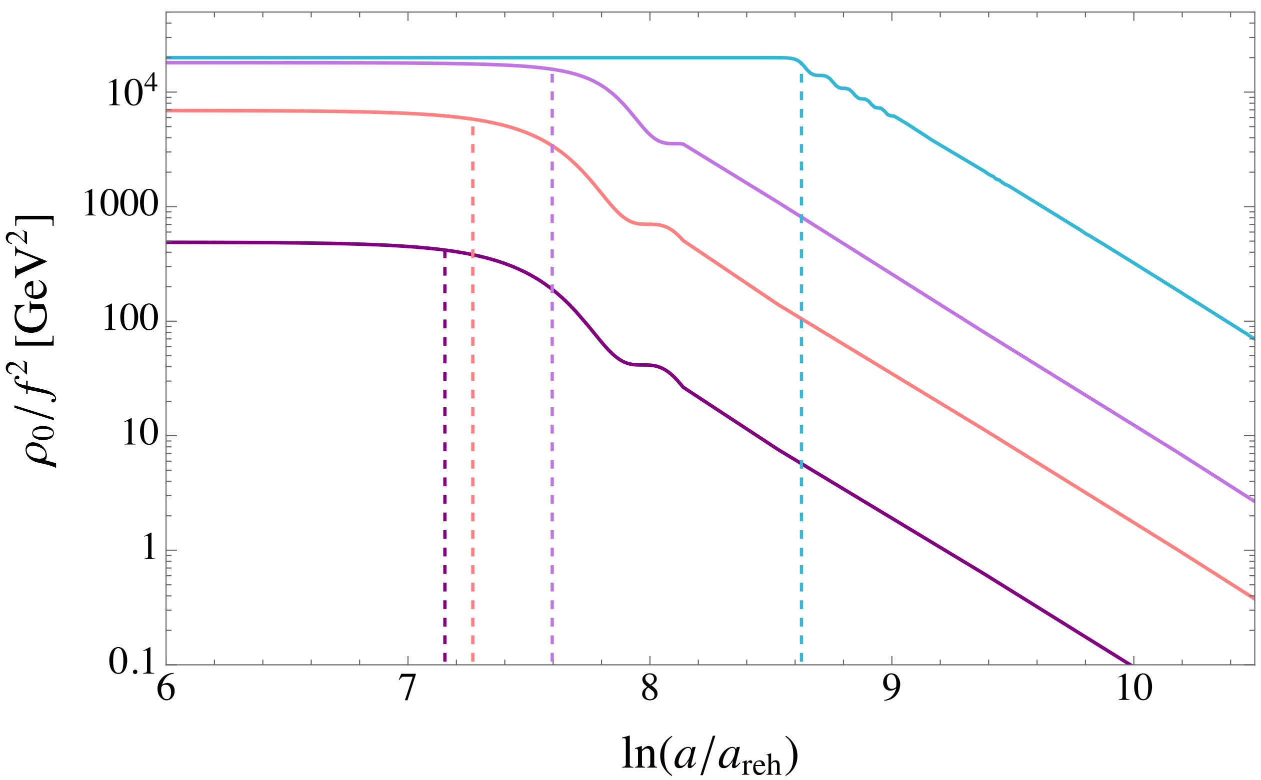




✓ The energy density is constant till the background field starts oscillating; thereafter it decays as a^{-3}

✓ The onset of the oscillations depend on the initial field value.

Axion Perturbations

✓ Consider the action for the perturbations.

✓ Compute the corresponding **Hamiltonian** (in Fourier space).

✓ Quantize the fields introducing **time-dependent ladder operators**.

$$\chi_{\mathbf{k}} = \frac{1}{\sqrt{2\omega_k}}\left(a_{\mathbf{k}}(\tau) + a^{\dagger}_{-\mathbf{k}}(\tau)\right)$$

$$p_{\mathbf{k}} = -i\sqrt{\frac{\omega_k}{2}}\left(a_{\mathbf{k}}(\tau) - a^{\dagger}_{-\mathbf{k}}(\tau)\right)$$

$$\boxed{\omega_k^2 = k^2 + m_{eff}^2 - \frac{a''}{a}}$$

✓ Time-dependent ladder operators are linked with time-independent ladder operators via **Bogoliubov transformation**:

$$\begin{cases} a_{\mathbf{k}}(\tau) = \alpha_k(\tau)\, a_{\mathbf{k}}(\tau_0) + \beta_k(\tau) a^{\dagger}_{-\mathbf{k}}(\tau_0) \\ a^{\dagger}_{-\mathbf{k}}(\tau) = \alpha_k^*(\tau)\, a^{\dagger}_{-\mathbf{k}}(\tau_0) + \beta_k^*(\tau) a_{\mathbf{k}}(\tau_0) \end{cases}$$

Axion Perturbations

✓ The fields $\chi_{\mathbf{k}}$ and $p_{\mathbf{k}}$ can be written alternatively in terms of the **time-independent ladder operators** directly:

$$\chi_{\mathbf{k}} = u_k(\tau)\, a^0_{\mathbf{k}} + u_k^*(\tau)\, a^{0\,\dagger}_{-\mathbf{k}}$$

$$p_{\mathbf{k}} = u_k'(\tau)\, a^0_{\mathbf{k}} + u_k^{*\,\prime}(\tau)\, a^{0\,\dagger}_{-\mathbf{k}}$$

✓ Comparing:

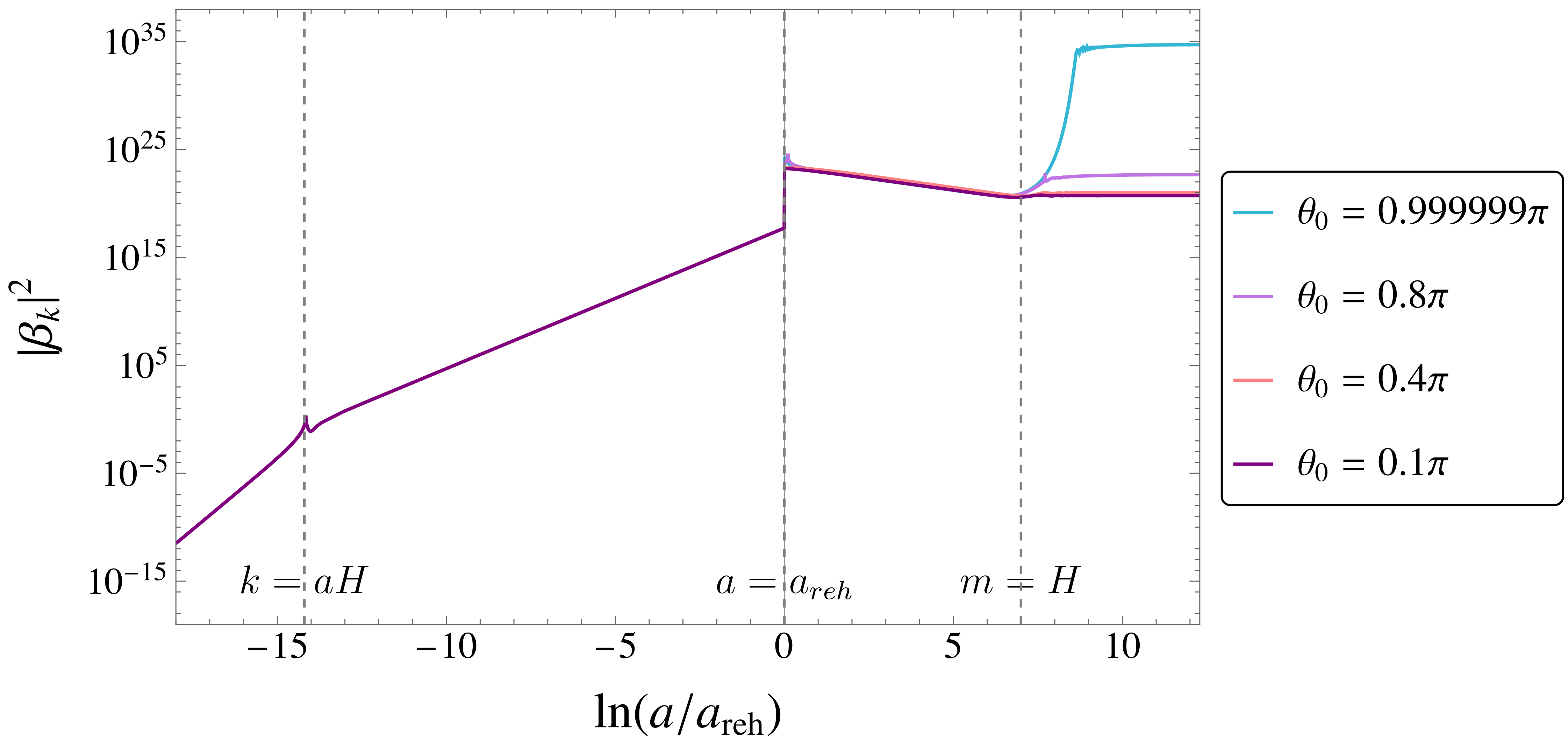
$$\boxed{\begin{aligned} \alpha_k &= \sqrt{\frac{\omega_k}{2}}\, u_k(\tau) - \frac{i}{\sqrt{2\,\omega_k}}\, u_k'(\tau) \\ \beta_k &= \sqrt{\frac{\omega_k}{2}}\, u_k^*(\tau) - \frac{i}{\sqrt{2\,\omega_k}}\, u_k^{*\,\prime}(\tau) \end{aligned}}$$

✓ The Bogoliubov coefficients can be parameterised by the **squeezing parameters**:

$$\begin{cases} \alpha_k(\tau) = e^{-i\vartheta_k(\tau)} \cosh r_k(\tau) \\ \beta_k(\tau) = e^{i\left[\vartheta_k(\tau) + 2\varphi_k(\tau)\right]} \sinh r_k(\tau) \end{cases}$$

Analysis of the Beta Coefficient

$$|\beta_k|^2 = \frac{\omega_k}{2}\,|f_k|^2 + \frac{1}{2\,\omega_k}\,|f_k'|^2 - \frac{1}{2}$$



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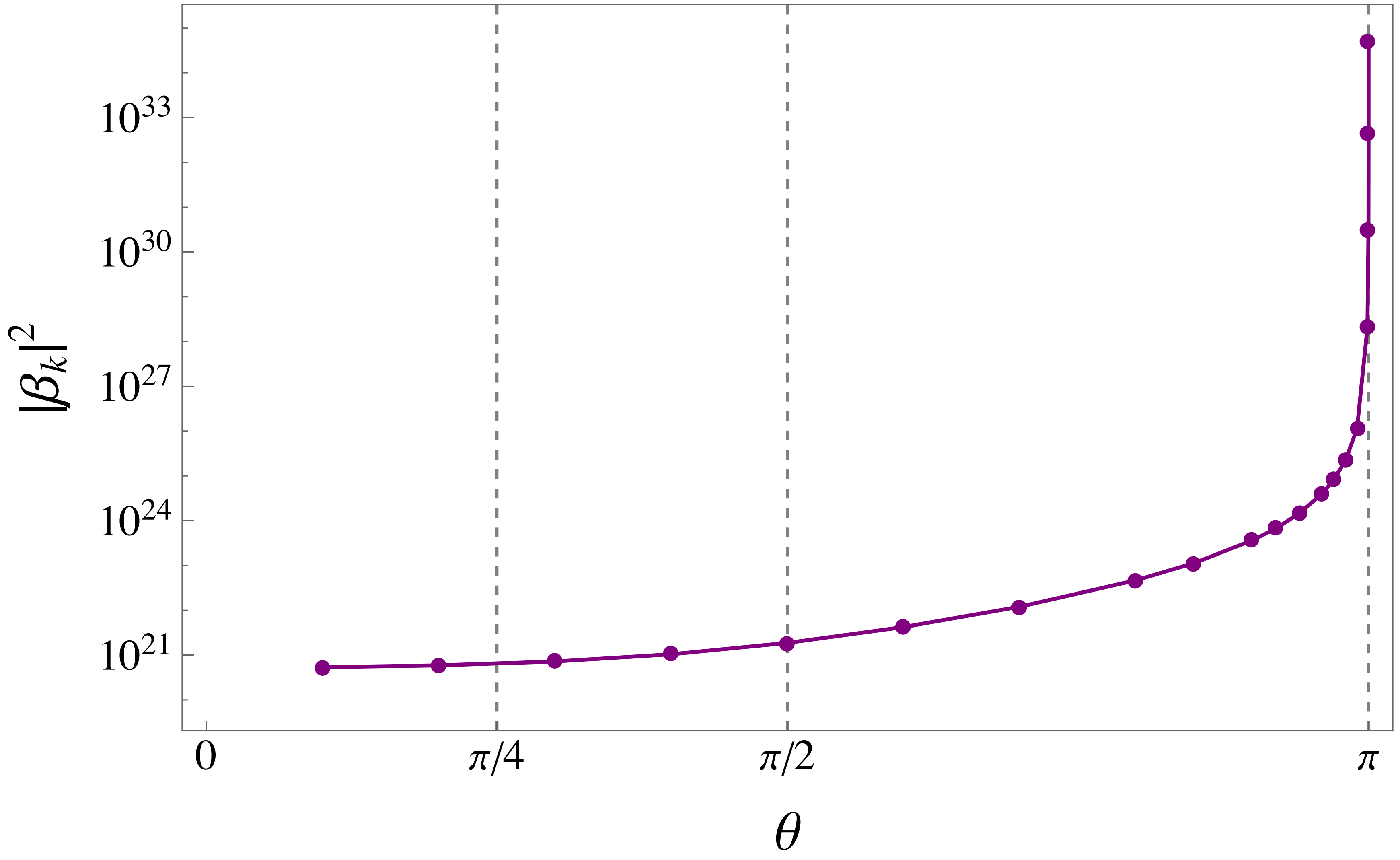

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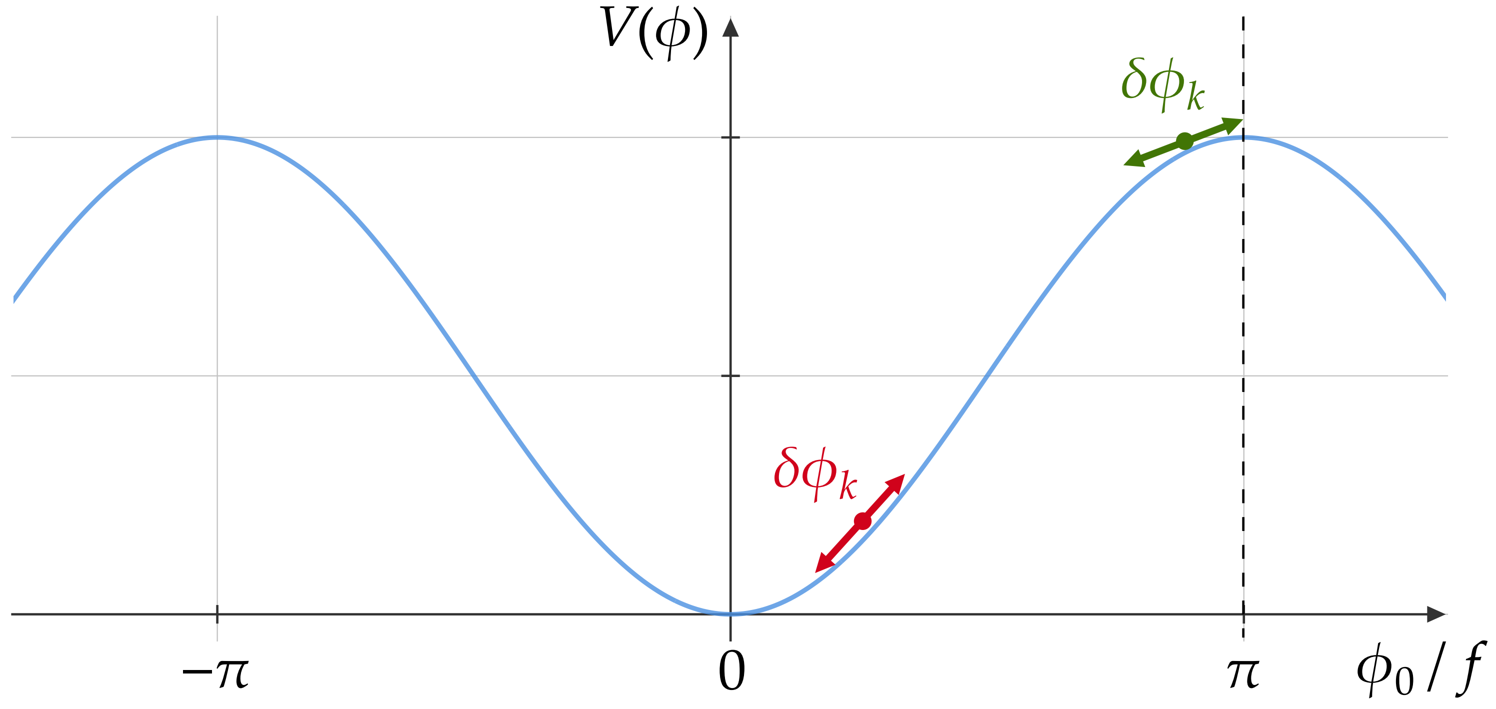


✓ The rolling down of the field is delayed increasing the initial field value.

✓ Near the hilltop, the field $\phi_0 - \delta\phi$ begins to oscillate much earlier than the $\phi_0 + \delta\phi$

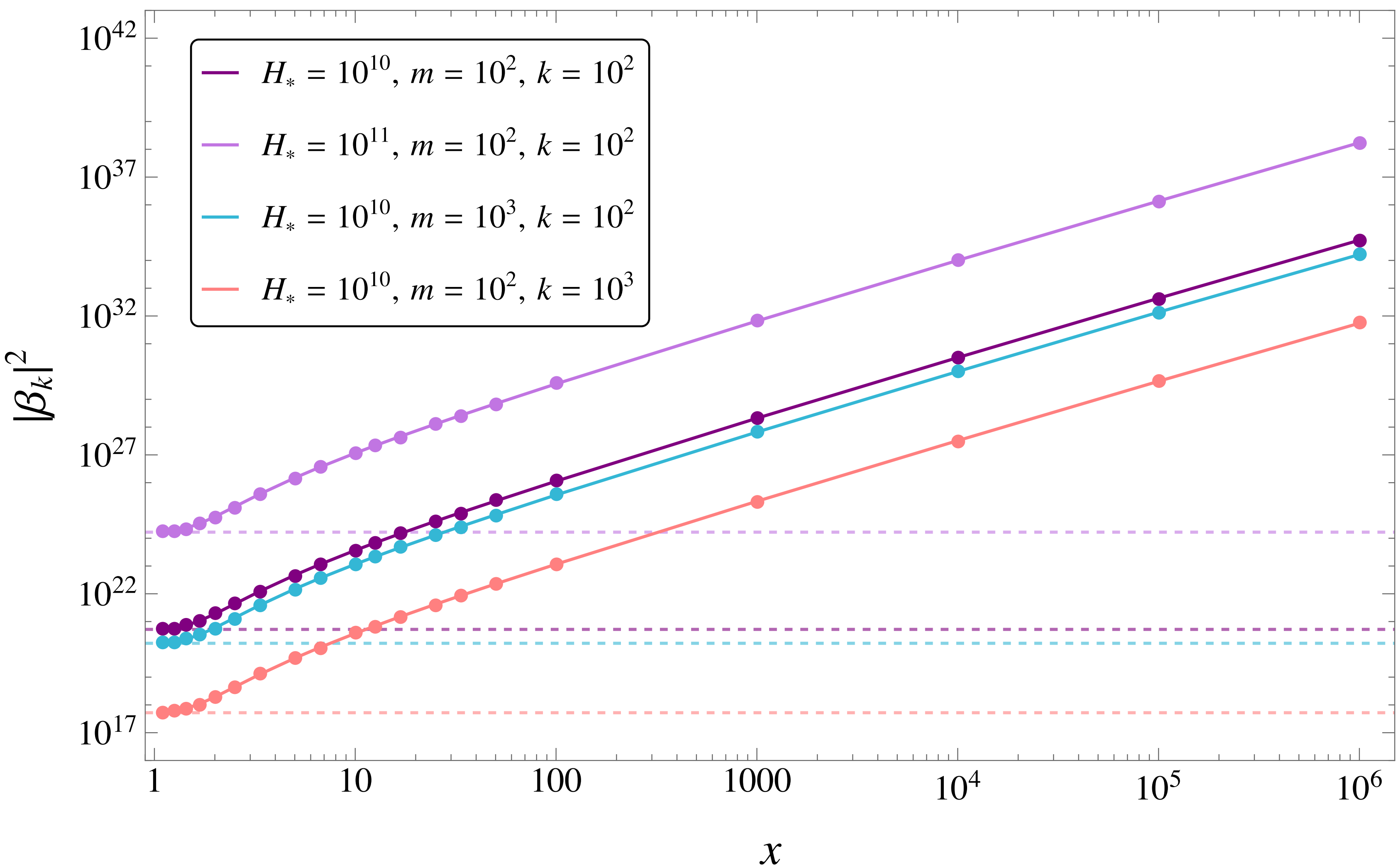
✓ This delay makes $\delta\phi_k$ larger and larger when evolving in time.

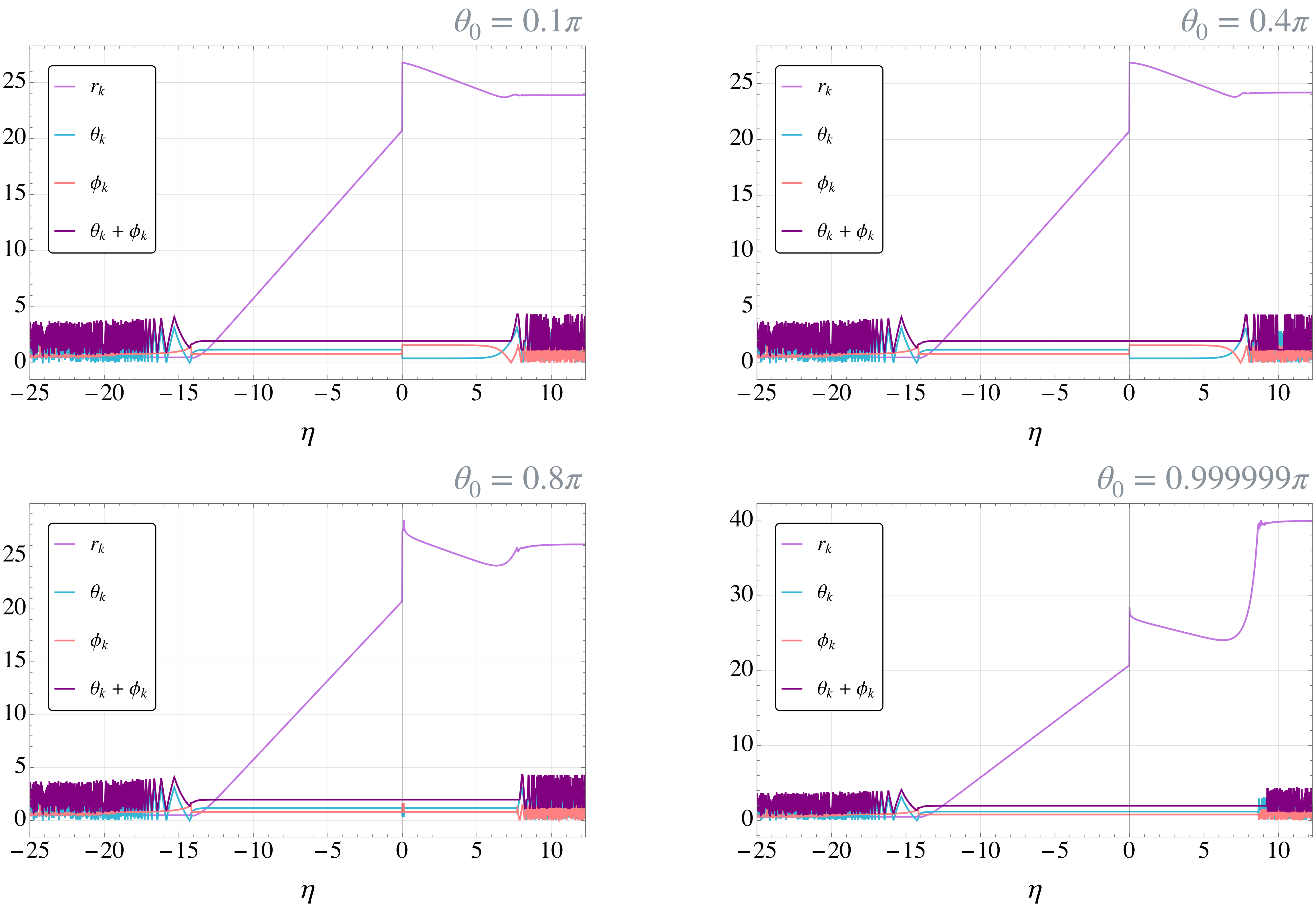
✓ In the limiting case where $\phi_{in} = \pi$, the field won't start oscillating at all.

Analysis of the Beta Coefficient

$$|\beta_k|^2 = \frac{1}{8\pi}\,\Gamma(1/4)^2\,\frac{H_{inf}^{7/2}}{\sqrt{m}\,k^3}$$

$$x = \frac{\pi}{\pi - \theta}$$





$\theta_0 = 0.1\pi$
$\theta_0 = 0.4\pi$
$\theta_0 = 0.8\pi$
$\theta_0 = 0.999999\pi$
r_k
θ_k
ϕ_k
$\theta_k + \phi_k$
η
−25 −20 −15 −10 −5 0 5 10
0 5 10 15 20 25
0 10 20 30 40

Squeezing Formalism

The process of particle creation can be equivalently described by means of the **squeezing formalism**, whose advantage is to give a clear phase space representation of the system's evolution.

The evolution in time of the ladder operators can be given by:

$$a_{\pm\mathbf{k}}(\tau) = U(\tau)\, a^0_{\pm\mathbf{k}}\, U^\dagger(\tau)$$

Where:

$$U = RS$$

$$R(\vartheta_k) = \exp\left[-i\vartheta_k \left(a^{\dagger\,0}_{\mathbf{k}}\, a^0_{\mathbf{k}} + a^{\dagger\,0}_{-\mathbf{k}}\, a^0_{-\mathbf{k}}\right)\right]$$

$$S(r_k, \varphi_k) = \exp\left[r_k \left(e^{-2i\varphi_k}\, a^0_{\mathbf{k}}\, a^0_{-\mathbf{k}} - e^{2i\varphi_k}\, a^{\dagger\,0}_{\mathbf{k}}\, a^{\dagger\,0}_{-\mathbf{k}}\right)\right]$$

The action of these two operators on $a_{\mathbf{k}}(\tau)$ can be computed:

$$RSa_{\mathbf{k}}(\tau)\,S^{\dagger}R^{\dagger} = e^{-i\,\vartheta_k}\,\cosh r_k\,a^{0}_{\mathbf{k}} - e^{i\,(\vartheta_k + 2\,\varphi_k)}\,\sinh r_k a^{0\,\dagger}_{-\mathbf{k}}$$

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Making a comparison we recognize:

$$\begin{cases} \alpha_k(\tau) = e^{-i\vartheta_k(\tau)}\cosh r_k(\tau) \\ \beta_k(\tau) = -\,e^{i\left[\vartheta_k(\tau) + 2\varphi_k(\tau)\right]}\,\sinh r_k(\tau) \end{cases}$$

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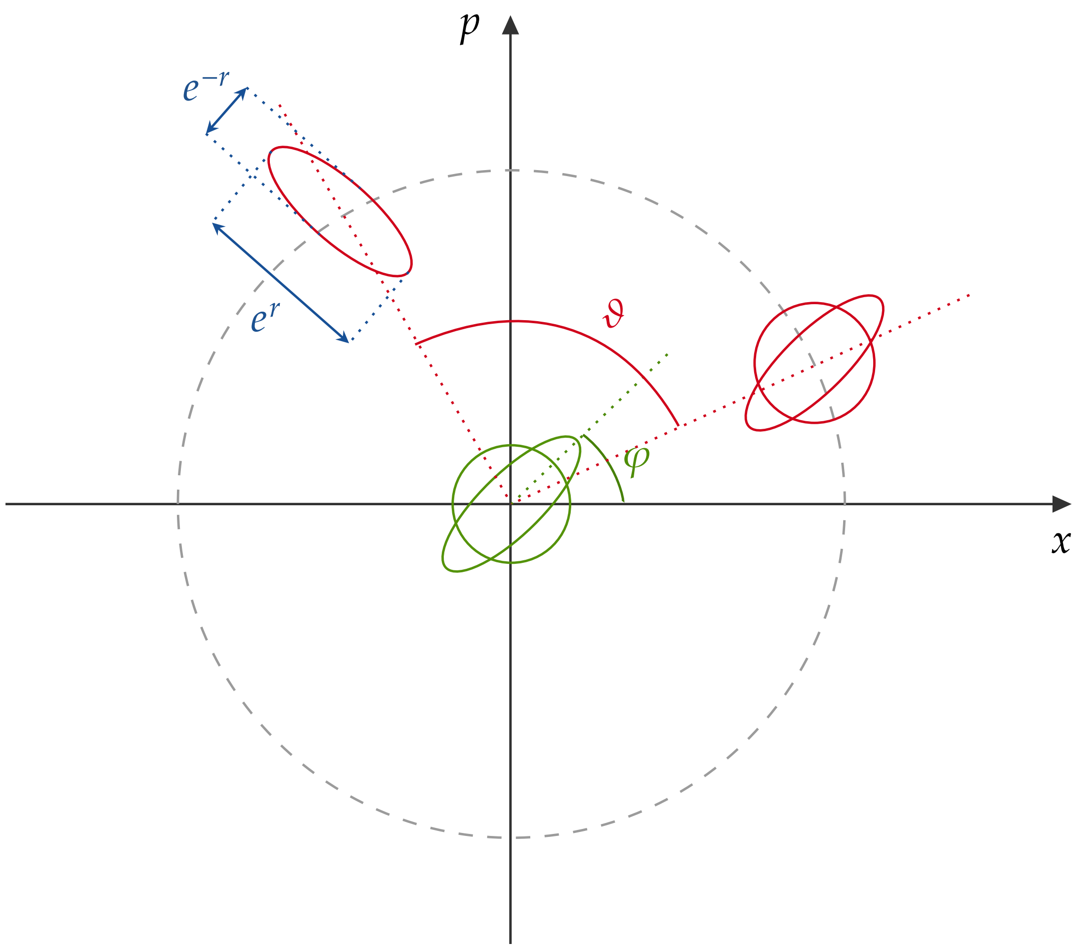


In the context of cosmological particle creation:

$$|\,\phi_{out}(\eta)\rangle = \frac{1}{2}\prod_{\mathbf{k}} S(r_k, \varphi_k)\,R(\vartheta_k)\,|\,0_{\mathbf{k}}, 0_{-\mathbf{k}}\rangle$$

$$|\,\phi_{out}(\eta)\rangle = \frac{1}{2}\prod_{\mathbf{k}}\frac{1}{\cosh r_k}\sum_{n=0}^{\infty}\left(-\tanh r_k\, e^{2i\varphi_k}\right)^n\,|\,n_{\mathbf{k}}, n_{-\mathbf{k}}\rangle$$

Conclusions and Future Developments

- ✓ Anharmonic effects produce an enhancement in the number of particles created due to the expansion
- ✓ The number of particles and the energy density increase exponentially when approaching the hilltop of the potential
- ✓ Anharmonic effects increase also the amount of squeezing of the perturbations
- ✓ Study the **observables** for this system, e.g. **power spectrum** and **bispectrum**
- ✓ Apply this machinery to the analysis of other physical systems, like **primordial electromagnetic fields**

THANKS FOR THE ATTENTION

Axion Perturbations

The **action** to consider is:

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_\phi^2 \cos\left(\frac{\phi_0}{f}\right) \phi^2 \right] =$$

$$= \int d^3x d\tau\, a^2 \left[\frac{1}{2}\phi'^2 - \frac{1}{2}(\partial_i \phi)^2 - \frac{1}{2} m_\phi^2\, a^2 \cos\left(\frac{\phi_0}{f}\right) \phi^2 \right]$$

Define:

$$u(\tau) = a(\tau)\phi(\tau)$$

We can compute the corresponding **Hamiltonian** (in Fourier space):

$$\mathscr{H} = \frac{1}{2(2\pi)^3} \int d^3k \left[p_{\mathbf{k}} p_{\mathbf{k}}^* + \left(k^2 + m_{eff}^2 a^2 - \frac{a''}{a} \right) \chi_{\mathbf{k}} \chi_{\mathbf{k}}^* \right]$$

$$\hat{p}_{\mathbf{k}} = \hat{\chi}'_{\mathbf{k}}$$

$$m_{eff}^2 = f m_\phi^2 \cos\left(\frac{\phi_0}{f}\right)$$

Axion Perturbations

We quantize the fields introducing **time-dependent ladder operators**:

$$\chi_{\mathbf{k}} = \frac{1}{\sqrt{2\omega_k}}\left(a_{\mathbf{k}}(\tau) + a^{\dagger}_{-\mathbf{k}}(\tau)\right)$$

$$p_{\mathbf{k}} = -i\sqrt{\frac{\omega_k}{2}}\left(a_{\mathbf{k}}(\tau) - a^{\dagger}_{-\mathbf{k}}(\tau)\right)$$

$$\omega_k^2 = k^2 + m_{eff}^2 - \frac{a''}{a}$$

Respecting canonical commutation relations:

$$\left[\chi_{\mathbf{k}}(\tau), p^{\dagger}_{\mathbf{k}'}(\tau)\right] = i\delta^{(3)}(\mathbf{k}-\mathbf{k}')\,, \qquad \left[a_{\mathbf{k}}(\tau), a^{\dagger}_{\mathbf{k}'}(\tau)\right] = \delta^{(3)}(\mathbf{k}-\mathbf{k}')$$

Time-dependent ladder operators are linked with time-independent ladder operators via **Bogoliubov transformation**:

$$\begin{cases} a_{\mathbf{k}}(\tau) = \alpha_k(\tau)\, a_{\mathbf{k}}(\tau_0) + \beta_k(\tau) a^{\dagger}_{-\mathbf{k}}(\tau_0) \\ a^{\dagger}_{-\mathbf{k}}(\tau) = \tilde{\alpha}_k(\tau)\, a^{\dagger}_{-\mathbf{k}}(\tau_0) + \tilde{\beta}_k(\tau) a_{\mathbf{k}}(\tau_0) \end{cases}$$

The fields $u_{\mathbf{k}}$ and $p_{\mathbf{k}}$ can be written alternatively in terms of the **time-independent ladder operators** directly:

$$\chi_{\mathbf{k}} = u_k(\tau)\, a^0_{\mathbf{k}} + u^*_k(\tau)\, a^{0\,\dagger}_{-\mathbf{k}}$$

$$p_{\mathbf{k}} = u'_k(\tau)\, a^0_{\mathbf{k}} + u^{*\,\prime}_k(\tau)\, a^{0\,\dagger}_{-\mathbf{k}}$$

Comparing:

$$\alpha_k = \sqrt{\frac{\omega_k}{2}}\, u_k(\tau) - \frac{i}{\sqrt{2\,\omega_k}}\, u'_k(\tau)$$

$$\beta_k = \sqrt{\frac{\omega_k}{2}}\, u^*_k(\tau) - \frac{i}{\sqrt{2\,\omega_k}}\, u^{*\,\prime}_k(\tau)$$

$$\longrightarrow \qquad |\beta_k|^2 = \frac{\omega_k}{2}\,|u_k|^2 + \frac{1}{2\,\omega_k}\,|u'_k|^2 - \frac{1}{2}$$

The Bogoliubov coefficients can be parameterised by the **squeezing parameters**:

$$\begin{cases} \alpha_k(\tau) = e^{-i\vartheta_k(\tau)} \cosh r_k(\tau) \\ \beta_k(\tau) = -\, e^{i\left[\vartheta_k(\tau) + 2\varphi_k(\tau)\right]} \sinh r_k(\tau) \end{cases}$$

Inverting these relations:

$$\begin{cases} r = \sinh^{-1} |\beta| \\ \vartheta = \arccos\left(Re \, \dfrac{\alpha}{|\alpha|} \right) \\ \varphi = -\dfrac{1}{2} \arccos\left(Re \, \dfrac{\alpha\beta}{|\alpha\beta|} \right) \end{cases}$$

Axion Mode Functions

In terms of **conformal time**:

$$\boxed{u'' + \left(k^2 + m_{eff}^2 a^2 - \frac{a''}{a}\right) u = 0}$$

Solution for **mass term potential**:

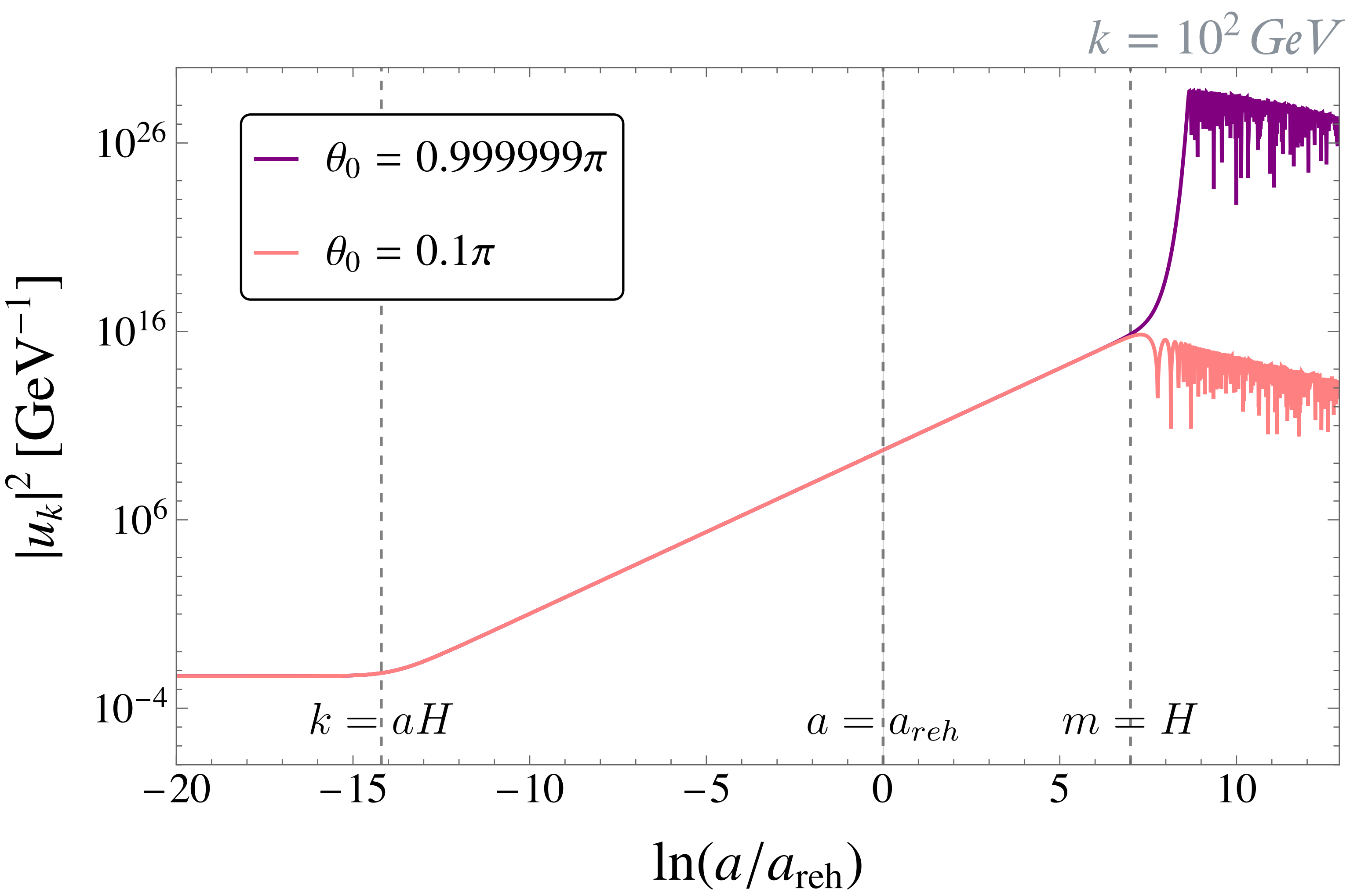
$$u_{DS}(\tau) = \frac{1}{4}\sqrt{\pi} e^{\frac{1}{2} i\pi\left(\nu^2 + \frac{1}{2}\right)} \sqrt{\frac{1}{H_*} - \tau}\, H_\nu^{(1)}\left(k\left(\frac{1}{H_*} - \tau\right)\right)$$

$$u_{RD}(\tau) = c_1\, D_{-\frac{ik^2 + H_* m}{2H_* m}}\left((1+i)\sqrt{\frac{m}{H_*}}\,(H_*\tau + 1)\right) + c_2\, D_{\frac{ik^2 - H_* m}{2\,H_*\, m}}\left((i-1)\sqrt{\frac{m}{H_*}}\,(H_*\tau + 1)\right)$$

Axion Mode Functions

In terms of **e-folding time**:

$$u'' + \left(1 + \frac{H'}{H}\right) u' + \left(\frac{k^2}{H^2} e^{-2\eta} - 2 - \frac{H'}{H} + \frac{m_{eff}^2}{H^2}\right) u = 0$$



Cosmological framework: the instantaneous vacuum defined by the time-dependent ladder operators $(a_{\mathbf{k}}(\eta), a^{\dagger}_{\mathbf{k}}(\eta))$ is filled with particles associated with the initial time-independent operators $(a^{0}_{\mathbf{k}}, a^{0\,\dagger}_{\mathbf{k}})$.

What is the correct choice for the initial ladder operators?

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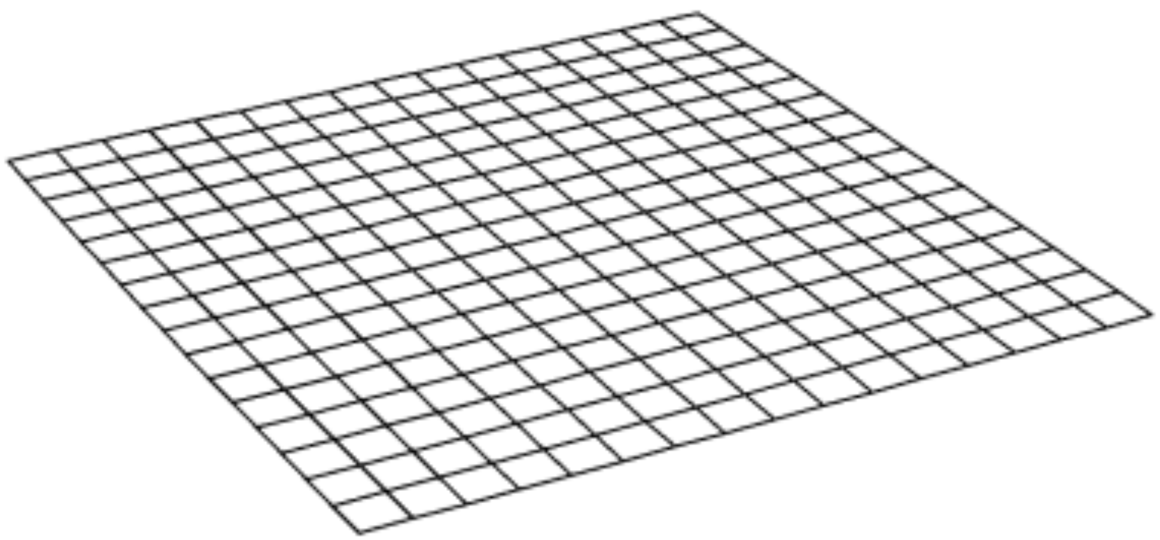

In **Minkowski** spacetime there is a **unique choice for the vacuum state**.

Particle Creation in Curved Spacetime

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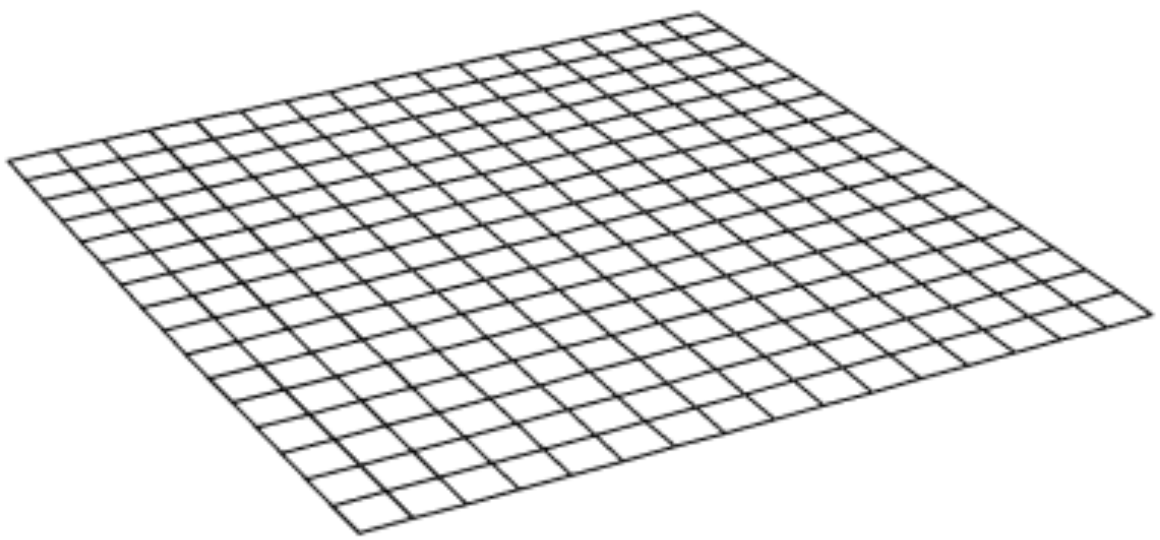
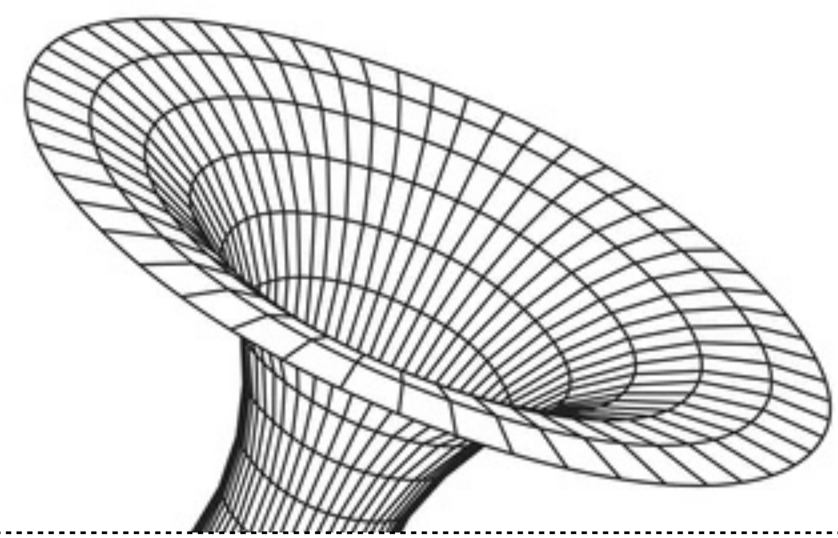


In **Minkowski** spacetime there is a **unique choice for the vacuum state**.

On an arbitrary spacetime, there are in general **no** isometries that allow to define **uniquely the vacuum state**.



Assuming **Minkowski in the asymptotic past and future:**

$$a_{\mathbf{k}}(\eta) \xrightarrow[\eta\to-\infty]{} a_{\mathbf{k}}^{in}, \qquad a_{\mathbf{k}}(\eta) \xrightarrow[\eta\to+\infty]{} a_{\mathbf{k}}^{out}$$

Linked via time-independent Bogoliubov coefficients A_k and B_k .

Time-dependent Bogoliubov coefficients are their late time limit:

$$\alpha_k(\eta) \xrightarrow[\eta\to+\infty]{} A_k, \qquad \beta_k(\eta) \xrightarrow[\eta\to+\infty]{} B_k$$

When the background felt by the fields can be approximated as **constant in time**?

Adiabaticity condition: $\left|\dfrac{\omega_k'}{\omega_k^2}\right|^2, \left|\dfrac{\omega_k''}{\omega_k^3}\right| \ll 1$

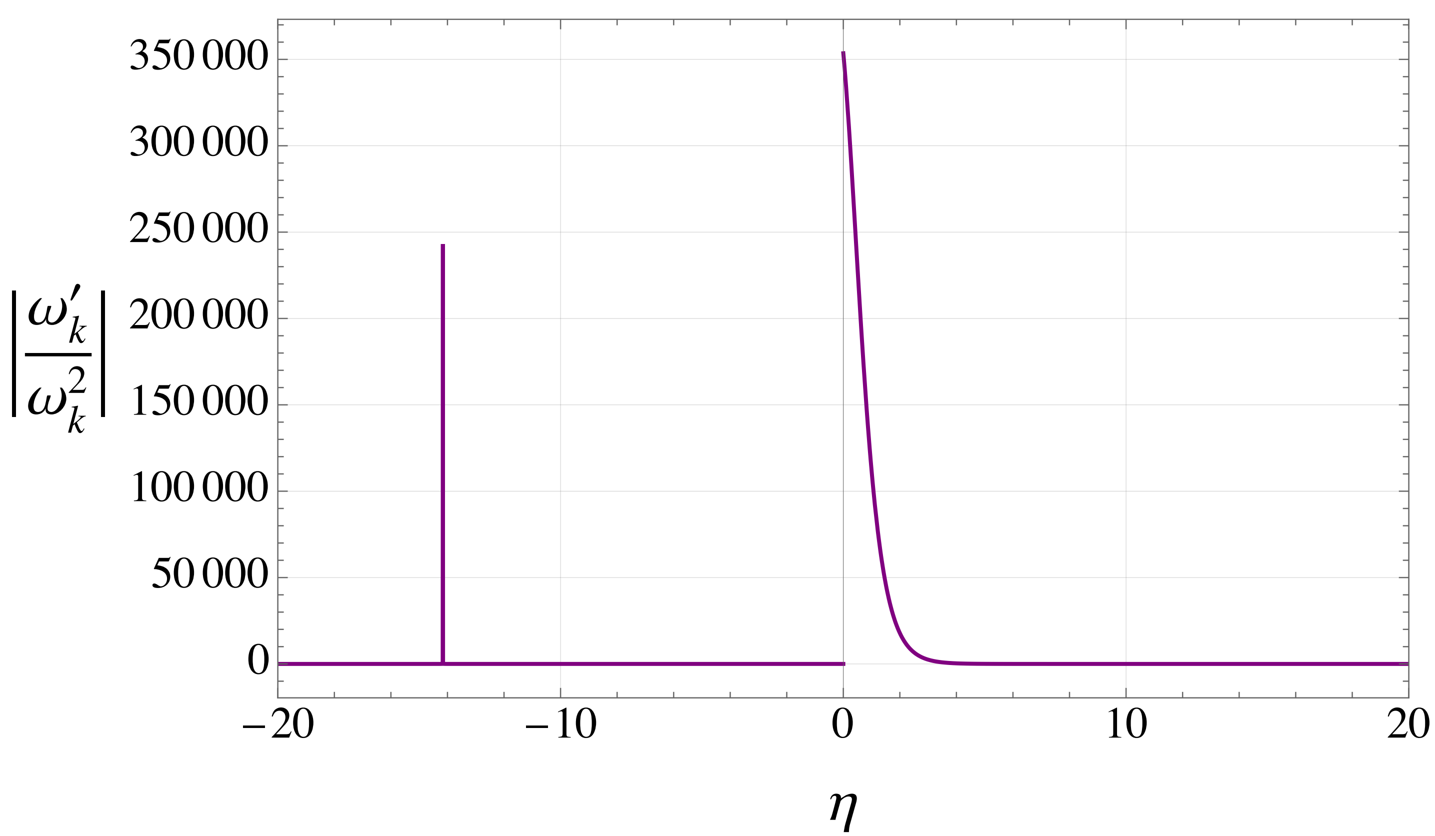
Adiabaticity Condition

The adiabaticity condition is defined as:

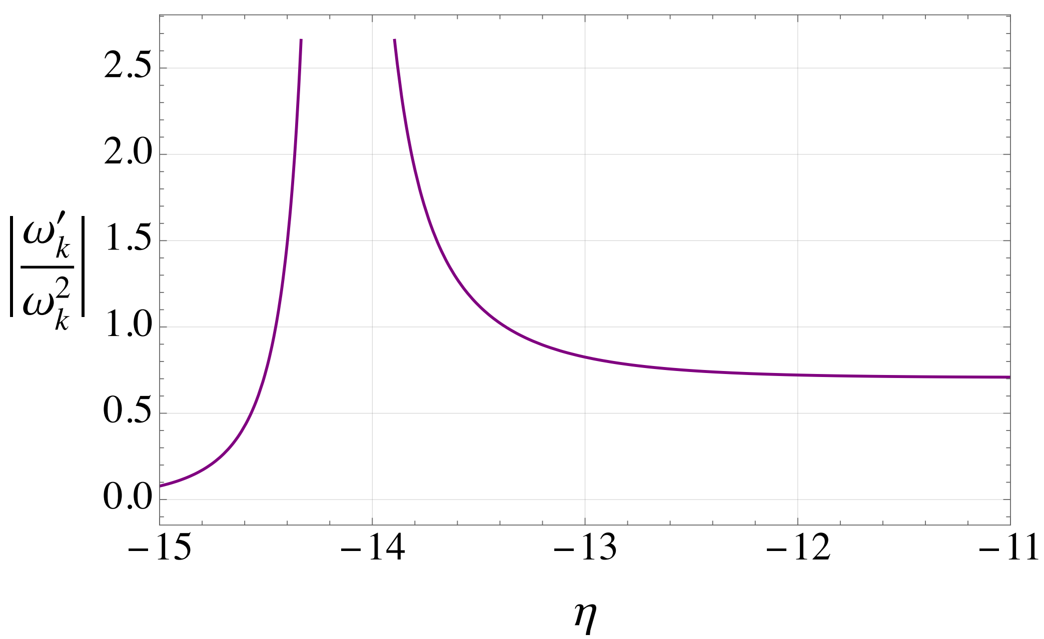
$$\left|\frac{\omega_k'}{\omega_k^2}\right|^2, \left|\frac{\omega_k''}{\omega_k^3}\right| \ll 1 \qquad \begin{cases} f'' + \omega_k^2 f = 0 \\ \omega_k^2 = k^2 + m^2 a^2 - \dfrac{a''}{a} \end{cases}$$

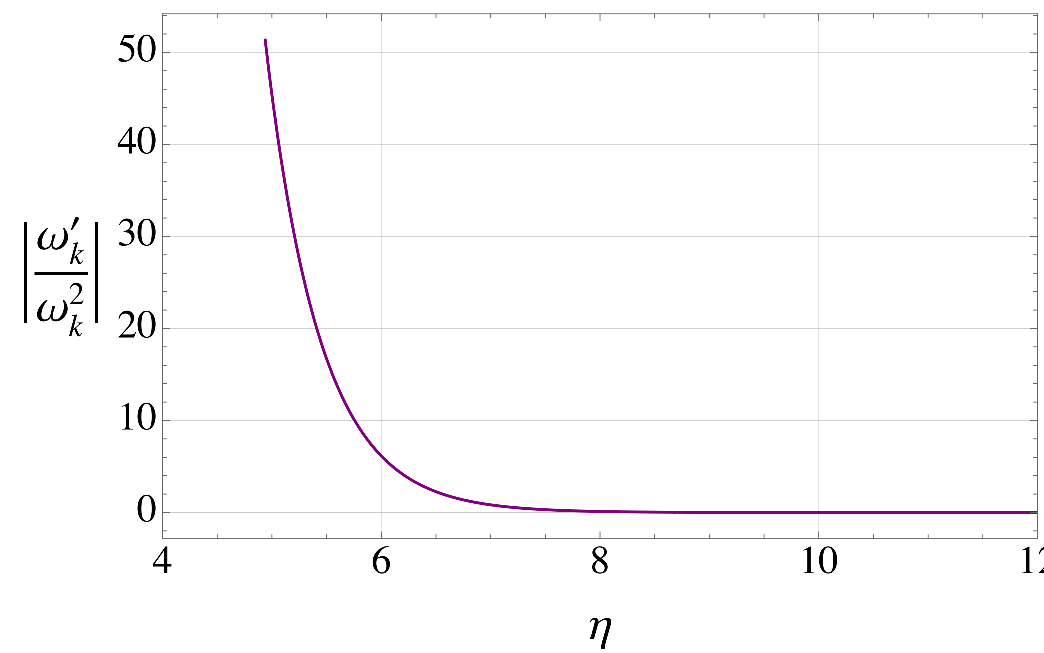
If the adiabaticity condition holds:

$$f(\tau) = \frac{A_k}{\sqrt{2k}}\, e^{+i\int^\tau \omega_k(\tau')d\tau'} + \frac{B_k}{\sqrt{2k}}\, e^{-i\int^\tau \omega_k(\tau')d\tau'}$$



Adiabaticity Condition





✓ Around $\eta \simeq -14$ the frequency starts changing rapidly in time.

✓ When the mode is far superhorizon it settles to a constant value given by

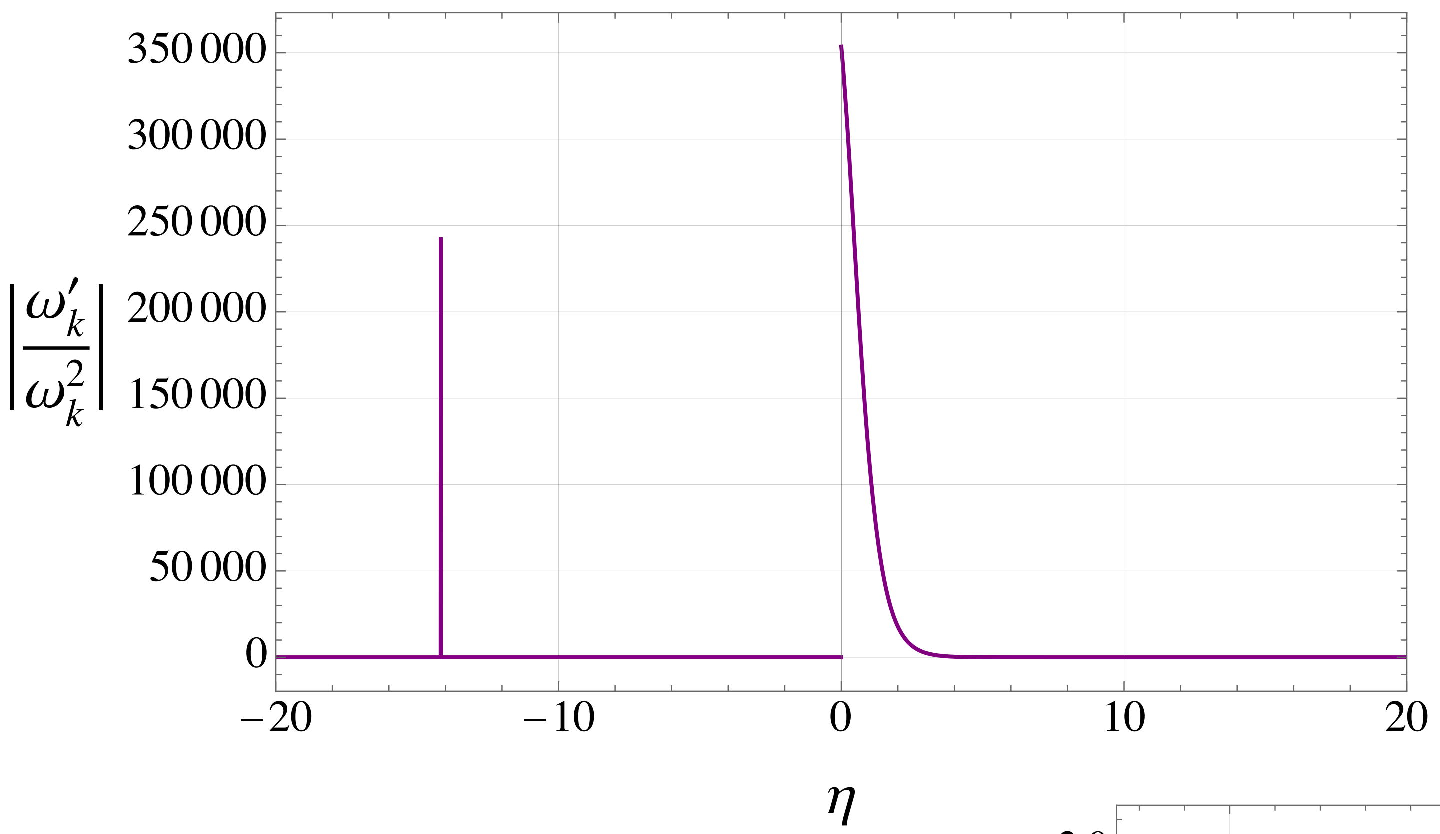
$$\frac{\omega'}{\omega^2} = \frac{a\,a'\left(m^2 - 2H^2\right)}{\left[k^2 + a^2\left(m^2 - 2\,H^2\right)\right]^{3/2}} \longrightarrow 1/\sqrt{2}$$

✓ The adiabaticity condition holds when the field starts oscillating, around $\eta \simeq 8$

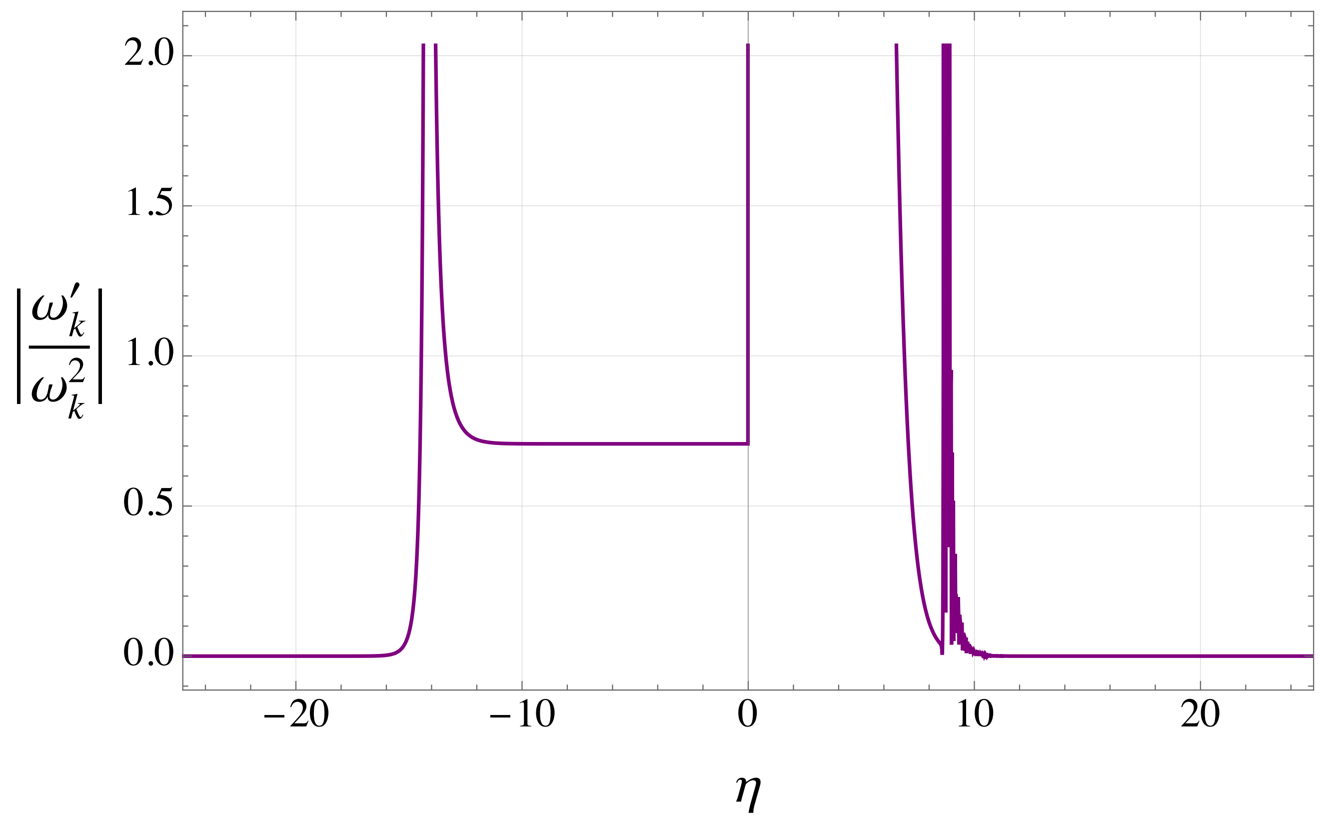
✓ It can be proved that

$$\frac{\omega'}{\omega^2} \to \begin{cases} \dfrac{a^3 H\, m^2}{k^3} & k \gg a\,m \\ \dfrac{H}{m} & k \ll a\,m \end{cases}$$

Adiabaticity Condition



$\phi_0^{in} = 0.999999\,\pi f$



To understand the physical meaning consider the **simple harmonic oscillator**

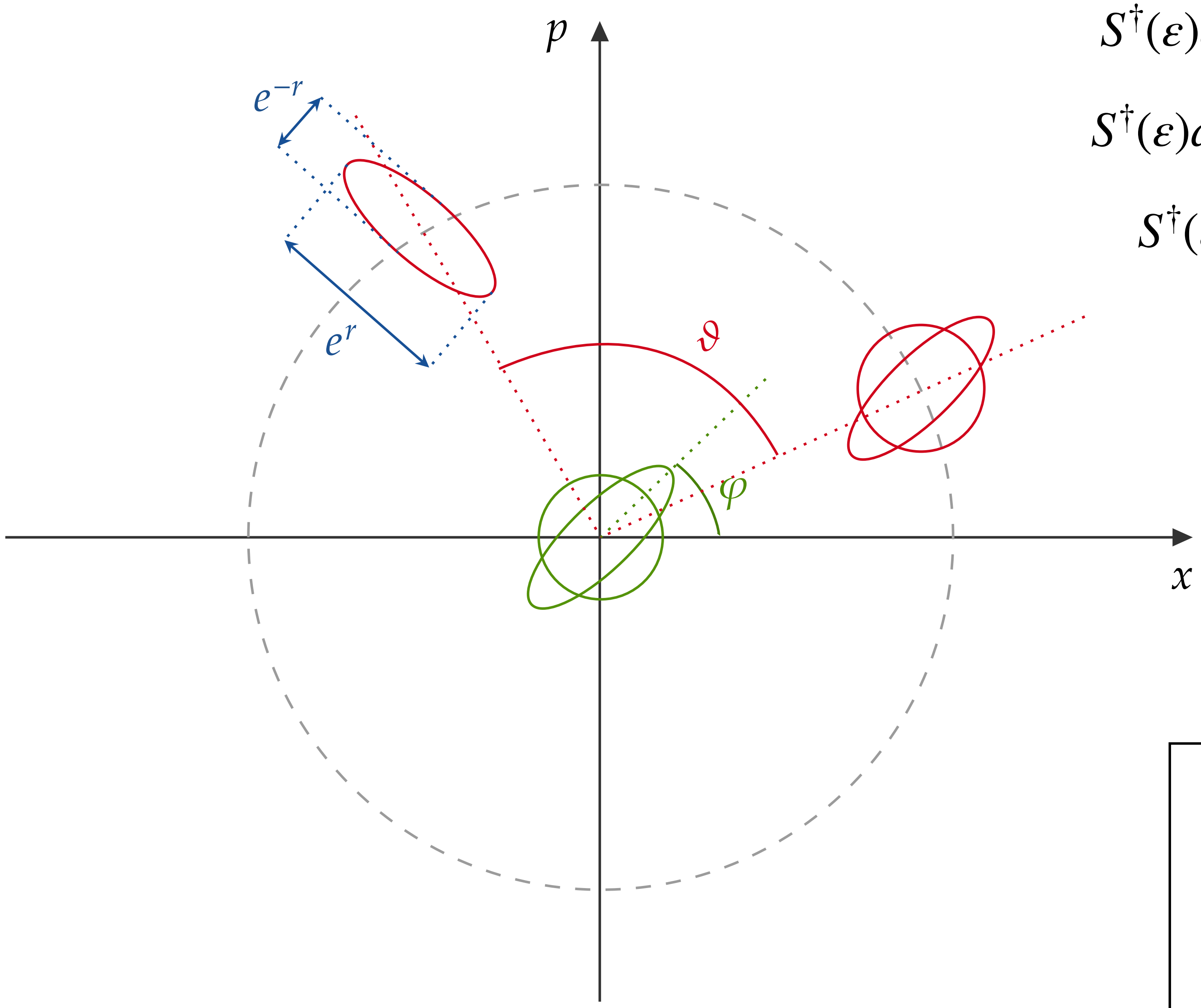
$$q = \sqrt{\frac{\hbar}{2\omega}}\left(a + a^{\dagger}\right) \qquad p = i\sqrt{\frac{\hbar\omega}{2}}\left(a - a^{\dagger}\right)$$

Define the Hermitian field quadrature operators:

$$X_1 = a + a^{\dagger} \qquad X_2 = -i\left(a - a^{\dagger}\right)$$

And the single-mode squeeze operator

$$S(\varepsilon) \equiv \exp\left[\frac{\varepsilon^*}{2}a^2 - \frac{\varepsilon}{2}a^{\dagger 2}\right] \qquad \varepsilon = re^{2i\phi}$$



$$S^{\dagger}(\varepsilon) a S(\varepsilon) = a\cosh(r) - a^{\dagger} e^{-2i\phi}\sinh(r)$$

$$S^{\dagger}(\varepsilon) a^{\dagger} S(\varepsilon) = a^{\dagger}\cosh(r) - a e^{-2i\phi}\sinh(r)$$

$$S^{\dagger}(\varepsilon)\left(Y_1 + iY_2\right) S(\varepsilon) = e^{-r} Y_1 + iY_2 e^{r}$$

$$Y_1 + iY_2 \equiv \left(X_1 + iX_2\right) e^{-i\phi}$$

$$\Delta Y_1 = e^{-r} \qquad \Delta Y_2 = e^{r}$$

The squeeze operator attenuates one component of the (rotated) complex amplitude while amplifying the other one.

Analysis of the Beta Coefficient

The beta coefficient can be tested analytically using the energy density.

1.

$$\rho_\phi = \rho_{\phi_0} + \frac{d\rho}{d\phi}\,\delta\phi + \frac{1}{2}\,\frac{d^2\rho}{d\phi^2}\,\delta\phi^2$$

$$\langle \delta\rho_\phi \rangle = \frac{1}{2}\,\frac{d^2\rho}{d\phi_*^2}\,\langle \delta\phi_*^2 \rangle \qquad \langle \delta\phi_*(\mathbf{x})\,\delta\phi_*(\mathbf{y}) \rangle = \int \frac{d^3k}{(2\pi)^3}\, e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\, \frac{H_*^2}{2\,k^3}$$

2.

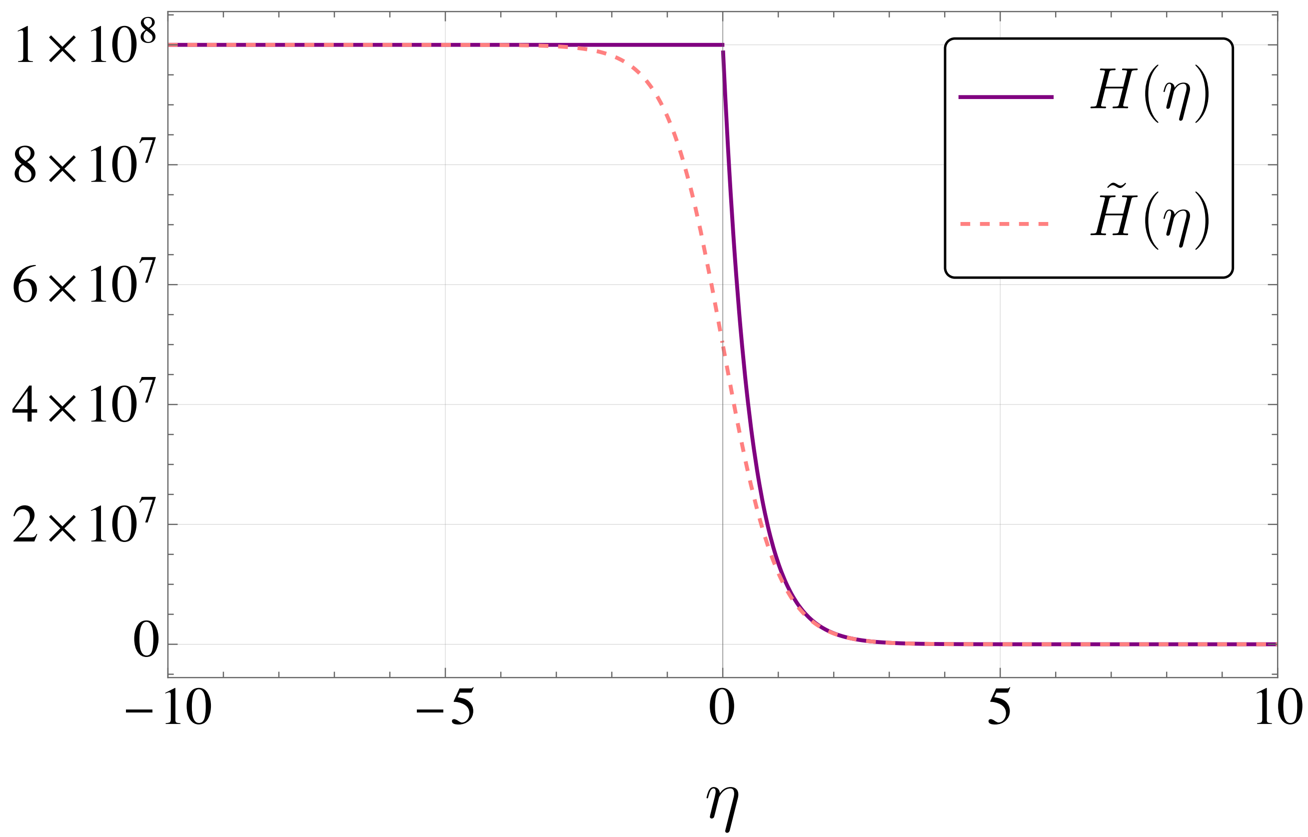
$$\delta\rho_\phi = \frac{\langle \mathscr{H} \rangle}{a^4\,V} = \frac{1}{a^4}\int d^3k\,\omega_k\,|\beta_k|^2$$

$$\longrightarrow \qquad \boxed{|\beta_k|^2 = \frac{1}{2}\,\frac{d^2\rho}{d\phi_*^2}\,\frac{H_*^2}{2\,k^3}\,\frac{a^3}{m}}$$

More Realistic Models: Smoothing the Hubble

We tried smoothing the Hubble in order to prove that the asymptotic behaviour is not affected by possible modifications to the Hubble during a non instantaneous reheating phase.

$$H(\eta) = H_* \frac{e^{-2\eta}}{e^{-2\eta} + 1}$$



More Realistic Models: Smoothing the Hubble

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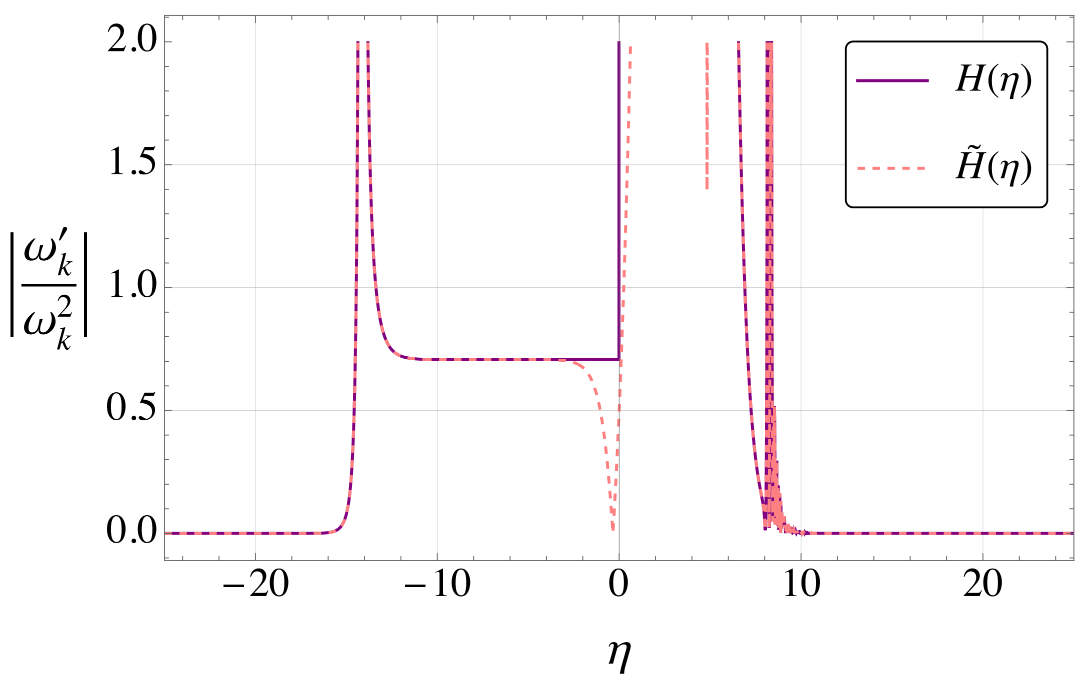
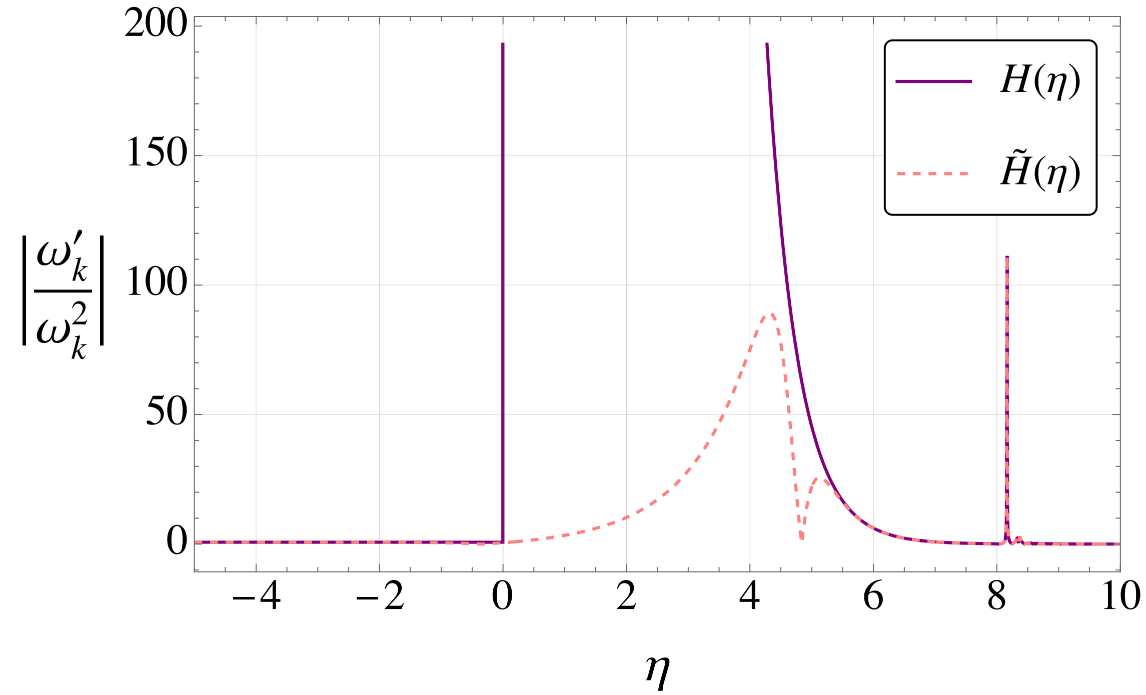
$$H(\eta) = H_* \frac{e^{-2\eta}}{e^{-2\eta} + 1}$$

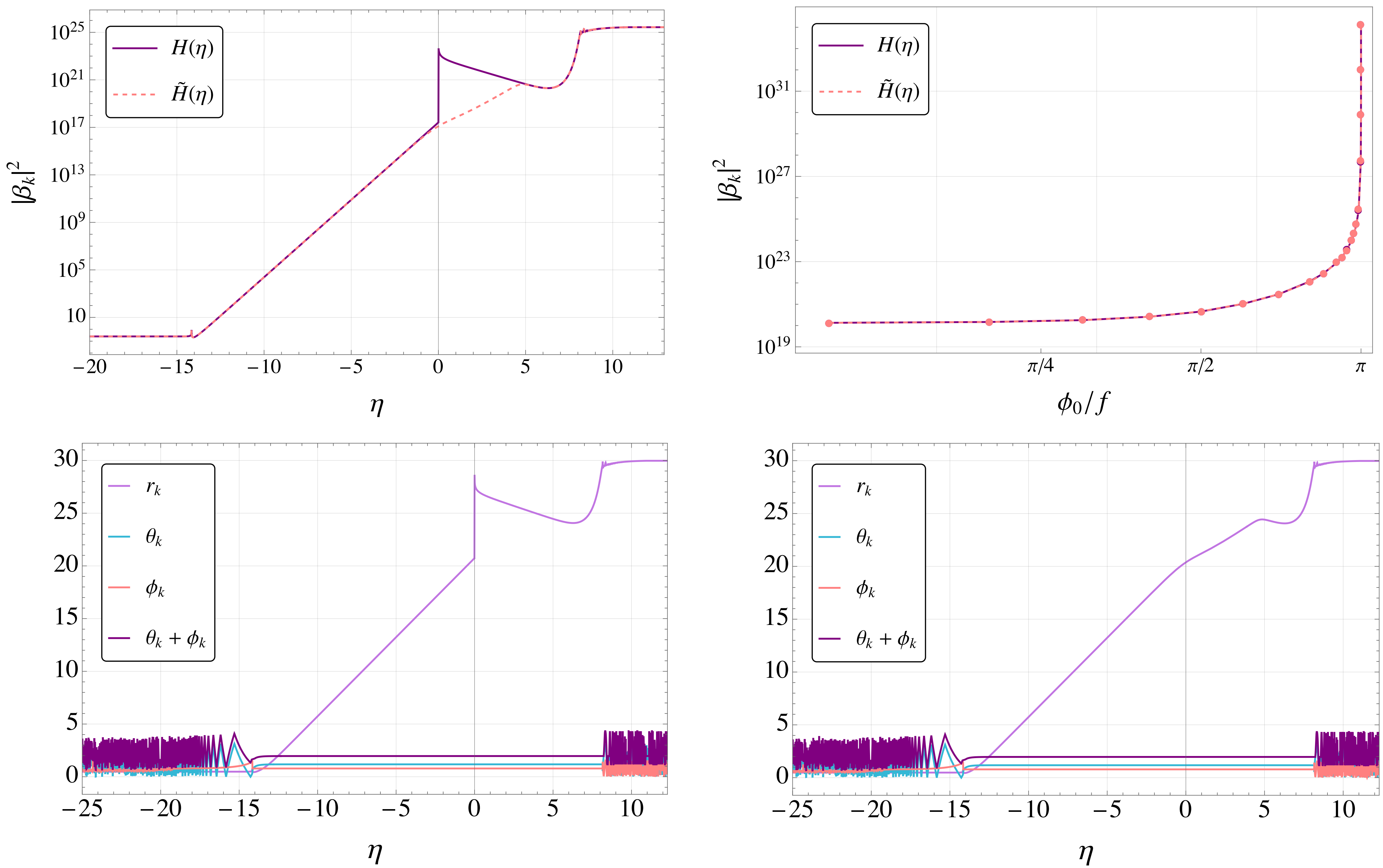
The adiabaticity condition still holds in the found regimes.





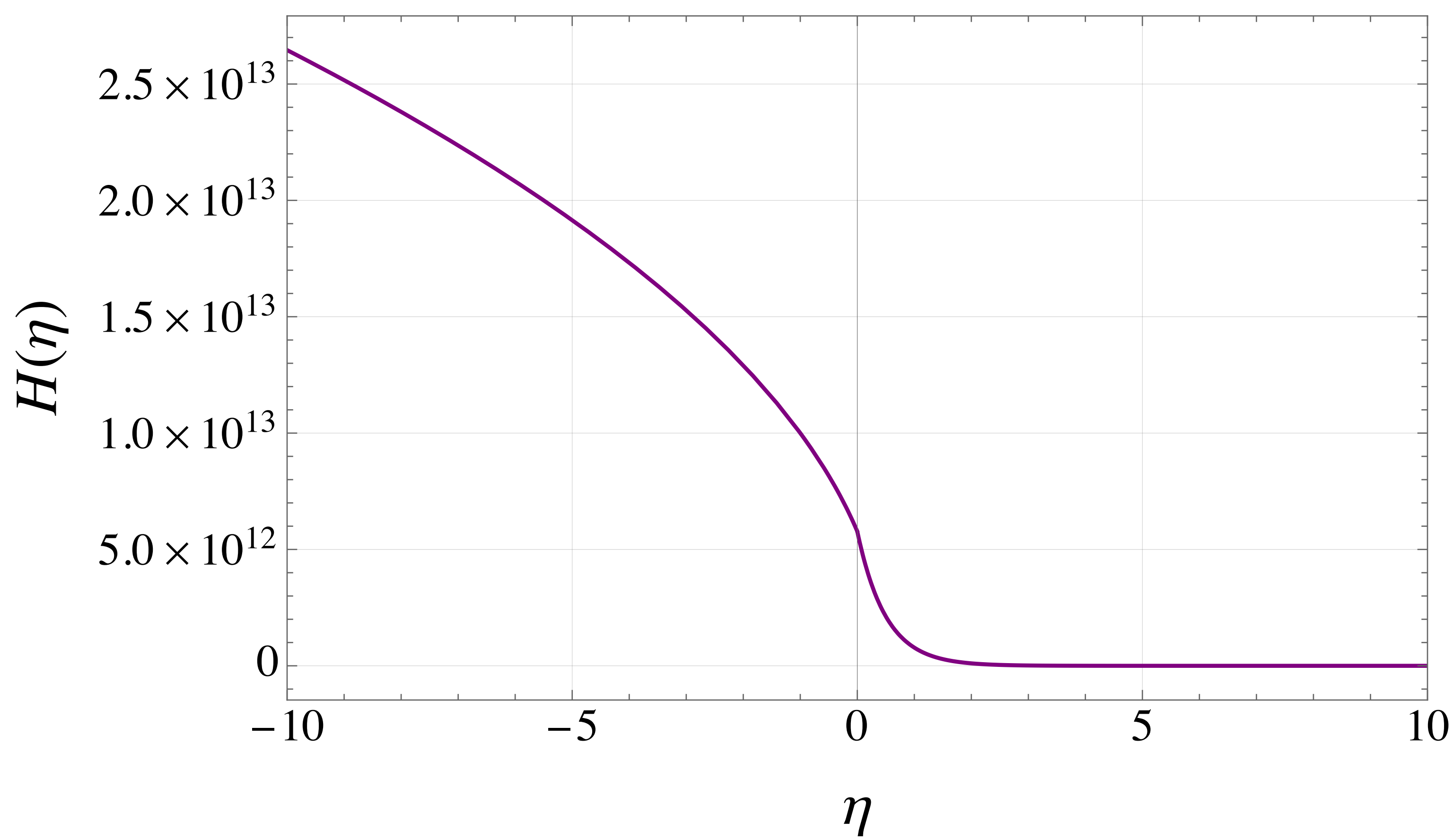
More Realistic Models: Smoothing the Hubble

Our results in the late time limit are not affected:



What happens if we consider the background evolution of the Universe as given by a single-field inflationary model?

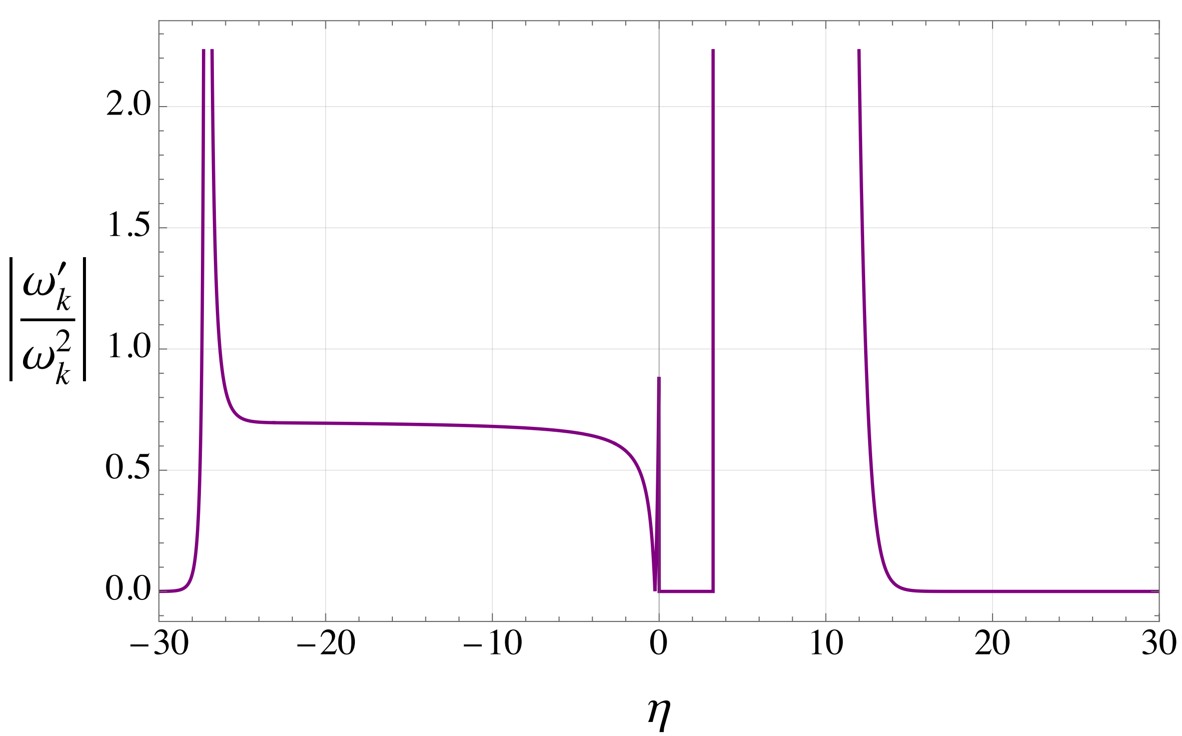
$$H(\eta) = \begin{cases} m_\varphi \sqrt{\frac{1}{3} - \frac{2}{3}\eta} & \textit{inflation} \\ \frac{m_\varphi}{\sqrt{3}} e^{-2\eta} & \textit{radiation} \end{cases}$$



More Realistic Models: Quasi DS Inflation

What happens if we consider the background evolution of the Universe as given by a single-field inflationary model?

$$H(\eta) = \begin{cases} m_\varphi \sqrt{\dfrac{1}{3} - \dfrac{2}{3}\,\eta} & \textit{inflation} \\ \dfrac{m_\varphi}{\sqrt{3}}\, e^{-2\eta} & \textit{radiation} \end{cases}$$



$m_\varphi = 10^{13} GeV$